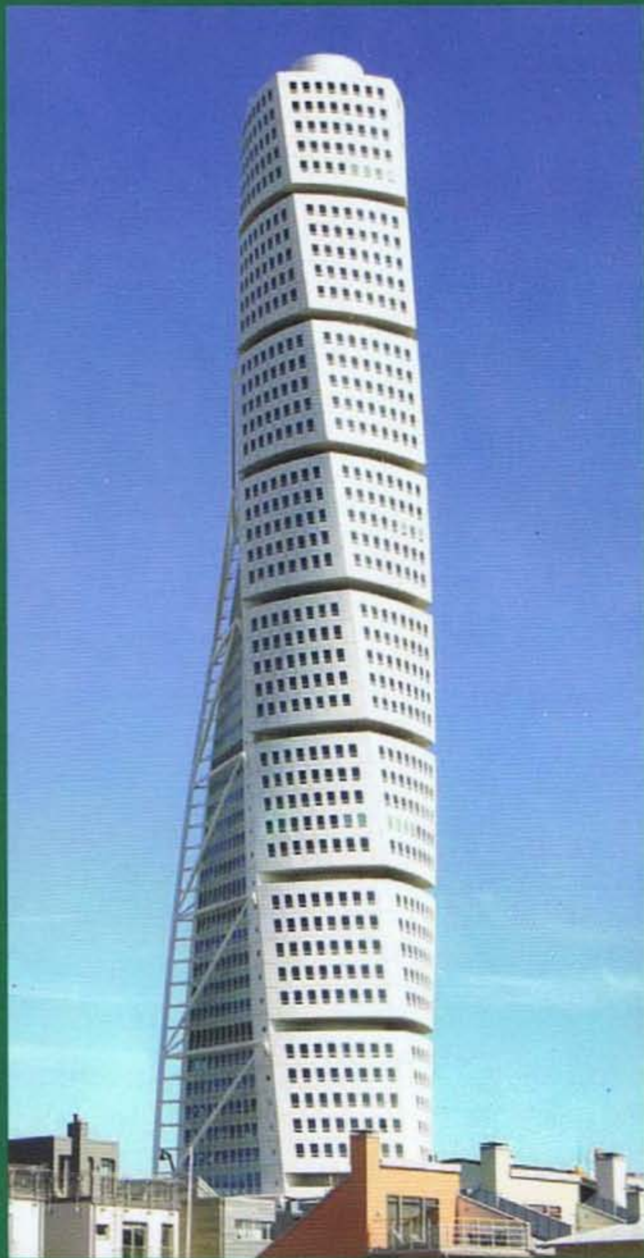
DESIGN OF REINFORCED CONCRETE STRUCTURES



Second Edition **Volume 1**

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DESIGN OF REINFORCED CONCRETE STRUCTURES

Prof. Mashhour Ghoneim   Dr. Mahmoud El-Mihilmy

Features

- Reflects the very latest Egyptian Code provisions (ECP 203 - 2007) and includes all major changes and additions.
- Numerous illustrations and figures for each topic.
- Good theoretical background for each topic with code provisions.
- Extensive examples in each chapter utilizing SI units.
- All examples are worked out step by step ranging from simple to advanced.
- Full reinforcement details for every example.
- Numerous design charts for sections subjected to flexure.

This volume covers the following topics:

- Reinforced Concrete Fundamentals
- Design of Singly Reinforced Sections
- Design of Doubly Reinforced Sections
- Design of T-Beams
- Bond and Development Length
- Design for Shear
- Design of Simple and Continuous Beams
- Design for Torsion
- Design for Combined Shear and Torsion
- Truss Models for R/C Beams

Second Edition   **Volume 1**

DESIGN OF REINFORCED CONCRETE STRUCTURES

Volume 1

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Second Edition

2008

PREFACE

Teaching reinforced concrete design, carrying out research relevant to the behavior of reinforced concrete members, as well as designing concrete structures motivated the preparation of this book. The basic objective of this book is to furnish the reader with the basic understanding of the mechanics and design of reinforced concrete. The contents of the book conform to the latest edition of the Egyptian Code for the Design and Construction of Concrete Structures ECP-203. The authors strongly recommend that the Code be utilized as a companion publication to this book.

The book is aimed at two different groups. First, by treating the material in a logical and unified form, it is hoped that it can serve as a useful text for undergraduate and graduate student courses on reinforced concrete. Secondly, as a result of the continuing activity in the design and construction of reinforced concrete structures, it will be of value to practicing structural engineers.

Numerous illustrative examples are given, the solution of which has been supplied so as to supplement the theoretical background and to familiarize the reader with the steps involved in actual design problem solving.

In writing the book, the authors are conscious of a debt to many sources, to friends, colleagues, and co-workers in the field. Finally, this is as good a place as any for the authors to express their indebtedness to their honorable professors of Egypt, Canada and the U.S.A. Their contributions in introducing the authors to the field will always be remembered with the deepest gratitude.

This volume covers the following topics

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- **Truss Models for the Behavior of R/C Beams**
- **Design for Torsion**

It also includes appendices containing design aids.

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REINFORCED CONCRETE FUNDAMENTALS

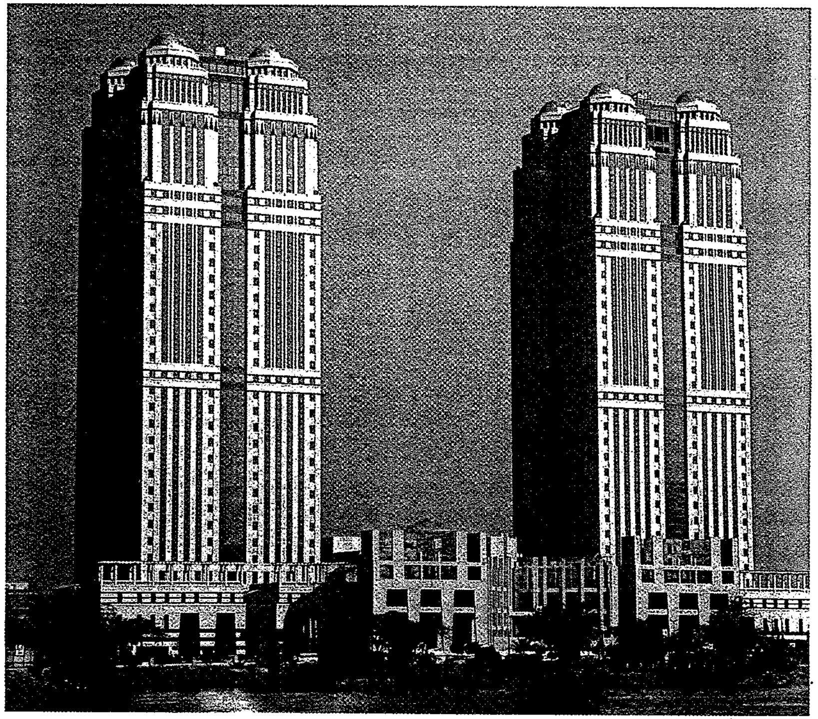


Photo 1.1 Nile City Towers, Cairo-Egypt.

1.1 Introduction

Reinforced concrete is one of the most important available materials for construction in Egypt and all over the world. It is used in almost all structures including; buildings, bridges, retaining walls, tunnels, tanks, shells and even ships.

Concrete is a mixture of sand and gravel held together with a paste of cement and water. Sometimes one or more admixture is added to change certain characteristic of the concrete such as its workability, durability, and time of hardening. Concrete has a high compressive strength and a very low tensile strength.

Reinforced concrete is a combination of concrete and steel wherein the steel reinforcement provides the tensile strength lacking in the concrete. Steel reinforcement is also capable of resisting compression forces and is used in columns as well as in other situations to be described later.

The tremendous success of reinforced concrete can be understood if its numerous advantages are considered. These include the following:

- It is a low maintenance material.
- It has great resistance to the action of fire provided that there is adequate cover over the reinforcing steel.
- A special nature of concrete is its ability to be cast in to a variety of shapes from simple slabs, beams, and columns to great arches and shells.
- A lower grade of skilled labor is required for erection as compared to other materials such as structural steel.
- In most areas, concrete takes advantage of inexpensive local materials (sand, gravel, and water) and requires a relatively small amount of cement and reinforcing steel.

To use concrete successfully, the designer must be completely familiar with its weak points and its strong ones. Among its disadvantages are the following:

- Concrete has a very low tensile strength, requiring the use of tensile reinforcing.
- Forms are required to hold the concrete in place until it hardens sufficiently. Formwork could be expensive.
- The properties of concrete could vary widely due to variations in its proportioning and mixing. Furthermore, the placing and curing of concrete is not as carefully controlled, as is the production of other materials such as structural steel.
- In general, reinforced concrete members are relatively large, as compared to structural members, an important consideration for tall buildings and long span bridges.

1.2 Reinforced Concrete Members

Reinforced concrete structures consist of a series of members. The first and the second floors of the building shown in Fig. 1.1 have a slab-and-beam system, in which the slab spans between beams, which in turn apply loads to the columns. Again, the columns' loads are applied to footings, which distribute the load over a sufficient area of soil.

The structure shown in Fig 1.2 is a typical framed structure. The slab carries its own weight, flooring and live loads. The load is then transferred to secondary beams. The reactions of the secondary beams are transferred to the girders, which in turn are supported by the columns. Finally, the columns' loads are applied to the footings, which distribute the load to the soil.

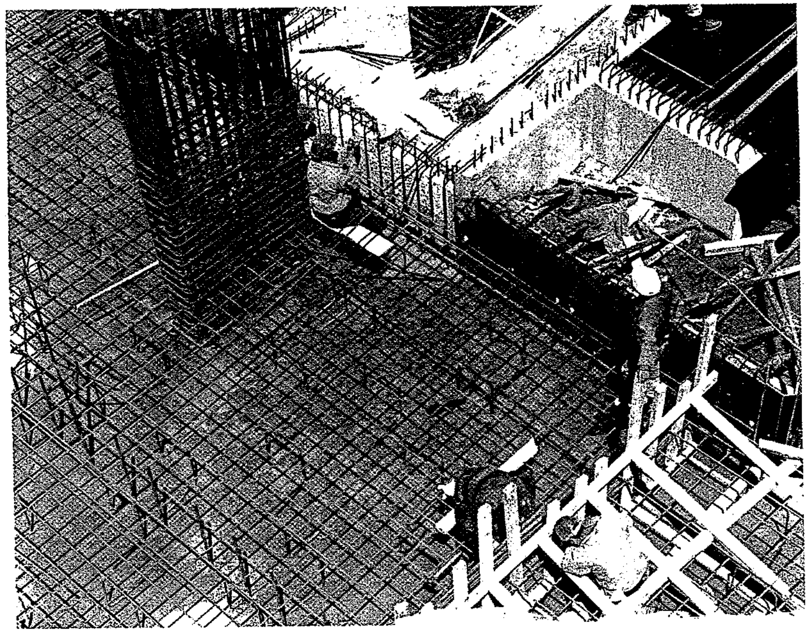


Photo 1.2 Reinforcement placement during construction

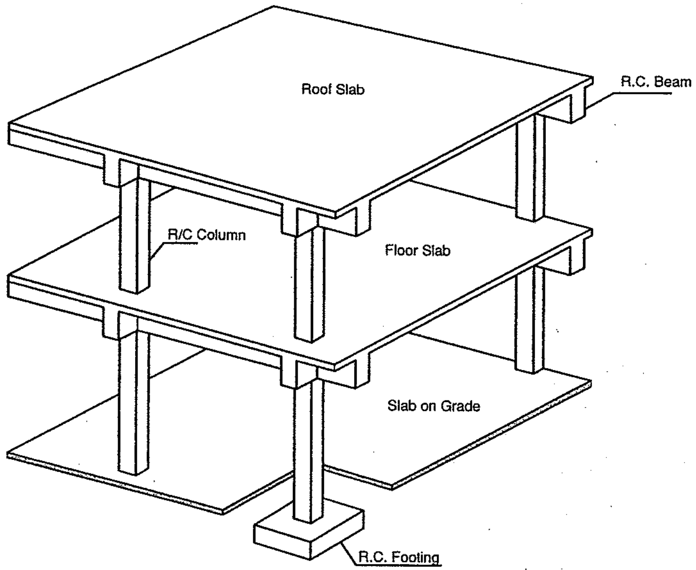


Fig. 1.1 Slab and beam system in a building

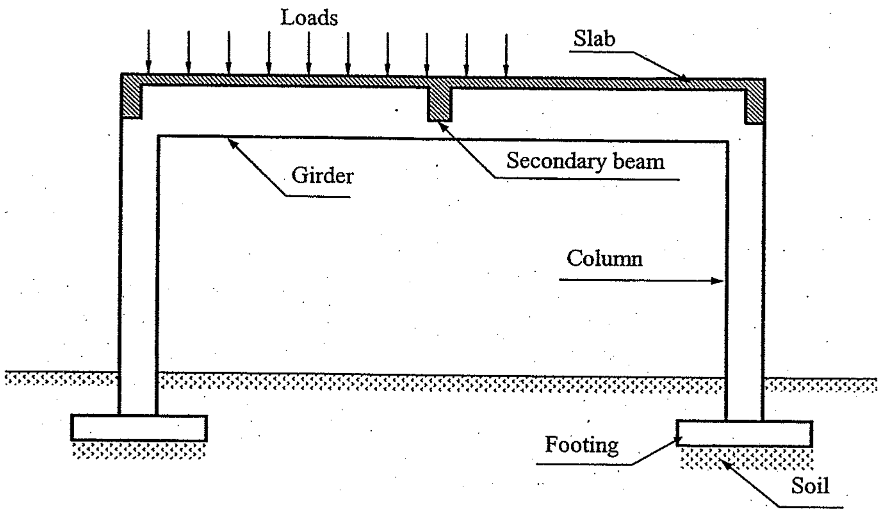


Fig. 1.2 Typical reinforced concrete structural framing system

1.3 Reinforced Concrete

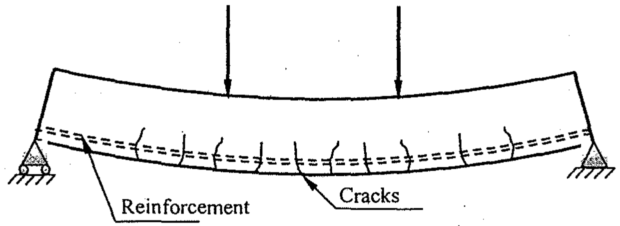
It is a well-known fact that plain concrete is strong in compression and very weak in tension. The tensile strength of concrete is about one-tenth its compressive strength. As a result, a plain concrete beam fails suddenly as soon as the tension cracks start to develop. Therefore, reinforcing steel is added in the tension zone to carry all the developed tensile stresses; this is called a *reinforced concrete beam.*

Concrete and steel work together beautifully in reinforced concrete structures. The advantages of each material seem to compensate for the disadvantages of the other. The great shortcoming of low concrete tensile strength is compensated for by the high tensile strength of the steel. The tensile strength of the steel is approximately equal to 100-140 times the tensile strength of the usual concrete mix. Also, the two materials bond together very well with no slippage, and thus act together as one unit in resisting the applied loads.

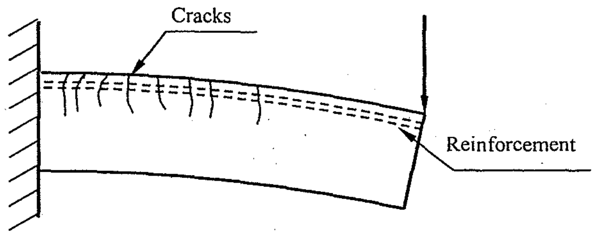
The disadvantage of steel is corrosion, but the concrete surrounding the reinforcement provides an excellent protection. Moreover, the strength of the exposed steel subjected to fire is close to zero, but again the enclosure of the reinforcement in the concrete produces very satisfactory fire protection. Finally, concrete and steel work very well together in temperature changes because their coefficients of thermal expansion are almost the same. The coefficient of thermal expansion for steel is $6.5\text{x}10^{-6}$, while that for the concrete is about $5.5\text{x}10^{-6}$.

1.4 Reinforced Concrete Behavior

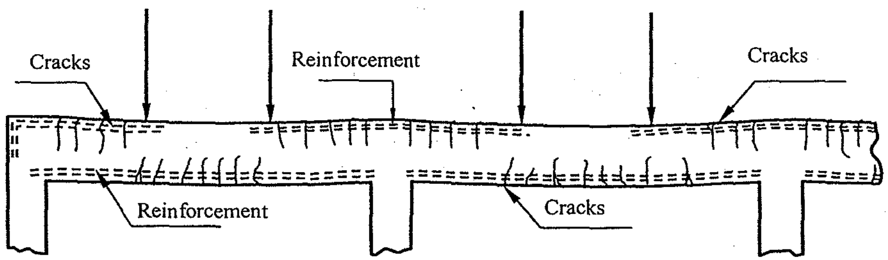
The addition of steel reinforcement that bonds strongly to concrete produces a relatively ductile material capable of transmitting tension and suitable for any structural elements, e.g., slabs, beam, columns. Reinforcement should be placed in the locations of anticipated tensile stresses and cracking areas as shown in Fig 1.3. For example, the main reinforcement in a simple beam is placed at the bottom fibers where the tensile stresses develop (Fig. 1.3A). However, for a cantilever, the main reinforcement is at the top of the beam at the location of the maximum negative moment (Fig.1.3B). Finally for a continuous beam, a part of the main reinforcement should be placed near the bottom fibers where the positive moments exist and the other part is placed at the top fibers where the negative moments exist (Fig. 1.3C).



A- Simple beam



B-Cantilever beam



C-Continuous beam

Fig. 1.3 Reinforcement placement for different types of beams

1.5. Mechanical Properties of Concrete

1.5.1 Compressive Strength

Many factors affect the concrete compressive strength such as the water cement ratio, the type of cement, aggregate properties, age of concrete, and time of curing. The most important factor of all is the water cement ratio. The lower water content with good workability leads to higher concrete compressive strength. Increasing the water cement ratio from 0.45 to 0.65 can decrease the compressive strength by 30-40 percent. Currently, high-range water-reducing admixtures (*super plasticizers*) are available and they allow engineers to produce fluid concrete mixes with a sharply reduced amount of water.

In Egypt, the compressive strength of concrete is usually determined by loading a 158 mm cube up to failure in uniaxial compression after 28 days of casting and is referred to as f_{cu}. Additional details covering the preparation and testing of cubes are covered by the Egyptian Code for Design and Construction of Concrete Structures (ECP-203) including correction factors that can be used if the tested specimen is not the same dimension or shape as the standard cube. This is the strength specified on the construction drawings and used in the design calculations.

It should be mentioned that in other countries such as the United States and Canada, the compressive strength is measured by compression tests on 150 mm x 300 mm cylinders tested after 28 days of moist curing. In the case of using specimens other than the standard cube, the ECP 203 gives the correction factors shown in Table 1.1 to obtain the equivalent compressive strength of the standard cube.

Table 1.1 Correction factors to obtain the equivalent $f_{cu}=f_c$ x *factor*

Shape	Size (mm)	Correction factor
Cube	100 x100 x 100	0.97
Cube	(158 x 158 x 158) or (150 x 150 x 150)	1.00
Cube	200 x 200 x 200	1.05
Cube	300 x 300 x 300	1.12
Cylinder	100 x 200	1.20
Cylinder	150 x 300	1.25
Cylinder	250 x 500	1.30
Prism	(150 x 150 x 300) or (158 x 158 x 316)	1.25
Prism	(150 x 150 x 450) or (158 x 158 x 474)	1.3
Prism	150 x 150 x 600	1.32

The ECP 203 states in clause (2.5.2) that a concrete strength of 18 N/mm^2 should be used to qualify for reinforced concrete category,15 N/mm^2 for plain concrete, and 30 N/mm^2 for prestressed concrete. Table 1.2 illustrates the grades of reinforced concrete R/C and prestressed concrete P/S as permitted by the code.

Table 1.2 Grades of reinforced and prestressed concrete (N/mm^2)

R/C	18	20	25	30	35	40	45			
P/S				30	35	40	45	50	55	60

Field conditions are not the same as those in the laboratory, and the specified 28-days strength might not practically be achieved in the field unless almost perfect mixture, vibration, and perfect curing conditions are present. As a result, section 2-5-3 of the ECP 203 requires that the target concrete compressive strength, f_m must exceed the *characteristic strength* f_{cu} by a safety margin *(M)*. The safety margin for a concrete mix design depends on the quality control of the concrete plant and can range from 4 N/mm^2 to 15 N/mm^2. Table 1.3 (2-15 of the Code) lists the values of the safety margin M according to the number of the performed tests and the characteristic strength f_{cu}. Therefore the targeted concrete compressive strength f_m is given by

$$f_m = f_{cu} + M \quad \text{..(1.1)}$$

Table 1.3 Value of the safety margin M (N/mm^2)

Statistical data	Safety margin M		
	f_{cu} < 20 N/mm^2	20-40 N/mm^2	40-60 N/mm^2
40 test data or more	1.64 SD ≥ 4 N/mm^2	1.64 SD ≥ 6 N/mm^2	1.64 SD ≥ 7.5 N/mm^2
less than 40 test data	Not less than 0.6 f_{cu}	≥ 12 N/mm^2	≥ 15 N/mm^2

One test data is an average of 3 cube tests
SD: Standard deviation

Since concrete is used mostly in compression, its compressive stress-strain curve is of a prime interest. Figure1.4 shows a typical set of such curves obtained from uniaxial compression test of cylinders. All curves have somewhat similar characteristics. They consist of an initial relatively straight elastic portion in which stresses and strains are closely proportional, then begin to curve to reach a maximum value at a strain of 0.002 to 0.003. There is a descending branch after the peak stress is reached. It can be noticed that the weaker grades of concrete are less brittle than the stronger ones. Thus, they will take larger stains and deformations before breaking.

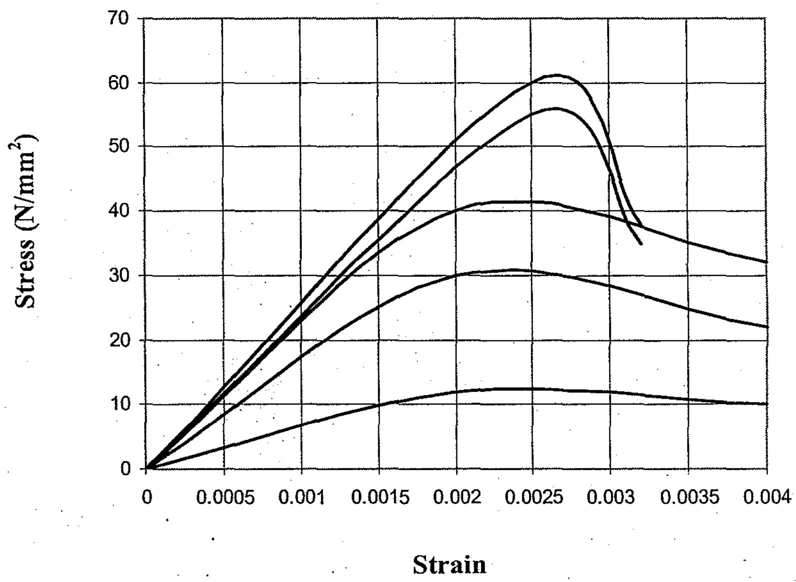


Fig. 1.4 Typical concrete stress-strain curves

For computational purposes, mathematical representations of the stress-strain curves of concrete in compression are available. For example, the stress-strain curve shown in Fig.1.5 may be used. The curve consists of a parabola followed by a sloping line. Such a curve has been used widely in research purposes.

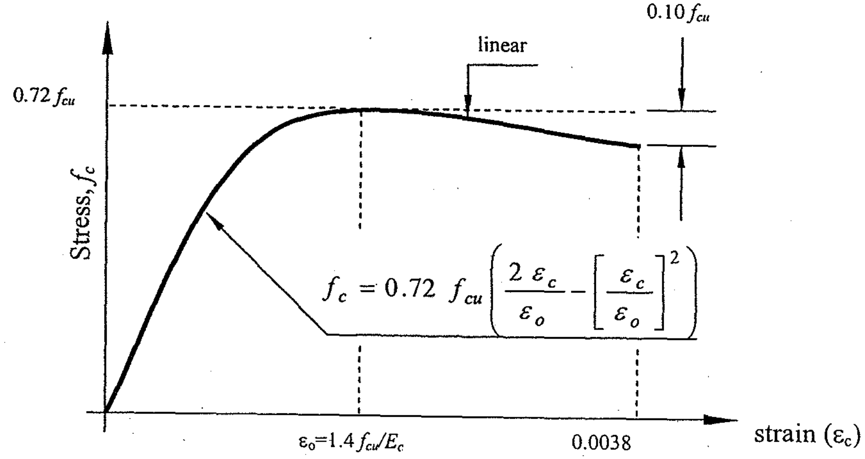


Fig. 1.5 Modified Hognestad curve for concrete stress-strain relation

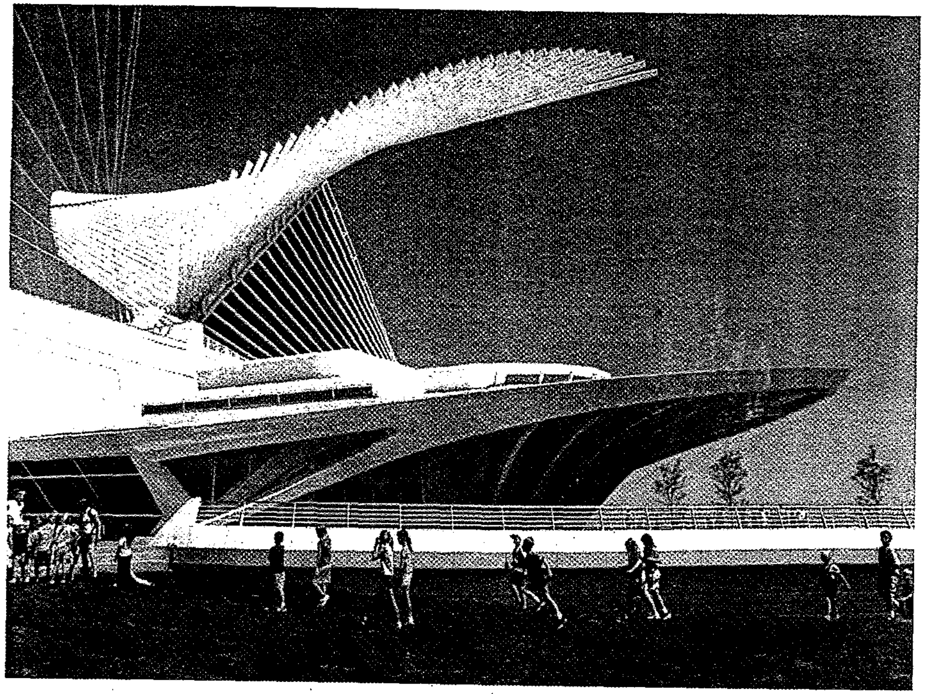


Photo 1.3 Milwaukee Art Museum, USA.

1.5.2 Tensile strength

Experimental tests indicate that the tensile strength of concrete is highly variable and ranges from about 8-12% of its compressive strength. The actual value depends on the type of test and crack propagation pattern at failure.

Tensile strength is usually determined by the bending test (Fig. 1.6) or by the split cylinder test (Fig 1.7). The ECP 203 states that the value of concrete tensile strength can be taken from experimental tests as follows:

60% from the concrete tensile strength determined from bending test.

85% from the concrete tensile strength determined from split cylinder test.

In the bending test (*modulus of rapture test*), a plain concrete beam is loaded in flexure up to failure as shown in Fig. 1.6. The flexure tensile strength or the modulus of rupture f_r is computed from the following equation

$$f_r = \frac{6\ M}{b \times t^2} \qquad (1.2)$$

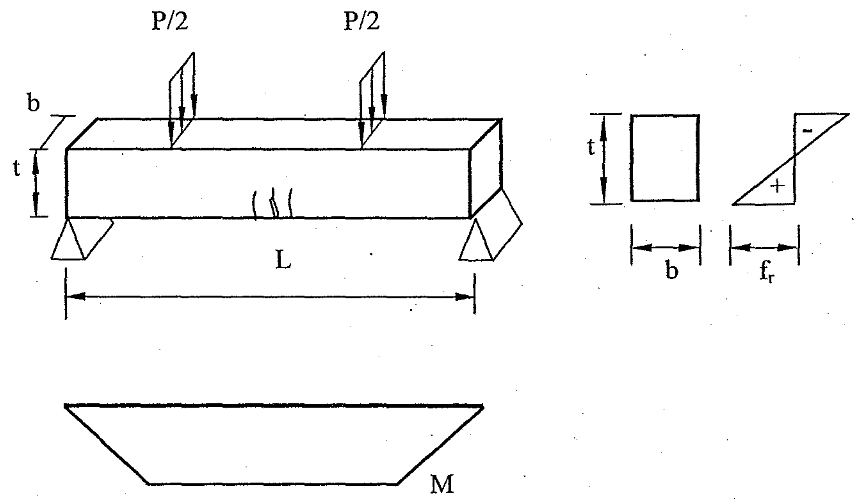


Fig. 1.6 Bending tensile test

The split cylinder test is performed on a 150x300 mm cylinder placed on its side and loaded in compression along its length as shown in Fig. 1.7.A The stresses along the diameter are nearly uniform tension perpendicular to the plan of loading as shown in Fig. 1.7.b The splitting tensile strength f_{ct} is calculated from the following expression

$$f_{ct} = \frac{2\,P}{\pi\ d\ L} \quad \text{.......................................} (1.3)$$

The parameters in Eq. 1.3 are defined in Fig. 1.7.

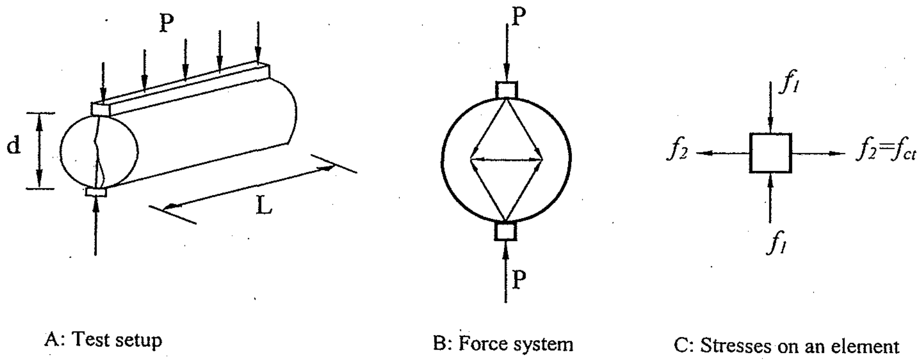


Fig 1.7 Split cylinder test

The tensile strength computed using the modulus of rupture is always higher than the split cylinder tension tests. The tensile strength of the concrete can be determined using its compressive strength. The tensile strength does not correlate well with the concrete compressive strength but rather with its square root. The ECP-203 gives an expression for estimating the concrete tensile strength f_{ctr} as a function of its compressive strength as follows:

$$f_{ctr} = 0.6\sqrt{f_{cu}} \quad \text{.......................................} (1.4)$$

1.5.3 Modulus of Elasticity

It is clear from the stress-strain curve of the concrete shown in Fig.1.3 that the relation between the stress and the strain is not linear. Thus, the modulus of elasticity changes from point to point. Furthermore, its value varies with different concrete strengths, concrete age, type of loading, and the characteristics of cement and aggregate. The initial tangent is sometimes used to estimate the concrete modulus of elasticity, in which the slope of the stress-strain curve of concrete at the origin is evaluated as shown in Fig. 1.8. The ECP-203 gives the following formula for estimating the concrete modulus of elasticity

$$E_c = 4400\sqrt{f_{cu}} \quad \text{.......................................} (1.5)$$

where f_{cu} is the concrete compressive strength in N/mm^2

The magnitude of the modulus of elasticity is required when calculating deflection, evaluating bracing condition, and cracking of a structure.

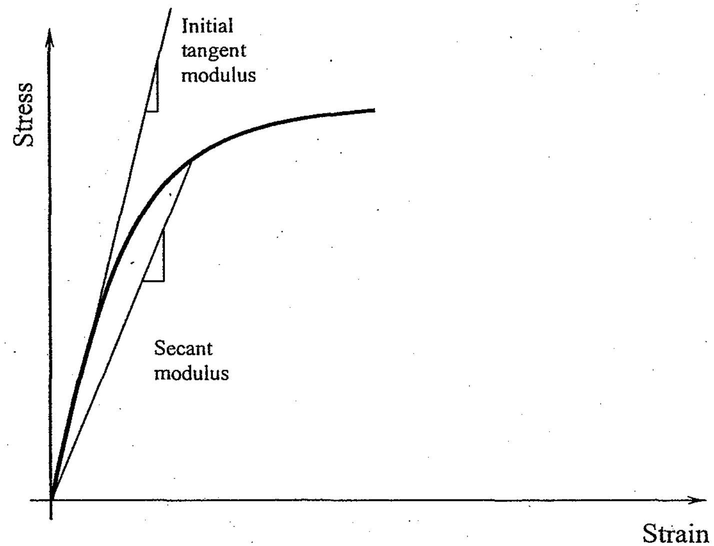


Fig. 1.8 Initial tangent modulus of concrete

1.5.4 Strength of Concrete Under Biaxial Loading

Portions of many concrete members may be subjected to stresses in two perpendicular directions (*biaxial state*). The strength of the concrete is affected greatly by the applied stress in the perpendicular direction as shown in Fig. 1.9.

In Fig. 1.9, all the stresses are normalized in terms of the uniaxial compressive strength f_{cu}. The curve has three regions; biaxial compression-compression, biaxial tension-tension, biaxial tension-compression.

In the compression-compression zone, it can be seen that the compressive strength of the concrete can be increased by 20-25% when applying compressive stress in the perpendicular direction.

In the tension-tension zone, it is clear that the tensile strength of the concrete is not affected by the presence of tension stresses in the normal direction. For example, a lateral tension of about half the value of the uniaxial tensile strength will reduce the compressive strength to 50% of the uniaxial compressive strength.

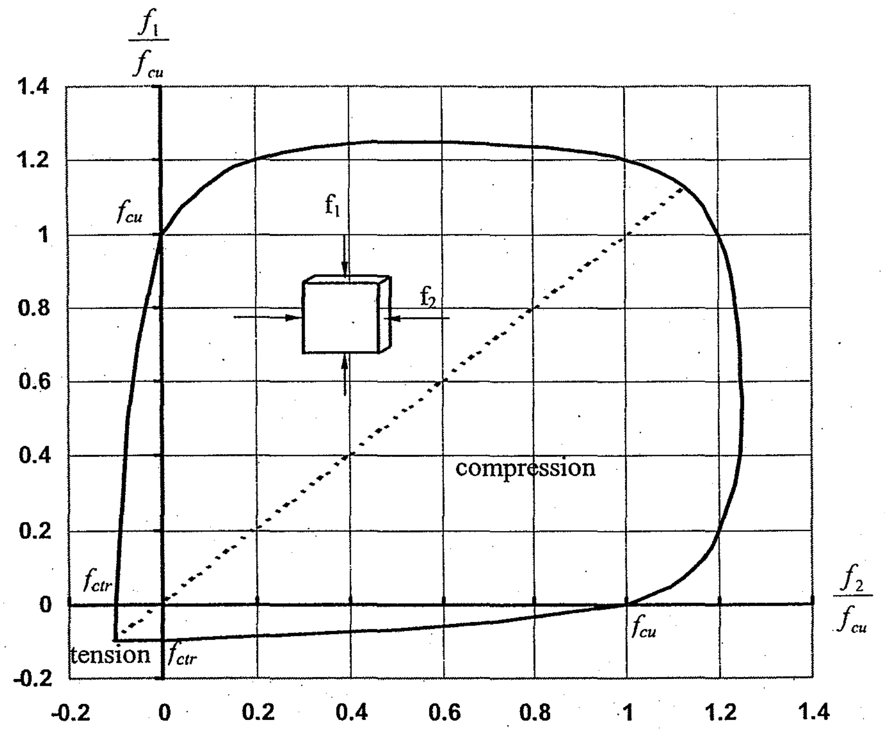


Fig. 1.9 Strength of concrete in biaxial stress

The biaxial state may occur in beams as shown in Fig. 1.10 where the principle tensile and compressive stresses lead to biaxial tension compression state of stress. The split cylinder test illustrated in Fig. 1.7C is a typical example of biaxial state of stress, where the compressive stresses develop in the vertical direction and tensile stresses develop in the horizontal direction. This is the main reason that splitting tensile strength is less than flexural tensile strength.

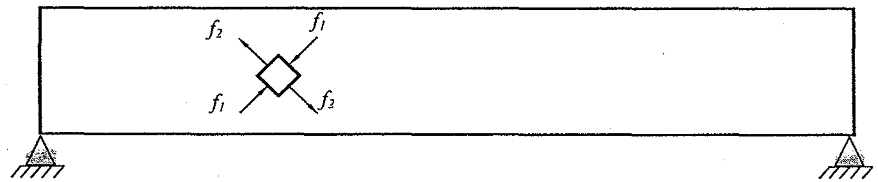


Fig. 1.10 Biaxial state of stress in beams

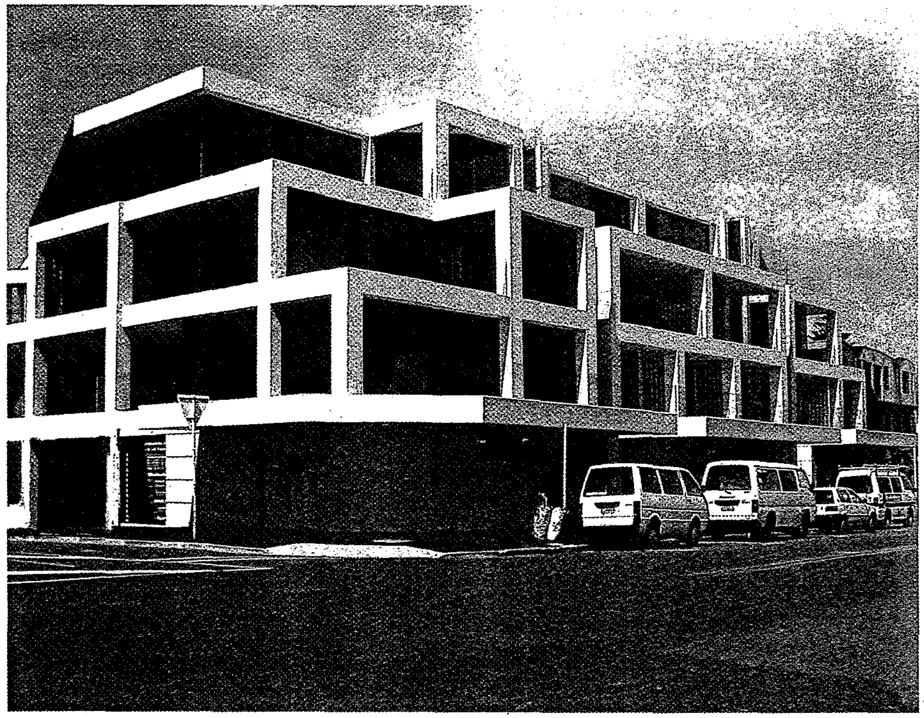


Photo 1.4 Typical reinforced concrete structure

1.5.5 Shrinkage

As the concrete dries it shrinks in volume due to the excess water used in concrete mixing. The shortening of the concrete per unit length due to moisture loss is called shrinkage strain. The magnitude of the shrinkage strain is a function of the initial water content, the composition of the concrete and the relative humidity of the surroundings. Shrinkage is also a function of member's size and shape. Drying shrinkage occurs as the moisture diffuses out of the concrete. As a result, the exterior shrinks more rapidly than the interior. This leads to tensile stresses in the outer skin of the concrete member and compressive stresses in its interior. The rate of the shrinkage increases as the exposed area to the volume increases.

The ECP-203 gives the following formula to estimate the virtual member thickness

$$B = \frac{2A_c}{P_c} \qquad (1.6)$$

where B is the virtual member thickness, A_c area of the cross section, P_c is the section perimeter subjected to shrinkage.

Although shrinkage continues for many years as shown in Fig. 1.11, approximately 90% of the ultimate shrinkage occurs during the first year.

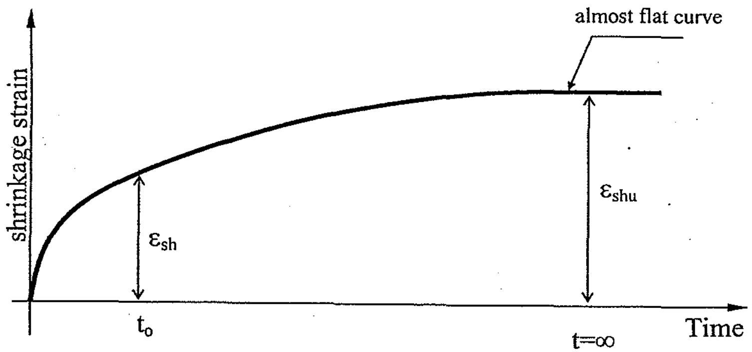


Fig. 1.11 Variation of shrinkage with time for a typical concrete mix

Values of final shrinkage for ordinary concrete are generally of the order of 0.00016 to 0.00030 and can be taken from table 1.4.

Table 1.4 Values of shrinkage strain for concrete (x 10^{-3})

weather condition	Dry weather Relative humidity ≈55%			Humid weather Relative humidity ≈ 75%		
Time by days	Virtual thickness B			Virtual thickness B		
	$B \geq 600$	$600 < B > 200$	$B \leq 200$	$B \geq 600$	$600 < B > 200$	$B \leq 200$
3-7	0.31	0.38	0.43	0.21	0.23	0.26
7-60	0.30	0.31	0.32	0.21	0.22	0.23
>60	0.28	0.25	0.19	0.20	0.19	0.16

1.5.6 Creep

When a reinforced concrete member is loaded, an initial deformation occurs as shown in Fig. 1.12. Experimental studies show that this initial deformation increases with time under constant loading.

The total deformation is usually divided into two parts:(1)initial deformation (2) a time dependent deformation named *creep.*

After the occurrence of the immediate deformation (point A_o to point A), the creep deformation starts rapidly (point A to pint B) and then continues at a much lower rate till almost it becomes a flat curve at infinity. More than 75% of the creep deformation occurs during the first year and 95% in the first five years. If the load is removed at point B, immediate recovery occurs *(point C)*, followed by a time dependent recovery till point D *(creep recovery)*. The member will never recover all the developed deformation and there will be a non-recoverable deformation called permanent deformation.

The creep deformations are within a range of one to three times the instantaneous elastic deformations. Creep causes an increase in the deflection with time that may lead to undesirable deformation of the member. Thus, the deflection must be investigated to ensure that the deformations are within the allowable limits of the code.

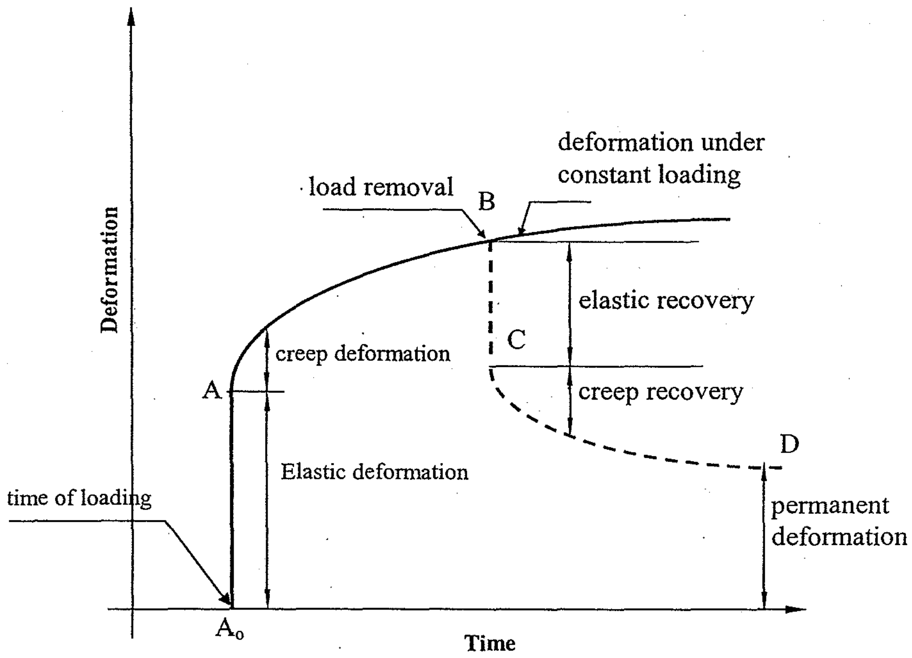


Fig. 1.12 Elastic and creep deformation of concrete

1.6 Reinforcing Steel

The most common types of reinforcing steel are bars and welded wire fabrics. Deformed bars are the most widely used type and manufactured in diameters from 10 mm to 40 mm. They are produced according to the Egyptian standards 262/1999. Bars are supplied in lengths up to 12m, however, longer bars may be specially ordered. Reinforcing bars are available in four grades with a yield strength of 240, 280, 360, and 400N/mm2. The cost of steel having a yield stress of 400 N/mm2 is slightly higher than that of steel with a yield point of 240 N/mm2. However, the gain in strength and accordingly the reduction in the required steel area is obvious. It should be mentioned that grade 400 N/mm2 is the highest steel grade allowed by the Code for reinforced concrete structures.
The ultimate tensile strength, the yield strength and the modulus of elasticity are determined from the stress-strain curve of a specimen bar loaded in uniaxial tension up to failure. The modulus of elasticity of *steel (the slope of the stress-strain curve in the elastic region)* is 200 GPa (200,000 N/mm^2). The specified strength used in design is based on the yield stress for mild steel, whereas for

high yield steel the strength is based on a specified proof stress of 0.2% as shown in Fig. 1.13.
The major disadvantage of using steel in beams and columns is corrosion. The volume of the corroded steel bar is much greater than that of the original one. The results are large outward pressure, which causes severe cracking and spalling of the concrete cover. The ECP-203 requires the increase of concrete cover in corrosive environments. Epoxy coated bars are a perfect solution for the problem of corrosion of the reinforcement. They are expensive and need to be handled very carefully to protect the coating layer from damage. However, they are not as efficient as uncoated bars in developing full bond with surrounding concrete.

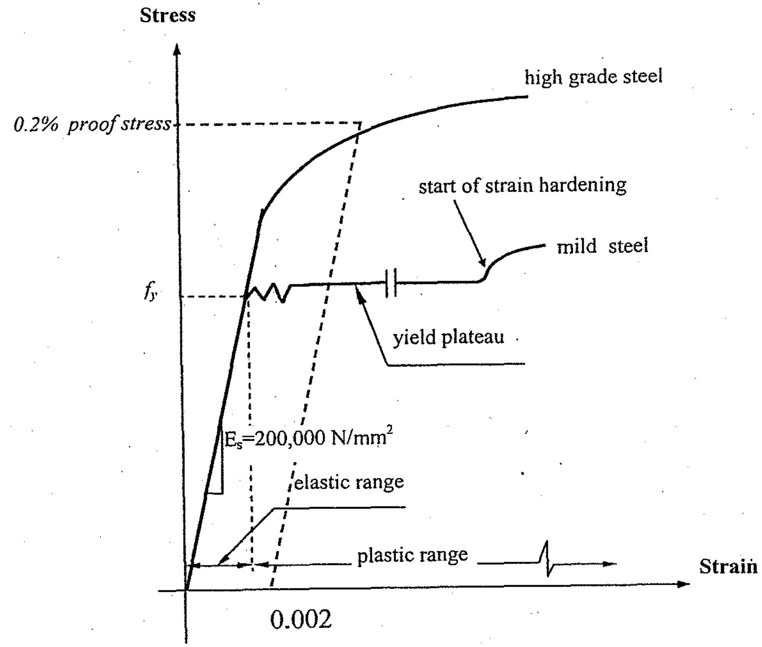


Fig. 1.13 Stress-Strain curve for mild and high grade steel

1.7 Limit States Design Method

Members are designed with a capacity that is much greater than required to support the anticipated set of loads. This extra capacity not only provides a factor of safety against failure by an accidental overload or defective construction but also limits the level of stress under service loads to control deflection and cracking. The Egyptian code permits the use of two design methods, namely, the allowable working stress design method and the ultimate limit states design method. In the present time, the former is the most commonly used in the design of reinforced concrete structures.
When a structure or a structural member becomes deficient for its planned use, it is said to have reached a limit state. The limit states of concrete structures can be divided into the following three groups:

A. Ultimate Limit states

These limit states are concerned with the failure of a structural member or the whole structure. Such a failure should have a very low probability of occurrence since it may lead to loss of human lives.

B. Serviceability limit states

These include all types that affect the functional use of the structure and can be classified as:

- **Deformation and Deflection Limit States**: Excessive deflections may be visually unacceptable and may lead to walls or partitions damage.
- **Cracking Limit States**: Excessive cracks may lead to leakage, corrosion of the reinforcement, and deterioration of concrete.
- **Vibration Limit States**: Vertical vibration of floors or roofs may cause unacceptable level of comfort for the users.

C. Stability limit states

These include buckling of compression members, overturning, sliding, formation of plastic hinge/mechanism, and general cases of instability. Also, in some cases, localized failure of a member may cause the entire structure to collapse. Such failure is called *progressive failure* and should be avoided.

1.8 Strength Reduction Factors

Strength reduction factors for both concrete and steel are introduced by the Egyptian code to account for several factors. These factors include simplifications, approximations, and small errors that may be encountered during calculations. They also consider variations between the actual strength and the design strength.

The strength reduction factors vary according to the applied compression force. As the compression force increases, the strength reduction factor in turn increases. One of the reasons for that, is the nature of the brittle failure that accompanies the compression forces. The strength reduction factor for concrete γc ranges from 1.73 for sections subjected to almost pure compression and 1.5 for sections subjected to pure bending. The strength reduction factor for steel reinforcement γs ranges from 1.32 for sections subjected to compression and 1.15 for section subjected to pure bending.

For sections subjected to combined compression forces and bending (eccentric compression sections) with at least 0.05t eccentricity, the ECP-203 gives the following values for the strength reduction factors

$$\gamma_c = 1.5 \times \left\{\frac{7}{6} - \frac{(e/t)}{3}\right\} \geq 1.5 \quad \text{.......... (1.7)}$$

$$\gamma_s = 1.15 \times \left\{\frac{7}{6} - \frac{(e/t)}{3}\right\} \geq 1.15 \quad \text{.......... (1.8)}$$

where e is the eccentricity and t is the member thickness and $\frac{e}{t} \geq 0.05$

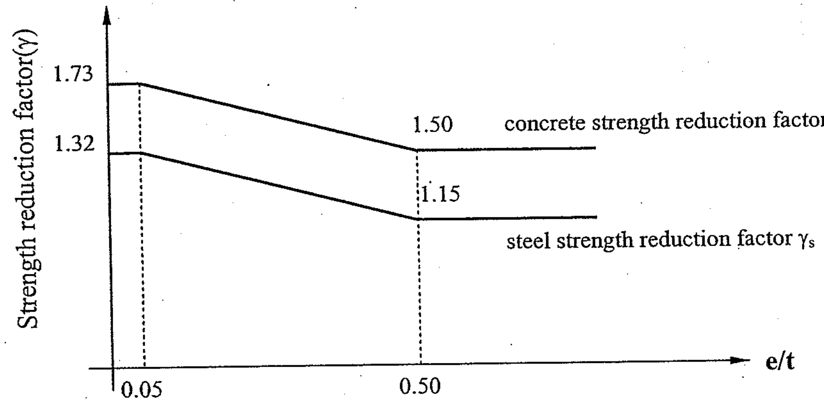


Fig. 1.11 Concrete and steel strength reduction factors

For other cases the strength reduction factors can be taken as

$$\left.\begin{array}{l}\gamma_c = 1.5 \\ \\ \gamma_s = 1.15\end{array}\right\} \begin{array}{l}\text{pure bending, shear and torsion} \\ \text{eccentric and concentric tensile forces} \\ \text{bond and bearing}\end{array}$$

for serviceability limit states the reduction factors can be taken as

$$\left.\begin{array}{l}\gamma_c = 1.0 \\ \gamma_s = 1.0\end{array}\right\} \text{for calculation of cracking, deflection and deformation}$$

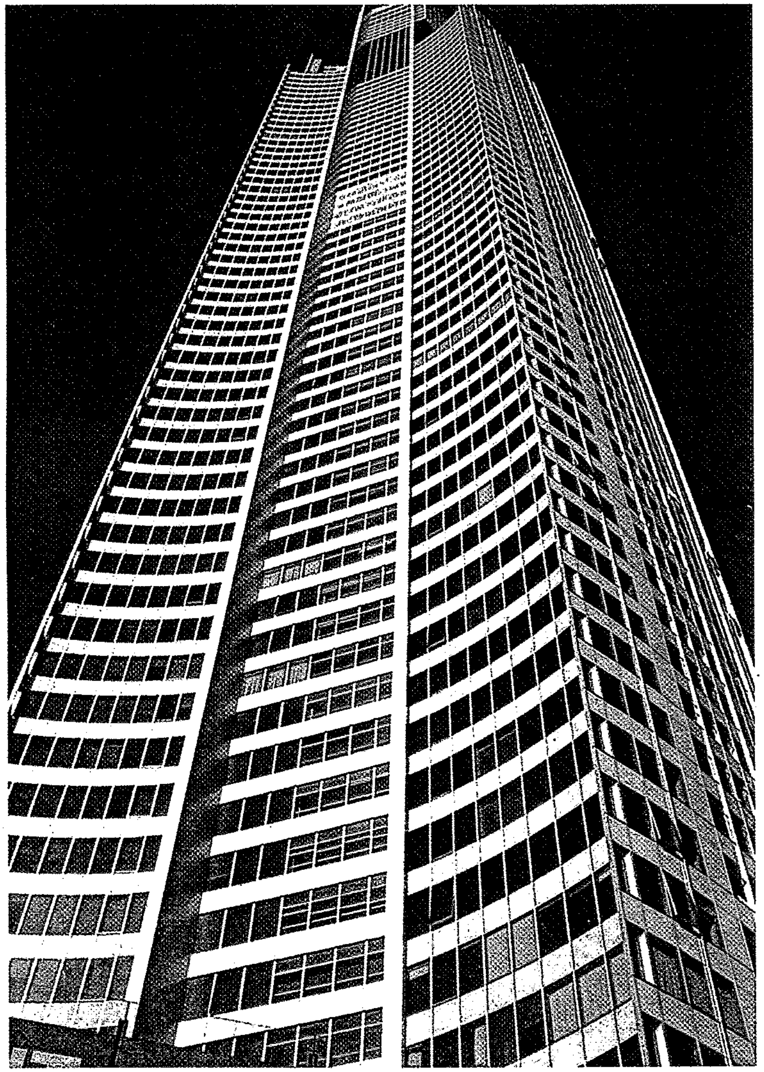


Photo 1.5 Queensland, Australia, 322 meters 78 stories (2005).

1.9 Classification of Loads

There are several types of loads that may act on a structure and can be categorized as:

Dead Loads: These are constant in magnitude and fixed in location for the lifetime of the structure. A major part of the dead loads results from the own weight of the structure itself. The dead loads also include sand required for leveling of the flooring, flooring material and brick walls.

Live loads depend mainly on the use of the structure. For buildings, live loads are the results of occupants and furniture. In bridges, vehicle loads represent the major live load. Their magnitude and location are variable. Live loads must be placed in such a way to produce the maximum straining actions on the structures. But rather by placing the live loads on the critical locations that cause maximum stresses for that member.

Table 1.5 gives examples of the values of live load on some structures as mentioned in the Egyptian Code for Calculation of loads on Structures.

Table 1.5 Live loads value according to building type (kN/m^2).

Structure Type	Location/usage	Live load
Residential buildings	Rooms	2
	Balconies , stairs, kitchen	3
Office buildings	Offices	2.5
	Archives	5-10
	Balconies and stairs	4
Hospitals	Patient rooms	2.5
	Surgery/lab	4 or more
	Balconies and stairs	4
Schools and faculties	Classrooms	3
	Labs	4 or more
	Sports centers	5
	Book shelf area	10
	Lecture rooms	4
	Balconies and stairs	4
Hotels	Gust rooms	2
	Public area/restaurants/stairs	4

Cinemas and theaters	Seated area	4
	Public area unseated	5
	Balconies	5
	Stairs and corridors	6
Mosque / church / Halls	Seated area	4
	Unseated area	5
Roofs	Inaccessible horizontal flexible roof	0.6
	Inaccessible horizontal rigid roof	1.0
	Accessible horizontal roof	2
garages	Parking area (small cars)	3
	Buses	4
	Garage corridor	5

For residential buildings with more than five stories, the live loads may be reduced according to the Table 1.6

Table 1.6 Reduction of live load in multistory residential buildings

Location of the floor	Live load value
Roof	P
From 1 to 4 under the roof	P
Fifth floor under the roof	0.9 P
Sixth floor under the roof	0.8 P
Seventh floor under the roof	0.7 P
Eighth floor under the roof	0.6 P
Ninth floor and more under the roof	0.5 P

Lateral loads These are the loads resulting from wind pressure, earthquake loads, soil pressure, and fluid pressure. In recent years, significant progress has been made to accurately estimate the horizontal forces due to wind or earthquake.
The ECP 203 states a series of load factors and load combination cases to be used in designing reinforced concrete sections.

1.10 Load Combinations

- For members that are subjected to live loads and where the lateral loads can be neglected, the ultimate factored loads U are computed from

$$U = 1.4\,D + 1.6\,L \quad \text{.........(1.9)}$$

where D are the working dead loads, and L are the working live loads Alternatively if the live loads are the less than 75% of the dead load, the following equation can be used

$$U = 1.5\,(D + L) \quad \text{.........(1.10)}$$

If the member is subjected to earth or fluid pressure (E), the ultimate load is given by

$$U = 1.4\,D + 1.6\,L + 1.6\,E \quad \text{.........(1.11)}$$

In the case of lateral pressure in closed spaces such as tanks and small pools, the ultimate load is taken from

$$U = 1.4\,D + 1.6\,L + 1.4\,E$$

- If the structure is subjected to wind loads W or earthquake loads S, the ultimate load U is taken as the largest from the following two equations

$$U = 0.8\,(1.4\,D + 1.6\,L + 1.6\,W) \quad \text{.........(1.12)}$$

$$U = 1.12\,D + \alpha\,L + S \quad \text{.........(1.13)}$$

Where α is a coefficient that takes into account the effect of live load that might exists on the building during an earthquake and is taken as follows

- ➢ α=1/4 in residentional buildings.
- ➢ α=1/2 in public buildings and structures such as malls, schools, hospitals, garages and theaters.
- ➢ α=1 in silos, water tanks, and structures loaded with sustained live loads such as public libraries, main storage areas and garages for public cars.

- In load cases in which reduction of live loads shall lead to increasing the value of maximum forces in some sections, the live load factor shall be taken to 0.9.

For cases in which the effects of the dead loads stabilize the structure, the ultimate loads should be taken from the following set of equations

$$U = 0.9\,D \quad \text{.......................} (1.14)$$

$$U = 0.9\,D + 1.6\,E \quad \text{.......................} (1.15)$$

$$U = 0.9\,D + 1.4\,E \text{ (for tanks and pools)} \quad \text{.......................} (1.16)$$

$$U = 0.9\,D + 1.3\,W \quad \text{.......................} (1.17)$$

$$U = 0.9\,D + 1.3\,S \quad \text{.......................} (1.18)$$

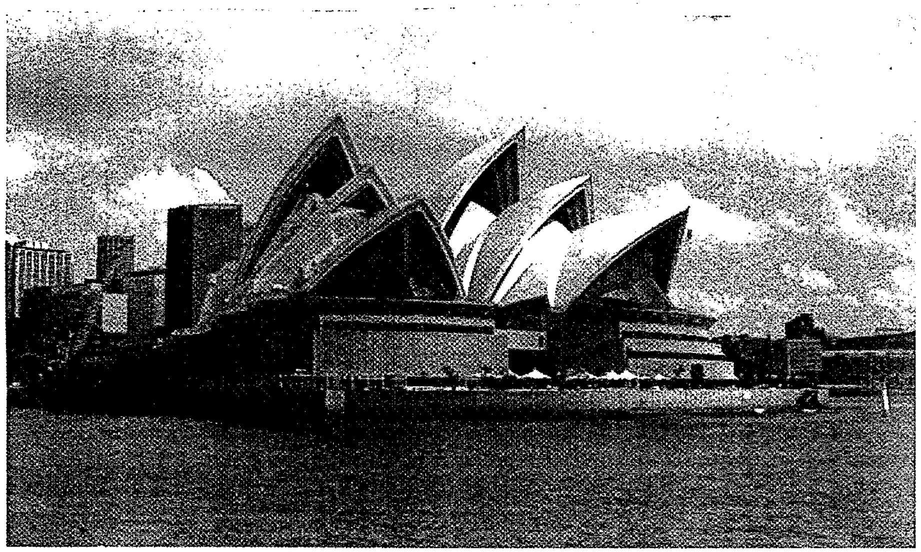


Photo 1.6: Opera Sydney in Australia

Table 1.7 Load factors according to ECP 203

Condition	Factored Load U
Basic	$U = 1.4\,D + 1.6\,L$
	$U = 1.5\,(D + L) \quad L \le 0.75\,D$
	$U = 0.9\,D$
	$U = 0.9\,D + 1.6\,L$
Wind	$U = 0.8\,(1.4\,D + 1.6\,L \pm 1.6\,W)$
	$U = 0.9\,D \pm 1.3\,W$
Earthquake	$U = 1.12\,D + \alpha\,L + S$
	$U = 0.9\,D \pm S$
Earth pressure	$U = 1.4\,D + 1.6\,L + 1.6\,E$
	$U = 0.9\,D + 1.6\,E$
Closed tanks	$U = 1.4\,D + 1.6\,L + 1.4\,E$
	$U = 0.9\,D + 1.4\,E$
Settlement, creep, or temperature	$U = 0.8\,(1.4\,D + 1.6\,L + 1.6\,T)$
	$U = 1.4\,D + 1.6\,T$
Dynamic loading	$U = 1.4\,D + 1.6\,L + 1.6\,K$
	$U = 0.9\,D + 1.6\,K$

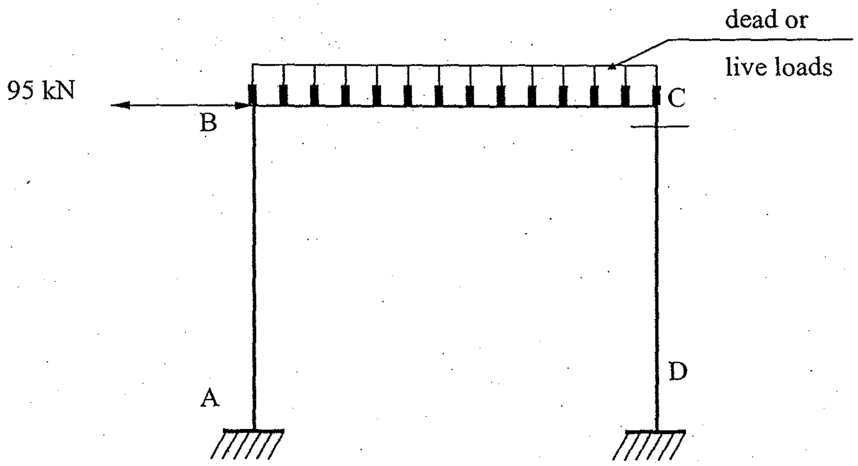
where D, L, W, S, E, T, K are the dead, live, wind, soil, earthquake, temperature and dynamic loads respectively.

Example 1.1

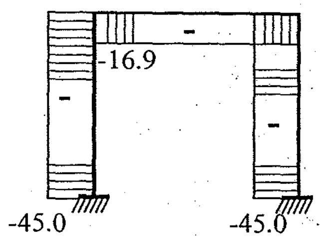
Using the load combinations of the ECP 203, determine the ultimate axial force and bending moment combinations for the column CD at point C. The frame is subjected to the following working loads

D=15 kN/m'(uniform)
L=30 kN/m'(uniform)

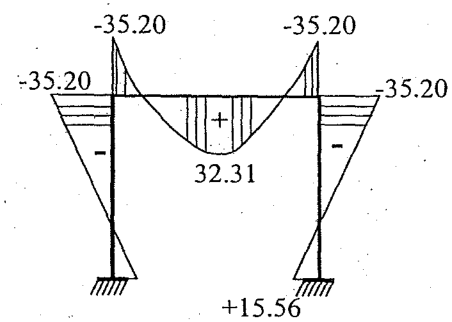
Wind load of 95 kN (may act in either direction)



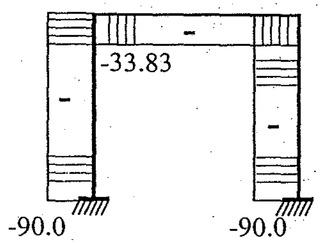
Solution:
since the structure is indeterminate, a computer program was used to calculate the axial and bending on the frame. The following figures summarize the results.



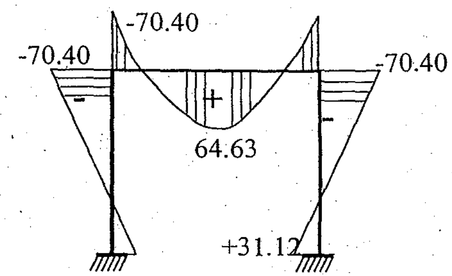
Normal force *(dead loads)*



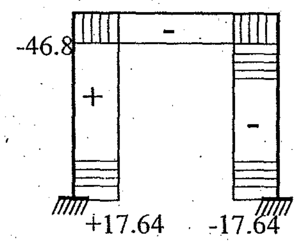
Bending moment *(dead loads)*



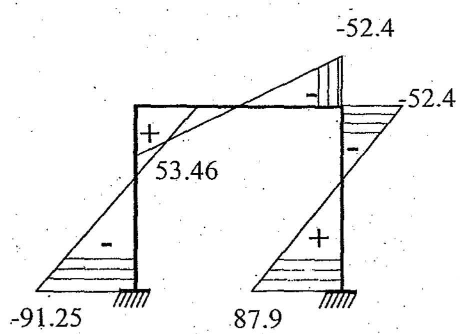
Normal force *(live loads)*



Bending moment *(live loads)*



Normal force *(wind loads)*



Bending moment *(wind loads)*

To compute the ultimate loads and according to the ECP-2003, five combinations were used as shown in the following table.

Load combinations for member CD

	Axial load	bending moment	case No.	Equation	Axial force combination	Bending combination
D	-45.0	-35.2	1	$U = 1.4\,D + 1.6\,L$	-207.0	-161.9
L	-90.0	-70.4	2	$U = 0.8\,(1.4\,D + 1.6\,L + 1.6\,W)$	-188.2	-196.6
W	-17.64	-52.4	3	$U = 0.8\,(1.4\,D + 1.6\,L - 1.6\,W)$	-143.1	-62.5
			4	$U = 0.9\,D + 1.3\,W$	-63.4	-99.8
			5	$U = 0.9\,D - 1.3\,W$	-17.6	+36.4

An example of the calculation for the axial force for the case of (D+L+W) is given by

$$U = 0.8\,[1.4\,D + 1.6\,L \pm 1.6W\,]$$

$$U = 0.8\,[1.4\,(-45) + 1.6\,(-90) \pm 1.6\,(-17.64)]$$

$$U = -165.60 \pm 22.60$$

$$= -188.2\,kN \;\; and \;\; -143.0\,kN$$

From the table, the maximum and minimum ultimate axial force on the column is -207.0 and -17.6 respectively. The maximum and minimum ultimate bending moment at C is –196.6 and +36.4.

It is very important to notice that the design should be carried out based on straining actions resulting from the same load combination not the maximum from each case. Thus, it is **wrong** to design the column for an axial compression force of –207.0 and bending moment of –196.6. Instead, the section must be designed to withstand (an axial compression force of –207 and a bending moment of –161.9) and (-axial -188.2, bending –196.6). In addition, it should be designed for an axial force of –17.6 and a bending moment of +36.4.

2

DESIGN OF SINGLY REINFORCED SECTIONS

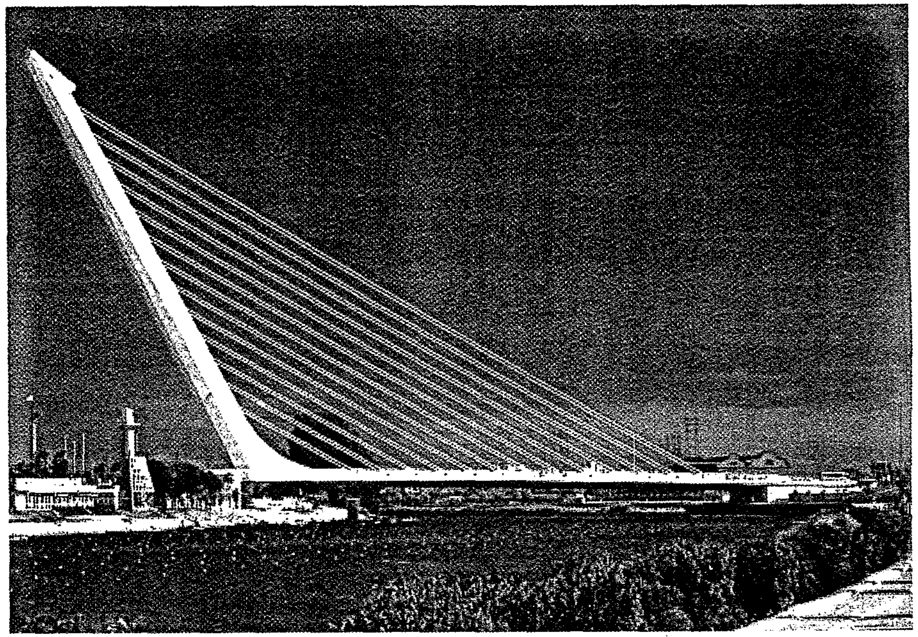


Photo 2.1: Alamillo Cartuja suspended bridge, Spain

2.1 Introduction

Until the late 1980s, nearly all reinforced concrete buildings in Egypt were designed according to the working-stress design method. However, since 1989 the ultimate limit states design method has gained popularity and has been adopted by the Egyptian Code for Design and Construction of Concrete Structures. In this chapter, the basic design concepts of the ultimate limit states design methods are discussed.

2.2 Reinforced Concrete Beam Behavior

Consider that a reinforced concrete beam as the one shown in Fig. 2.1, is subjected to an increasing load that will cause the beam to fail. Several stages of behavior can be clearly identified.

At low loads, below the cracking load, the whole of the concrete section is effective in resisting compression and tension stresses. In addition, since the steel reinforcement deforms the same amount as the concrete, it will contribute in carrying the tension stresses. At this stage, the distributions of strains and stresses are linear over the cross section.

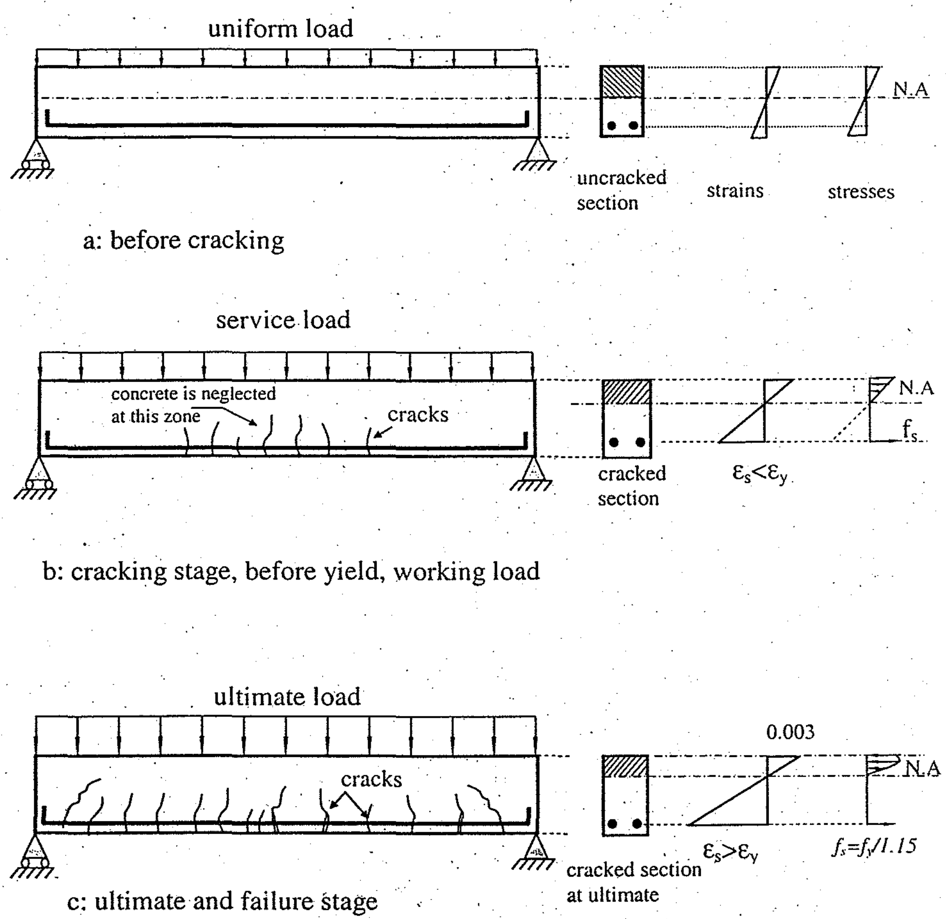


Fig. 2.1 Reinforced concrete beam behavior at different stages of loading

When the load is further increased, the developed tensile stresses in the concrete exceed its tensile strength and tension cracks start to develop. Most of these cracks are so small that they are not noticeable with the naked eye. At the location of the cracks, the concrete does not transmit any tension forces and steel bars are placed in the tension zone to carry all the developed tensile forces below the neutral axis. The neutral axis is an imaginary line that separates the tension zone from the compression zone. Therefore, by definition the stress at the neutral axis is equal to zero as shown in Fig. 2.1. Thus, the part of the concrete below the neutral axis is completely neglected in the strength calculations and the reinforcing steel is solely responsible for resisting the entire tension force.

At moderate loads (if the concrete stresses do not exceed approximately one-third the concrete compressive strength), stresses and strains continue to be very close to linear. This is called the working loads stage, which was the basis of the working-stress design method. When the load is furthered increased, more cracks are developed and the neutral axis is shifted towards the compression zone. Consequently, the compression and tension forces will increase and the stresses over the compression zone will become nonlinear. However, the strain distribution over the cross section is linear. This is called the ultimate stage. The distribution of the stresses in the compression zone is of the same shape of the concrete stress-strain curve. The steel stress f_s in this stage reaches yielding stress f_y. For normally reinforced beams, the yielding load is about 90%-95% of the ultimate load.

At the ultimate stage, two types of failure can be noticed. If the beam is reinforced with a small amount of steel, ductile failure will occur. In this type of failure, the steel yields and the concrete crushes after experiencing large deflections and lots of cracks. On the other hand, if the beam is reinforced with a large amount of steel, brittle failure will occur. The failure in this case is sudden and occurs due to the crushing of concrete in the compression zone without yielding of the steel and under relatively small deflections and cracks. This is not a preferred mode of failure because it does not give enough warning before final collapse.

2.3 Flexure Theory of Reinforced Concrete

2.3.1 Basic Assumptions of the Flexure Theory

In order to analyze beams subjected to pure bending, certain assumptions have to be established. These assumptions can be summarized as follows

1. Strain distribution is assumed to be linear. Thus, the strain at any point is proportional to the distance from the neutral axis. This assumption can also be stated as plane sections before bending remain plane after bending.
2. The strain in the reinforcement is equal to the strain in the concrete at the same level.
3. The tension force developed in the concrete is neglected. Thus, only the compression force developed in the concrete is considered, and all the tension force is carried by the reinforcement.
4. The stresses in the concrete and steel can be calculated using the idealized stress-strain curves for the concrete and steel after applying the strength reduction factors.
5. An equivalent rectangular stress block may be used to simplify the calculation of the concrete compression force.

The above assumptions are sufficient to allow one to calculate the moment capacity of a beam. The first of these assumptions is the traditional assumption made in the development of the beam theory. It has been proven valid as long as the beam is not deep. The second assumption is necessary because the concrete and reinforcement must act together to carry the load and it implies a perfect bond between concrete and steel. The third assumption is obviously valid since the strength of concrete in tension is roughly 1/10 of the compressive strength and the tensile force in the concrete below the neutral axis will not affect the flexural capacity of the beam. The fourth and fifth assumptions will be discussed in items 2.3.2 and 2.3.3.

2.3.2 Stress-Strain Relationships

2.3.2.1 Concrete in Compression

The stress-strain curve for concrete is non-linear with a descending branch after reaching the maximum stress as shown in Fig. 1.4, presented in Chapter 1. The recorded maximum compressive stress in a real beam differs from that obtained in a cylinder or a cube test. Several studies have indicated that the ratio of the maximum compression stress in beams or columns to the cylinder compressive strength f_c' can be taken equal to 0.85 for most practical purposes. This accounts for the size effect and the fact that the beam is subject to a sustained load while the cylinder is tested during a short period. Furthermore, since the cylinder strength f_c' is about 0.80 of cube strength f_{cu}, the maximum value of the stress strain curve for beams or columns is 0.85 x 0.80 f_{cu}= 0.67 f_{cu}. For design purposes, the previous value is divided by the concrete safety factor (γ_c=*1.5 in case of pure bending*) to account for the uncertainties explained in section 2.3. Hence the design compressive strength of the concrete as adopted by the Egyptian Code (ECP 203) is $0.67\, f_{cu}/\gamma_c = 0.45\, f_{cu}$.

The Egyptian Code presents an idealization for the stress-strain curve in compression. The first part of the curve is a parabolic curve up to a strain of 0.002 and the second part is a straight horizontal line up to a strain of 0.003, as shown in Fig.2.2. Referring to Fig. 2.2, the equation of the concrete stress f_c in terms of the concrete strain (ε_c) can be expressed as:

$$f_c = \begin{cases} f_c^* \left[\dfrac{2\,\varepsilon_c}{0.002} - \left(\dfrac{\varepsilon_c}{0.002} \right)^2 \right] & for \quad \varepsilon_c < 0.002 \\ f_c^* & for \quad 0.002 \le \varepsilon_c \le 0.003 \end{cases} \qquad \text{.................(2.1.A)}$$

where $f_c^* = \dfrac{0.67\, f_{cu}}{\gamma_c}$

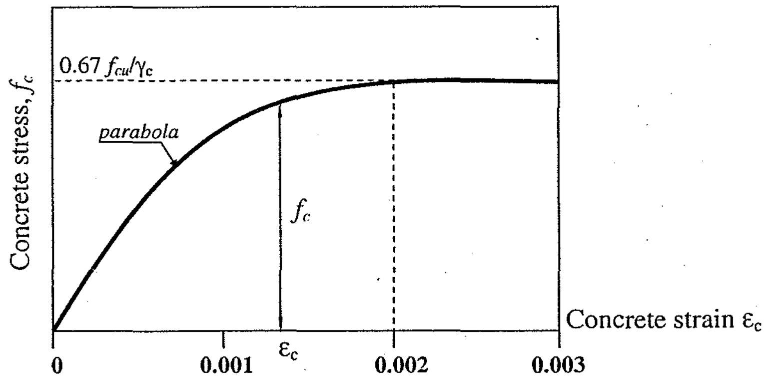


Fig 2.2 ECP 203 idealized stress-strain curve for concrete

2.3.2.2 Reinforcing Steel

The behavior of the steel reinforcement is idealized by the Egyptian code (section 4.2.1.1)as an elastoplastic material as shown in Fig 2.3. The reinforcing steel stress can be calculated using Eq. 2.1.B.

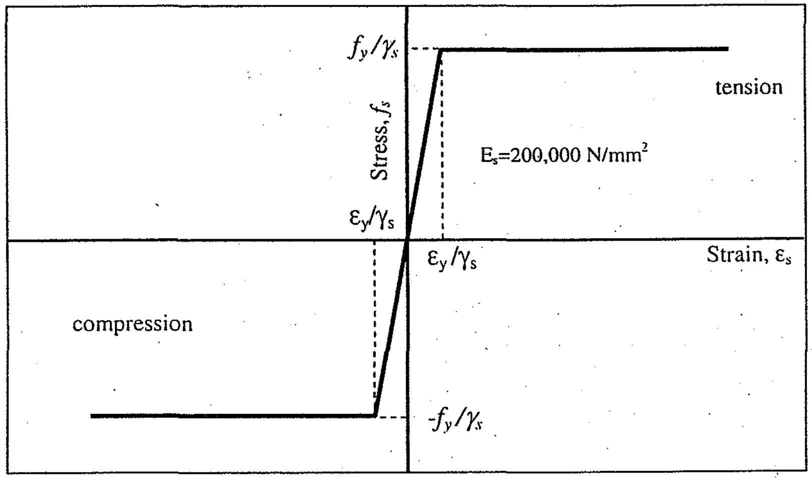


Fig 2.3 Idealized stress-strain curve for steel

$$f_s = \varepsilon_s \times E_s \quad when\ \varepsilon_s < \varepsilon_y/\gamma_s$$
$$f_s = f_y/\gamma_s \quad when\ \varepsilon_s \geq \varepsilon_y/\gamma_s \qquad \text{(2.1.B)}$$

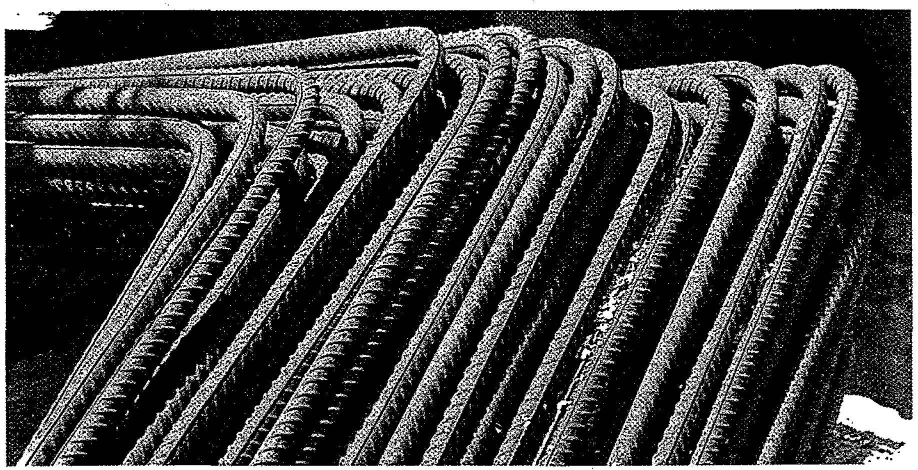


Photo 2.2 High grade steel Reinforcement

2.3.3 The Equivalent Rectangular Stress Block

To compute the compression force resisted by concrete, the Egyptian Code replaces the curved stress block shown in Fig 2.4C by an equivalent stress block of an average intensity of 0.67 f_{cu}/γ_c and a depth a= β c as shown in Fig. 2.4D. The magnitude and location of the force calculated using the equivalent stress block should be equal to that of the curved one.

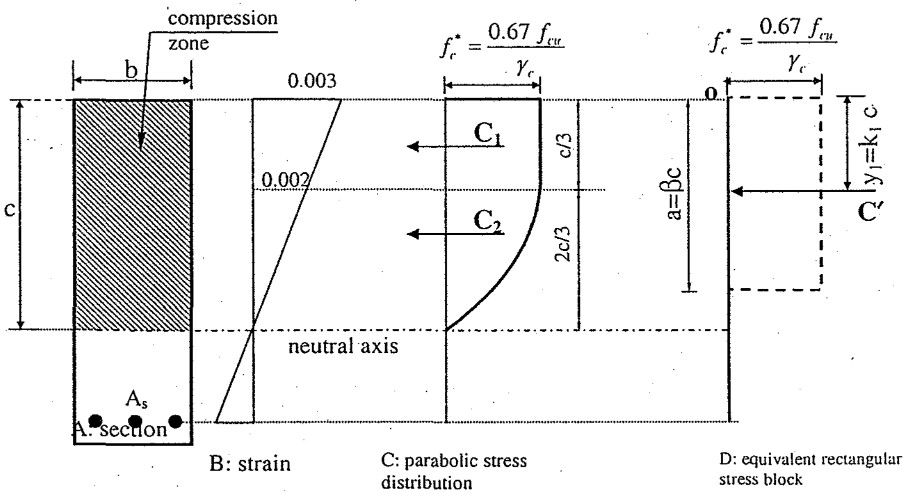


Fig. 2.4 Equivalent rectangular stress block calculation.

To calculate the depth "a" of the stress block, one equates the compression force obtained using the stress-strain curve of the Egyptian Code, shown in Fig. 2.4C, to that using the equivalent stress block (Fig. 2.4D).
The total compression force ($C=C_1+C_2$) obtained using the stress-strain curve of the Egyptian Code can be calculated as follows:

$$C_1 = b \times \left(\frac{c}{3} \times f_c^*\right) \qquad \text{(2.2)}$$

$$C_2 = b \times \frac{2}{3} \times \frac{2\,c}{3} \times f_c^* = b \times \left(\frac{4}{9}\, c \times f_c^*\right) \qquad \text{(2.3)}$$

$$C = C_1 + C_2 = \frac{c}{3} \times b \times f_c^* + \frac{4\,c}{9} \times b \times f_c^* = \frac{7}{9}\, c \times b \times f_c^* \qquad \text{(2.4)}$$

The compression force obtained using the stress block C′ equals

$$C' = b \times a \times f_c^* = \beta\, c \times b \times f_c^* \qquad \text{(2.5)}$$

By definition, C must be equal to C′, thus solving Eq. 2.4 and Eq. 2.5 for β gives

$$\beta = \frac{7}{9} = 0.777$$

The code approximates the previous value to β=0.8, thus the rectangular stress block depth (a=0.8 c).

To find the location of the total compression force C′, take the moment of the forces at point "o" and note that the C.G of the force F_2 is at 3/8 of the distance (2/3c)

$$C' \times k_1\ c = C_1\left(\frac{1}{2}\times\frac{c}{3}\right) + C_2\times\left(\frac{3}{8}\times\frac{2c}{3}+\frac{c}{3}\right) \qquad (2.6)$$

$$\frac{7}{9}c\times b\times f_c^{*}\times k_1\ c = \frac{c}{3}\times b\times f_c^{*}\left(\frac{c}{6}\right) + \frac{4}{9}c\times b\times f_c^{*}\left(\frac{7}{12}c\right) \qquad (2.7)$$

k_1=0.404

The code simplifies the value of k_1 with β/2=0.4 (i.e. the resultant is at the middle of the stress block)

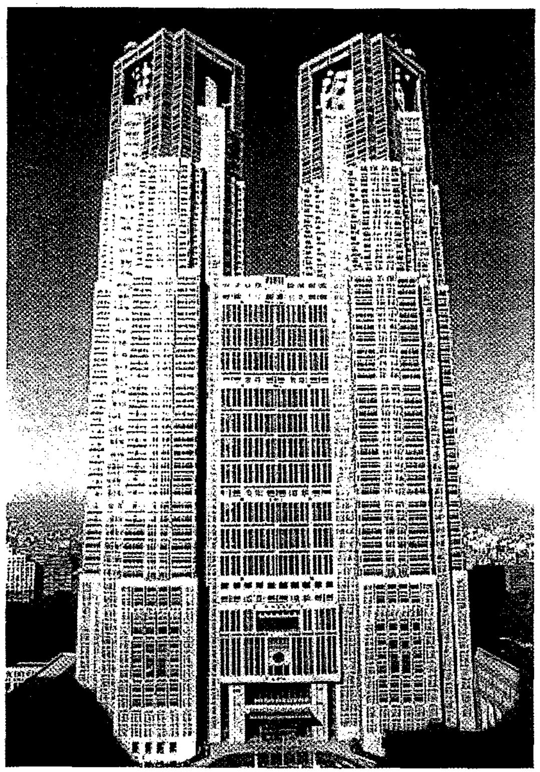


Photo 2.3 Metropolitan Government Building in Tokyo

2.4 Analysis of Singly Reinforced Sections

Concrete beams subjected to pure bending must resist both tensile and compressive stresses. However, concrete has very low tensile stresses, and therefore tension steel is placed in these locations (below neutral axis) as shown Fig. 2.5. The most economic solution is to place the steel bars as far as possible from the neutral axis except for the concrete cover, which is normally assumed 50 mm from the external surface.

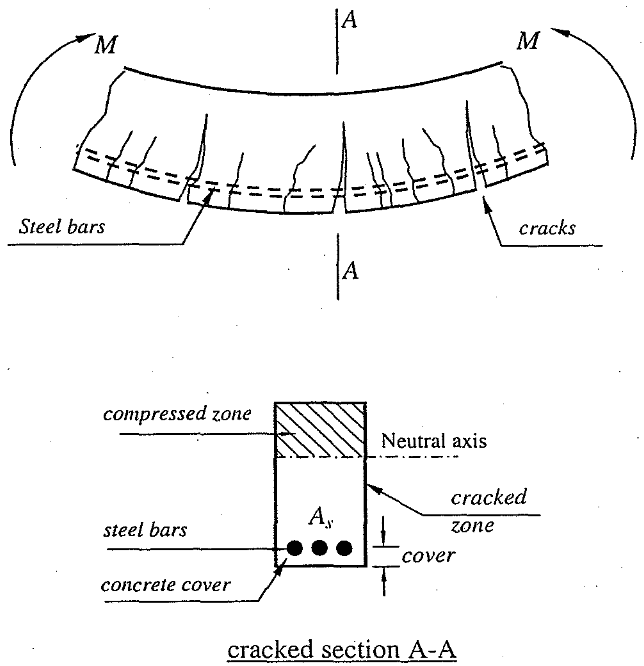


Fig. 2.5 Reinforcement placement in reinforced concrete beam

The compressive stresses in concrete are replaced by a uniform stress block as suggested by the Egyptian Code (section 4.2.1.1.9) with distance "a" from the concrete surface as shown in Fig. 2.6.

The analysis of the cross section is carried out by satisfying two requirements:

- **Equilibrium**

1. $\sum \text{Forces (internal)} = \sum \text{Forces (external)}$

   For sections subjected to pure bending, the external forces equal to zero. This leads to

   $\sum \text{Forces (internal)} = 0 \;\Rightarrow\; T - C = 0 \;\Rightarrow\; T = C$

2. $\sum M_u \text{(internal)} = \sum M_u \text{(external)}$ (taken about any point in the section)

- **Compatibility of Strains**

1. The strain at any point is proportional to its distance from the neutral axis.

Therefore, if the design problem has more than two unknowns, assumptions have to be made to reduce them to exactly two. The stress in the tension steel is assumed to be equal to the yield strength f_y. This assumption should be verified after determining the neutral axis position. The equilibrium of the internal forces is used to determine the stress block distance "a" as follows:

$$C = T \qquad \text{(2.8)}$$

$$\frac{0.67\, f_{cu}\, b\, a}{1.5} = \frac{A_s\, f_y}{1.15} \qquad \text{(2.9.A)}$$

If the tension steel does not yield Eq. 2.9.A becomes

$$\frac{0.67\, f_{cu}\, b\, a}{1.5} = A_s \times f_s \qquad \text{(2.9.B)}$$

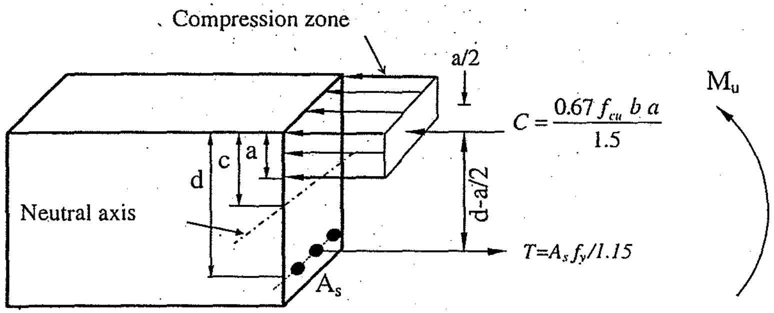


Fig. 2.6 Equilibrium of forces in a singly reinforced section

Having determined the stress block distance a, the assumption of the tension steel yielding can be verified using compatibility of strains as follows ($c = a/0.8$ and $E_s = 200{,}000$ N/mm^2)

$$f_s = E_s \times \varepsilon_s \qquad \text{(Hook's Law)}$$

$$\varepsilon_s = 0.003\,\frac{d-c}{c} \qquad \text{(compatibility of strains)}$$

$$f_s = 600\,\frac{d-c}{c} \le \frac{f_y}{1.15} \qquad \text{(2.10)}$$

If the steel stress f_s calculated by Eq. 2.10 exceed $f_y/1.15$, then the assumption of the yielding of the tension steel is valid ($f_s = f_y/1.15$) as used in Eq. 2.9.A.

The second equilibrium equation is used to determine the moment capacity of the section by equating the internal moment to the external applied moment M_u. The internal moment capacity is computed by taking the moment of the internal forces about any point. Normally, this point is taken at the resultant of the compression force C to simplify the calculations. The internal moment in this case is the product of the tension force multiplied by the distance to the compression force. This distance is called the lever arm (d-a/2) as shown in Fig. 2.7. The equation for the moment is:

$$M_u = \frac{A_s\, f_y}{1.15}\left(d - \frac{a}{2}\right) \qquad \text{(2.11.A)}$$

If the tension steel does not yield, Eq. 2.11.A becomes

$$M_u = A_s\, f_s\left(d - \frac{a}{2}\right) \qquad \text{(2.11.B)}$$

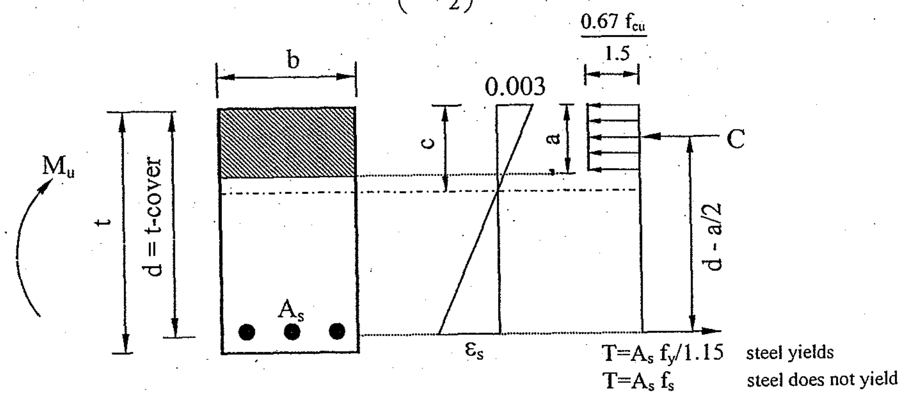


Fig. 2.7 Stress and strain distributions of a singly reinforced section

2.5 Maximum Area of Steel of a Singly Reinforced Section

The balanced failure occurs when the concrete strain reaches a value of 0.003 at the same time that the steel reaches the yield strain divided by the reduction factor (ε_y/γ_s) as shown in Fig. 2.8.

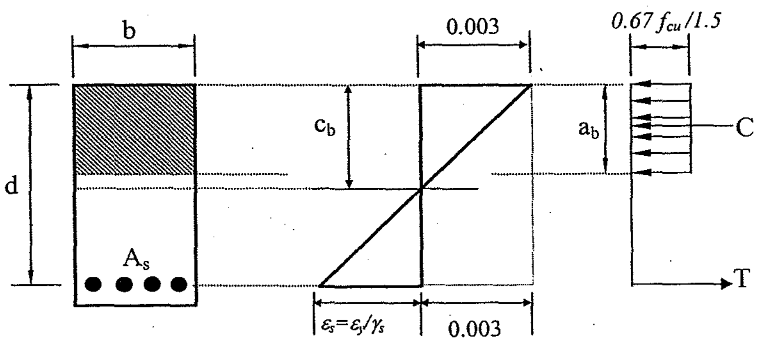


Fig.2.8 Neutral axis position at the balanced condition

From similar triangles shown in Fig. 2.8, one can conclude that

$$\frac{c_b}{d}=\frac{0.003}{0.003+\dfrac{\varepsilon_y}{\gamma_s}} \quad \text{..........(2.12)}$$

where c_b is the neutral axis at the balanced failure. The steel Young's modulus E_s equals

$$E_s=\frac{f_y}{\varepsilon_y}=\frac{f_y/\gamma_s}{\varepsilon_y/\gamma_s} \quad \text{..........(2.13)}$$

Substituting with steel Young's modulus E_s =200,000 N/mm^2 and γ_s=1.15 gives

$$\frac{c_b}{d}=\frac{690}{690+f_y} \quad \text{..........(2.14)}$$

If $c < c_b$, then the strain in the tension steel is greater than ε_s/γ_s and that the tension steel yields. To ensure ductile failure the ECP 203 requires that the value of c_{max} be limited to 2/3 c_b. Substitution in Eq. 2.14 and referring to Fig. 2.9 gives the following equation

$$\frac{c_{max}}{d}=\frac{460}{690+f_y} \quad \text{..........(2.15)}$$

$$\frac{a_{max}}{d}=\frac{368}{690+f_y} \quad \text{..........(2.16)}$$

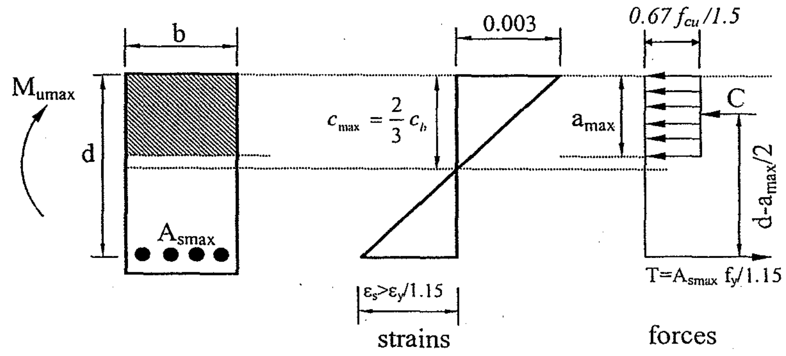


Fig. 2.9 Neutral axis position for calculating the maximum values allowed by the code

The ratio of the reinforcement in the concrete section (μ) is an indication to show if the section is lightly reinforced or heavily reinforced and can be expressed as:

$$\mu=\frac{A_s}{b\,d} \quad \text{..........(2.17)}$$

After finding the maximum neutral axis position c_{max}, it is beneficial to compute the maximum area of steel $A_{s,max}$ recommended by the code. To find the maximum area of steel, apply the equilibrium equation (C=T) with neutral axis at c_{max} as shown in Fig. 2.9.

$$\frac{0.67\,f_{cu}\,b\,a_{max}}{1.5}=\frac{A_{s,max}\,f_y}{1.15} \quad \text{..........(2.18)}$$

Diving both sides by (b x d) gives

$$\frac{0.67\,f_{cu}}{1.5}\,\frac{a_{max}}{d}=\frac{\mu_{max}\,f_y}{1.15} \quad \text{..........(2.19)}$$

substituting with Eq. 2.16 into Eq. 2.19 gives

$$\boxed{\mu_{max} = \frac{189}{690\, f_y + f_y^2}\, f_{cu}} \quad \text{.......................................(2.20)}$$

The ECP 203 limits the reinforcement ratio μ to μ_{max} given by Eq. 2.20 to ensure ductile failure. Moreover, it is a good practice, from the economic point of view, to limit the area of steel reinforcement in beams to only 0.5-0.7 μ_{max}. It can be noticed that steel with smaller f_y will have smaller yield strain ε_y leading to larger neutral axis distance c_{max} as shown in Table 2.1. Thus, the smaller the steel yield strength, the larger the maximum permissible steel ratio μ_{max} as shown in Fig. 2.10.

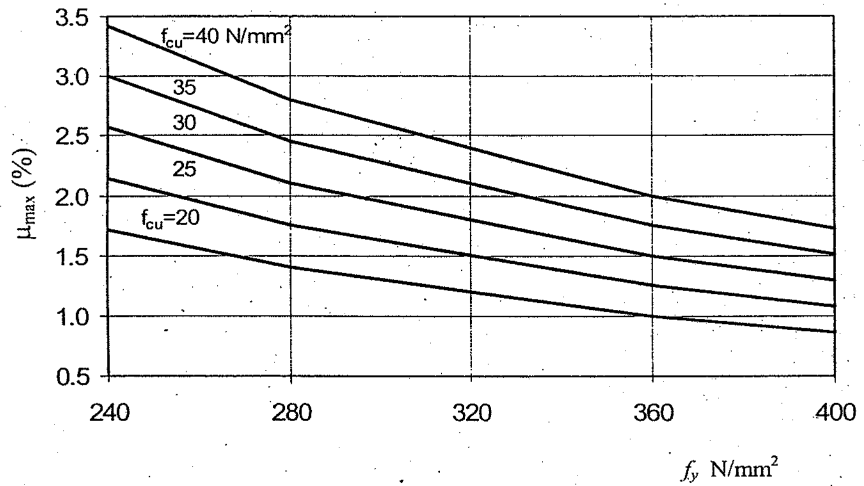


Fig. 2.10 Effect of f_{cu} and f_y on μ_{max}

It should be clear that if for a given section the neutral axis distance "c" is less than neutral axis maximum value c_{max}, then the steel is yielded, the actual area of steel A_s, and the applied moment M_u is less than code maximum limits as indicated in Eq. 2.21.

$$\boxed{If\ \frac{c}{d} < \frac{c_{max}}{d}\quad then \left(\begin{array}{l} f_s = \frac{f_y}{1.15} \\ \mu < \mu_{max} \\ A_s < A_{s,max} \\ M_u < M_{u,max} \end{array} \right.} \quad \text{.............................(2.21)}$$

Defining $\omega = \mu \frac{f_y}{f_{cu}}$

$$\omega_{max} = \mu_{max} \frac{f_y}{f_{cu}} \quad \text{.......................................(2.22)}$$

Substituting with the value of μ_{max} determined from Eq. 2.20 gives

$$\omega_{max} = \frac{189}{690 + f_y} \qquad (2.23)$$

Table 2.1 Values of c_{max}/d, μ_{max}, ω_{max}

Steel	c_b/d	c_{max}/d	a_{max}/d	R_{max}	$R1_{max}$	μ_{max}*	ω_{max}
240/350	0.74	0.50	0.40	0.214	0.143	$8.56\times10^{-4} f_{cu}$	0.205
280/450	0.71	0.48	0.38	0.208	0.139	$7.00\times10^{-4} f_{cu}$	0.196
360/520	0.66	0.44	0.35	0.194	0.129	$5.00\times10^{-4} f_{cu}$	0.180
400/600	0.63	0.42	0.34	0.187	0.125	$4.31\times10^{-4} f_{cu}$	0.172
450/520**	0.61	0.40	0.32	0.180	0.120	$3.65\times10^{-4} f_{cu}$	0.164

* f_{cu} in N/mm²
** for welded mesh

Maximum Moment Capacity

To determine the maximum moment for a singly reinforced section, one can compute the moments of the tension force about the compression force (refer to Fig. 2.9) at $c = c_{max}$

$$M_{u,\max} = \frac{A_{s,\max}\, f_y}{1.15}\left(d - \frac{a_{\max}}{2}\right) \quad \text{.............................}(2.24)$$

Defining R_{max} as $R_{\max} = \dfrac{1.5 \times M_{u,\max}}{f_{cu}\, b\, d^2}$

$$R_{\max} = \frac{1.5}{f_{cu}\, b\, d^2} \times \frac{A_{s,\max}\, f_y}{1.15}\left(d - \frac{a_{\max}}{2}\right) \quad \text{.......................}(2.25)$$

$$R_{\max} = 1.304 \frac{\mu_{\max}\, f_y}{f_{cu}}\left(1 - 0.4\frac{c_{\max}}{d}\right) \quad \text{.......................}(2.26a)$$

$$R_{\max} = 1.304\; \omega_{\max}\left(1 - 0.4\frac{c_{\max}}{d}\right) \quad \text{.............................}(2.26b)$$

Substituting with the value of μ_{max} calculated from Eq. 2.20 gives

$$R_{\max} = \frac{246}{690 + f_y}\left(1 - 0.4\frac{c_{\max}}{d}\right) \quad \text{.............................}(2.27)$$

$$R1_{\max} = \frac{M_{u\,\max}}{f_{cu}\, b\, d^2} = \frac{R_{\max}}{1.5} \quad \text{.............................}(2.28)$$

2.6 Balanced, Under, and Over Reinforced Sections

In general, an under-reinforced section is the one in which reinforcing steel yields before the crushing of concrete. An over-reinforced section is the one in which failure occurs due to the crushing of concrete in the compression zone before the yielding of the steel. On the other hand, a balanced section is the one in which yielding of steel and crushing of concrete occur simultaneously.

According to the analysis carried out in section 2.5, one can conclude that if the section is reinforced with μ less than μ_b ($=1.5\ \mu_{max}$) it is called *"under reinforced"*. On the contrary, if the section is reinforced with μ greater than μ_b, it is called *"over reinforced"*. The under-reinforced sections are preferred because they fail in a ductile manner, in which the member will experience large deflections, large strains, and wide cracks. This gives enough warning so that repair can be performed on that member. On the other hand, over reinforced sections will fail suddenly without enough warnings. Figure 2.11 gives the strain distributions and the related values of the three sections

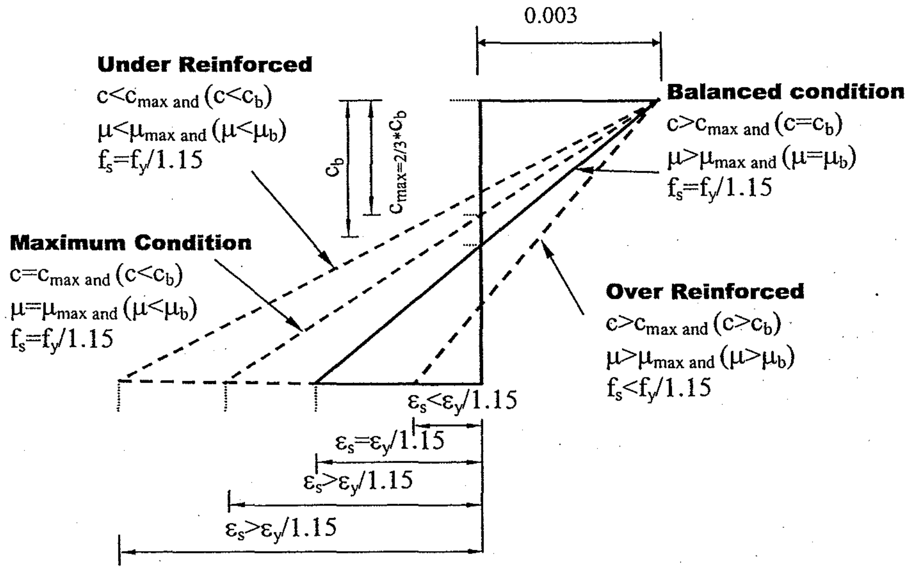


Fig 2.11 Strain distributions for over, under and balanced sections

2.7 Minimum Area of Steel

In some cases, and mainly due to architectural considerations, the member could be chosen with concrete dimensions bigger than those required by strength calculations. Accordingly, the required area of steel could be very small. This may lead to situations where the strength of the section using cracked section analysis is less than the strength of the uncracked section computed using the tensile strength of concrete.

The failure of such sections is brittle and wide cracks tend to develop. Thus, to control cracks, to ensure ductility, and to avoid sudden failure in tension, the Egyptian code (4.2.1.2.g) requires that the actual area of steel A_s in any section should be greater than A_{smin} given by:

$$A_{s\,min} = smaller\ of \begin{cases} \dfrac{0.225\sqrt{f_{cu}}}{f_y}\, b\ d \geq \dfrac{1.1}{f_y}\, b\ d \\ 1.3\,A_s \end{cases} \qquad (2.29)$$

$$but\ not\ less\ than \begin{Bmatrix} \dfrac{0.25}{100}\, b\ d\,(mild\ steel) \\ \dfrac{0.15}{100}\, b\ d\,(high\ grade) \end{Bmatrix}$$

If f_{cu} is greater or equal to 25 N/mm^2 the term ($0.225\sqrt{f_{cu}}/f_y$ b d) is bigger than (0.25% b d) and (0.15% b d). Thus, there is no need to check the third condition in Eq. 2.29, if $0.225\sqrt{f_{cu}}/f_y < 1.3A_s$. The minimum area of steel in this case can be simplified to:

$$\boxed{A_{s\,min} = smaller\ \ of \begin{cases} \dfrac{0.225\sqrt{f_{cu}}}{f_y}\, b\ d \\ 1.3A_s \end{cases}} \qquad (2.30)$$

2.8 Factors Affecting Ultimate Strength

There are several factors that affect the ultimate strength of a beam subjected to bending. These factors can be summarized as

- Yield strength of reinforcing steel, f_y.
- Concrete compressive strength, f_{cu}
- Beam depth, d
- Beam width, b
- Reinforcement ratio, μ.

The effect of steel yield strength on ultimate strength is shown in Fig. 2.12A. It is clear that steel yield strength has a big impact on its ultimate capacity. Increasing the steel yield strength from 240 N/mm^2 to 400 N/mm^2 increases the ultimate capacity by 55%. On the other hand, concrete compressive strength has a little effect on the ultimate strength as shown in Fig. 2.12B. Changing concrete compressive strength from 20 N/mm^2 to 40 N/mm^2 increases the ultimate strength by only 5%.

Comparing Fig. 2.12C and Fig. 2.12D shows that increasing beam depth affects the ultimate capacity more than increasing beam width. Increasing beam depth from 500 mm to 1000 mm increases the capacity of the beam by almost three times. Finally, increasing steel reinforcement ratio has a significant effect on the ultimate capacity as illustrated in Fig. 2.12E.

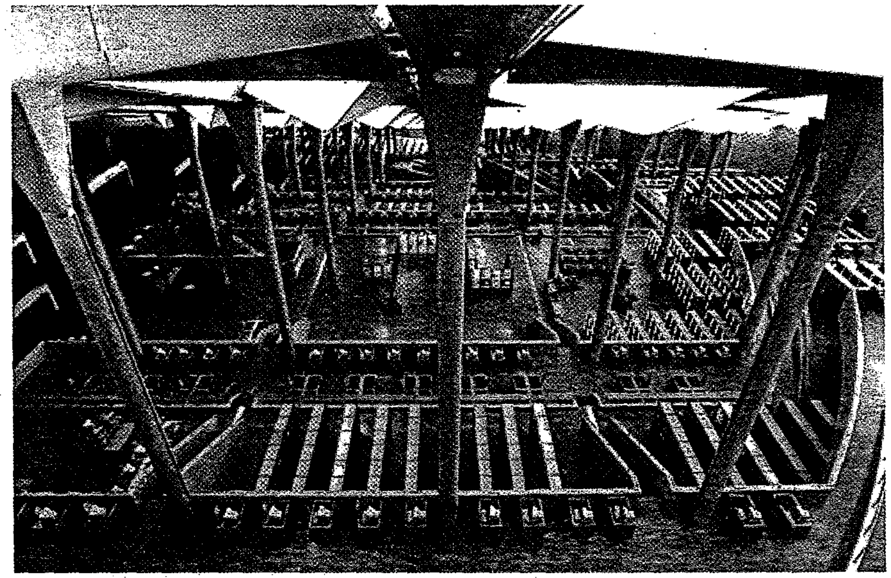
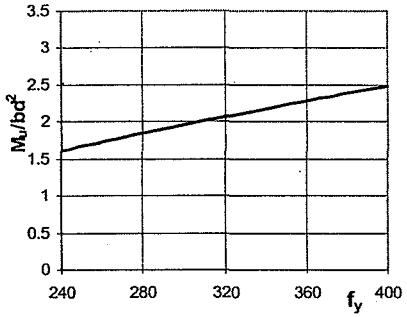
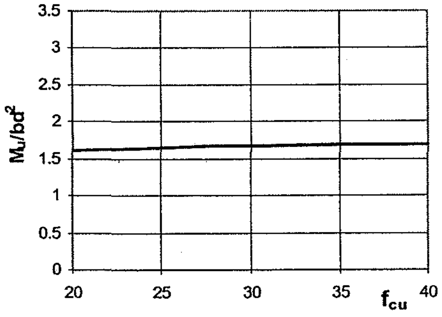


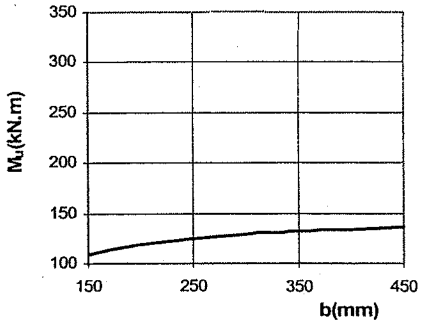
Photo 2.4 Interior reading halls in the Library of Alexandria



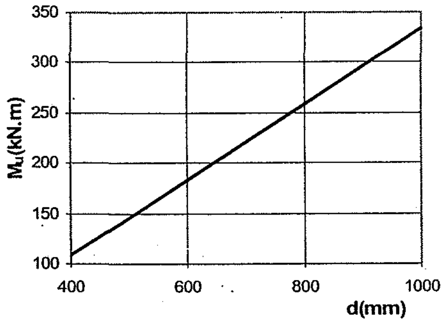
A-Effect of f_y



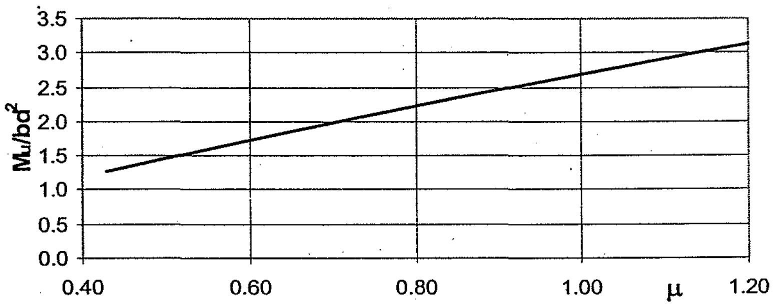
B-Effect of f_{cu}



C-Effect of beam width (b)



D-Effect of beam depth (d)



E-Effect of reinforcement ratio μ

Fig. 2.12 Parametric study on the ultimate moment capacity

Analysis Summary

In this type of problem all the cross section information is known including beam cross section dimensions, steel yield strength and concrete strength. It is required to calculate the moment capacity M_u.

ANALYSIS PROBLEM	
Given	**: b, t , A_s f_{cu}, f_y**
Required	**: M_u**
Unknowns	**: a, M_u**

Procedure

- <u>Step 1:</u> Apply the equilibrium equation T=C to find the depth of the stress block, "a" and the neutral axis depth "c" assuming that tension steel has yielded $f_s=f_y/1.15$.
- <u>Step 2:</u> Check that tension steel has yielded ($f_s \geq f_y/1.15$) by ensuring the $c<c_b$ or by using Eq.2.10.
- <u>Step 3:</u> Compute the bending moment capacity M_u by taking the moment about the concrete compression force.

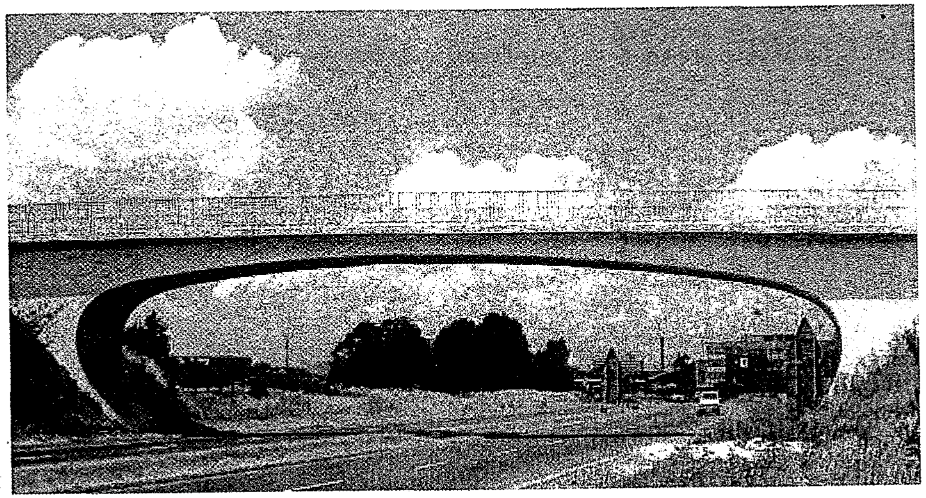


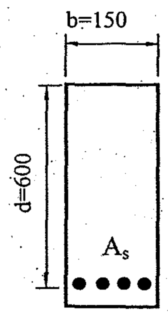
Photo 2.5 Cantilever box section in a reinforced concrete bridge

Example 2.1

Determine whether the section shown in figure is under-or-over-reinforced section and check code maximum permissible area of steel for the following cases

1. $A_s = 500\ mm^2$
2. $A_s = 1000\ mm^2$
3. $A_s = 1500\ mm^2$
4. $A_s = 2000\ mm^2$

$f_{cu}=25\ N/mm^2$, $f_y=360\ N/mm^2$



Solution

To determine whether the section is under-or-over-reinforced, one has to calculate the balanced area of steel A_{sb}.

From the code Table 4-1 or Table 2.1 in this text one can get :

$\mu_{max}=5 \times 10^{-4}\ f_{cu}=5 \times 10^{-4} \times 25= 0.0125$

$$A_{s\,max} = \mu_{max}\ b\ d = 0.0125 \times 150 \times 600 = 1125\ mm^2$$

but $c_{max}= 2/3\ c_b$ or $A_{smax}= 2/3\ A_{sb}$

$A_{sb} = 3/2 \times 1125=1687.5\ mm^2$

$$A_{s\,min} = smaller\ \ of \begin{cases} \dfrac{0.225\sqrt{25}}{360} \times 150 \times 600 = 281\ mm^2 \\ 1.3 \times (500) = 650\ mm^2 \end{cases} = 281\ \ mm^2\ \ \ldots < A_s \ldots o.k$$

Thus, all sections satisfy the minimum area steel requirements

Case 1: A_s= 500 mm^2	under reinforced (A_s<A_{sb})	Code limit safe (A_s<A_{smax})
Case 2: A_s=1000 mm^2	under reinforced (A_s<A_{sb})	Code limit safe (A_s<A_{smax})
Case 3: A_s=1500 mm^2	under reinforced (A_s<A_{sb})	Code limit unsafe (A_s>A_{smax})
Case 4: A_s=2000 mm^2	over reinforced (A_s>A_{sb})	Code limit unsafe (A_s>A_{smax})

Note 1: An alternative method for calculating the balanced area of steel is as follows:

$$\frac{c_b}{d} = \frac{690}{690+f_y} = \frac{690}{690+360} = 0.657 \longrightarrow c_b=(\ 0.657 \times 600)=394.2\ mm$$

$a_b=0.8\ c_b = 0.8\ (394.2) = 315.36\ mm$

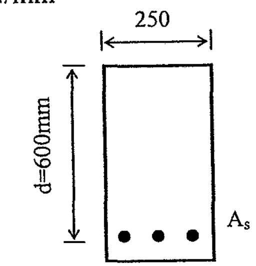
$$\frac{0.67\ f_{cu}\ b\ a_b}{1.5} = \frac{A_{sb}\ f_y}{1.15}$$

$$\frac{0.67 \times 25 \times 150 \times 315.36}{1.5} = \frac{A_{sb} \times 360}{1.15}$$

$A_{sb}=1687.4\ mm^2$

Example 2.2

For the cross-section shown in figure:
A- Determine the bending moment that the reinforced concrete section can carry if A_s=1200 mm^2.
B- Determine the maximum area of steel that can be used in this section
C- Determine the maximum moment that can be resisted by the section
f_{cu}=25 N/mm^2 and f_y=400 N/mm^2



Solution

Step1: Apply equilibrium equation T=C

Assume tension steel yields

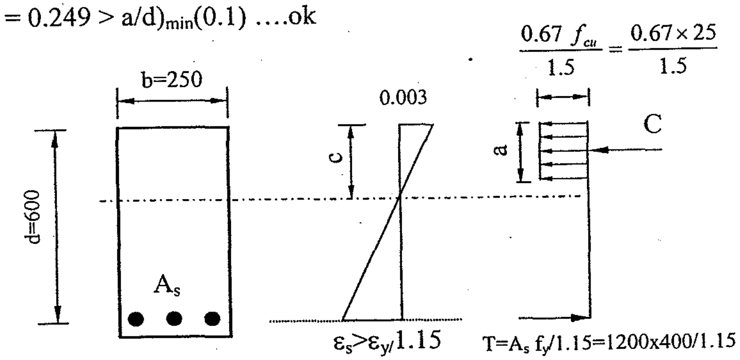
$$\frac{0.67\, f_{cu}\, b\, a}{1.5} = \frac{A_s\, f_y}{1.15}$$

$$\frac{0.67 \times 25 \times 250 \times a}{1.5} = \frac{1200 \times 400}{1.15}$$

a= 149.51 mm

$$\frac{c}{d} = \frac{a/0.80}{d} = 0.311$$

a/d = 0.249 > a/d)$_{min}$(0.1) ….ok



Stress and strain distribution in the

Step 2: Check f_s

From Table 2.1 for f_y=400N/mm^2 → c_b/d=0.63

Since c/d(0.311) < c_b/d (0.63) then $f_s=f_y/1.15$

Since c/d (0.311)<c_{max}/d (0.42) then the beam satisfies code requirements ($A_s<A_{smax}$ and $M_u < M_{umax}$) as will be shown in step 4.1 and 4.2

Step 3: Calculate bending moment M_u

$$M_u = \frac{A_s\, f_y}{1.15}\left(d - \frac{a}{2}\right)$$

$$M_u = \frac{1200 \times 400}{1.15}\left(600 - \frac{149.51}{2}\right) = 219.23 \times 10^6 = 219.23\ kN.m$$

Step 4.1: Calculate Maximum Area of steel $A_{s,max}$

From Table 2.1, μ_{max}= 4.31x10^{-4} f_{cu}

μ_{max} = 4.31x10^{-4} (25) =0.01077

$A_{s,max}$ = μ_{max} b d =0.01077 x 250 x 600

$A_{s,max}$ = 1616 mm^2

Step 4.2: Calculate Maximum Moment $M_{u,max}$

From Table 2.1, R_{max}= 0.187

$$M_{u,max} = \frac{R_{max}\, f_{cu}\, b\, d^2}{1.5} = \frac{0.187 \times 25 \times 250 \times 600^2}{1.5 \times 10^6} = 280.5\ kN.m$$

Or, alternatively

From Table 2.1, c_{max}/d =0.42

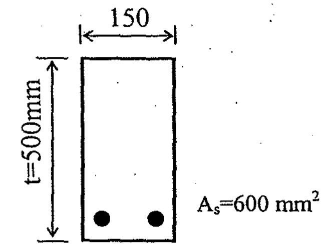
a_{max} =0.8 x 0.42x 600 = 201.6 mm

$$M_{u,max} = \frac{A_{s,max}\, f_y}{1.15}\left(d - \frac{a_{max}}{2}\right) = \frac{1616 \times 400}{1.15 \times 10^6}\left(600 - \frac{201.6}{2}\right) = 280.6\ kN.m$$

Final results M_u=219 kN.m, $A_{s,max}$=1616 mm^2 and M_{umax}=280.6 kN.m

Example 2.3

Determine whether the cross-section shown in the figure below can withstand an applied bending moment of 80 kN.m.
f_{cu}=30 N/mm^2 and f_y=240 N/mm^2



Solution

Step1: Apply equilibrium equation T=C

Assume concrete cover of 50 mm

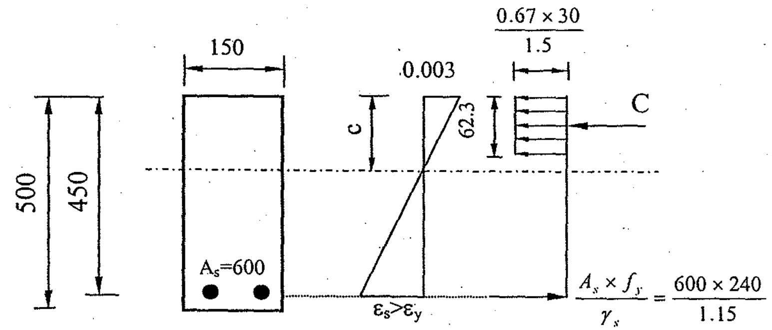
$$d = t - cover = 500 - 50 = 450\ mm$$

$$\frac{0.67\ f_{cu}\ b\ a}{1.5} = \frac{A_s\ f_y}{1.15}$$

$$\frac{0.67 \times 30 \times 150 \times a}{1.5} = \frac{600 \times 240}{1.15}$$

a= 62.30 mm c=a/0.8=77.8 mm

$a/d = 0.138 > a/d)_{min}(0.10)$ ....ok



Stress and strain distribution in the beam

Step 2: Check f_s

From Table 2 → c_b/d=0.74 and c_{max}/d=0.5

$c/d = 0.173 < c_b/d\ (0.74)$ then $f_s=f_y/1.15$

Since $c/d\ (0.173)<c_{max}/d\ (0.50)$ then the beam satisfies code requirements

Step 3: Calculate bending moment M_u

$$M_u = \frac{A_s\ f_y}{1.15}\left(d - \frac{a}{2}\right)$$

$$M_u = \frac{600 \times 240}{1.15}\left(450 - \frac{62.3}{2}\right) = 52.44 \times 10^6 = 52.44\ kN.m$$

Since M_u(52.44 kN.m) is less than the applied moment (80 kN.m), the cross-section **can not** withstand the applied moment ***unsafe).***

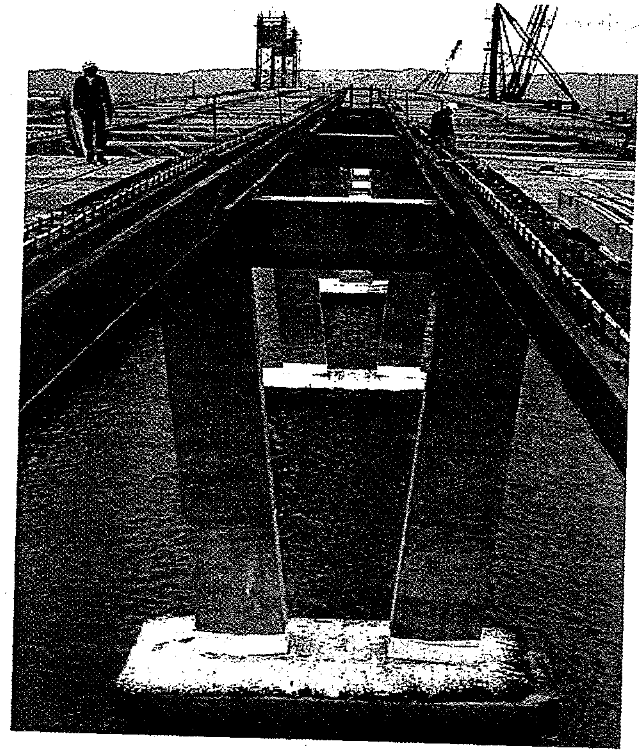
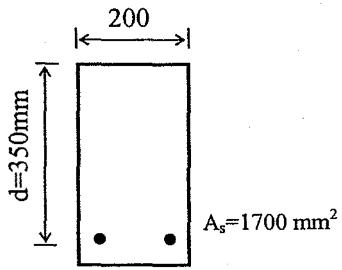


Photo 2.6 Reinforced concrete bridge during construction

Example 2.4

Calculate the maximum moment that the beam shown in figure can sustain.
Check whether the cross-section meets the code requirements regarding the maximum area of steel.
The material properties are
f_{cu}=25 N/mm^2 and f_y=400 N/mm^2



Beam cross section

Solution

Step1: Apply equilibrium equation T=C

$$\frac{0.67\ f_{cu}\ b\ a}{1.5}=\frac{A_s\ f_y}{1.15}$$

$$\frac{0.67\times 25\times 200\times a}{1.5}=\frac{1700\times 400}{1.15}$$

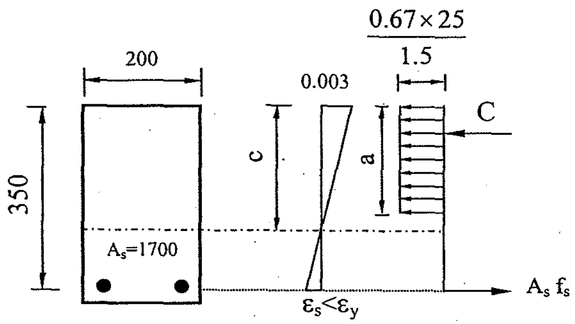
a= 264.76 mm   c=330.95 mm

a/d = 0.756 > a/d)$_{min}$(0.10) ….ok

Step 2: Check f_s

c/d = 0.94 > c_b/d (0.63) tension reinforcement ***does not yield***, we have to recalculate "a" thus fs equals

$$f_s=600\,\frac{d-c}{c}=600\,\frac{0.8\times 350-a}{a}=\frac{168000-600\times a}{a}$$



Step 3: Re-calculate a

$$\frac{0.67\ f_{cu}\ b\ a}{1.5}=A_s\ f_s$$

$$\frac{0.67\times 25\times 200\times a}{1.5}=1700\times\frac{168000-600\,a}{a}$$

The above equation is a second order equation, solving for "a" gives

a=195.94 mm   c=244.93mm   →c/d=0.69

$$f_s=600\,\frac{350-244.93}{244.93}=257.4\ N/mm^2 \ldots\left(<\frac{400}{1.15}\right)$$

Step 4: Calculate bending moment M_u

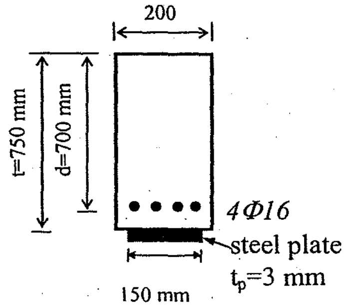
$$M_u=A_s\ f_s\left(d-\frac{a}{2}\right)$$

$$M_u=1700\times 257.4\left(350-\frac{195.94}{2}\right)=110.28\times 10^6=110.28\ kN.m$$

Note: Since the steel does not yield, the cross-section is considered over-reinforced. Thus, the cross-section ***<u>does not</u>*** meet code requirements (c/d(0.69)>c_{max}/d(0.42)).

Example 2.5

A 3 mm steel plate with a yield strength of 400 N/mm^2 is glued to a concrete beam reinforced with steel bars (*4Φ16*, f_y=360 N/mm^2) as shown in figure. Determine the bending moment that the reinforced concrete section can resist. The concrete compressive strength of the beam is 20 N/mm^2.



Solution

Step1: Apply equilibrium equation T=C

Area of the plate A_p= 3 x 150 = 450 mm^2

Area of the steel bars = 4Φ16 = 804 mm^2

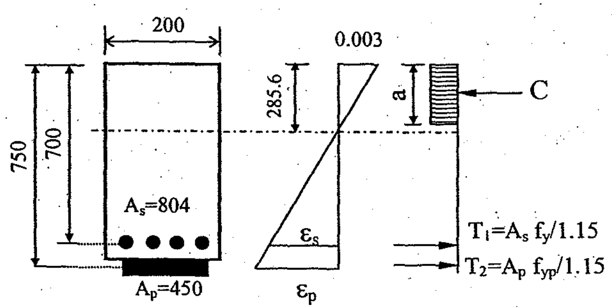
$C = T_1 + T_2$

Assume that both the plate and steel bars yield

$$\frac{0.67\, f_{cu}\, b\, a}{1.5} = \frac{A_s\, f_y}{1.15} + \frac{A_p\, f_{yp}}{1.15}$$

$$\frac{0.67 \times 20 \times 200 \times a}{1.5} = \frac{804 \times 360}{1.15} + \frac{450 \times 400}{1.15}$$

a= 228.48 mm , $c = \dfrac{a}{0.8} = \dfrac{228.480}{0.8} = 285.6\, mm$



Step 2: Check c_{max}/d

From the code c_{max}/d for (f_y=*360* N/mm^2)=0.44

c_{max} = 0.44 x 700 = 308 mm

$c<c_{max}$ ...o.k (steel yields)

The depth of the plate d_p= h+t_p/2=750+3/2=751.5 mm

The stress in the plate $f_{sp} = 600\,\dfrac{d_p - c}{c}$

$$f_{sp} = 600\frac{751.5-285.6}{285.6} = 978.78\, N/mm^2 > \frac{400}{1.15} (steel\ \ plate\ yields\ f_{sp} = 400/1.15)$$

Step 3: Calculate the ultimate moment M_u

Take the moment about the concrete force C

$$M_u = T_1\left(d - \frac{a}{2}\right) + T_2\left(d_p - \frac{a}{2}\right)$$

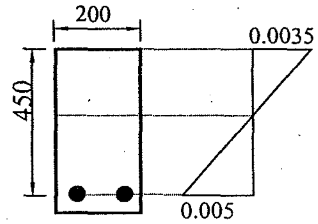
$$M_u = \frac{A_s\, f_y}{1.15}\left(d - \frac{a}{2}\right) + \frac{A_p\, f_{yp}}{1.15}\left(d_p - \frac{a}{2}\right)$$

$$M_u = \frac{804 \times 360}{1.15}\left(700 - \frac{228.48}{2}\right) + \frac{450 \times 400}{1.15}\left(751.5 - \frac{228.48}{2}\right) = 247.17\ kN.m$$

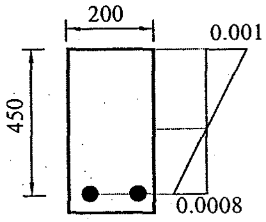
Final results M_u=247.7 kN.m

Example 2.6

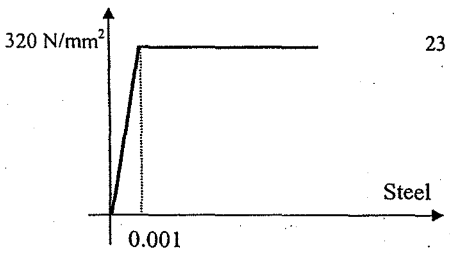
A reinforced concrete beam has a cross section of concrete dimensions b=200mm and d=450 mm. Calculate the moment capacity and the area of steel using the idealized curves for concrete & steel, without applying safety factors ($\gamma_s=\gamma_c=1$) for the strain distribution shown in cases A&B. The idealized stress-strain curve for the concrete and steel is given below.

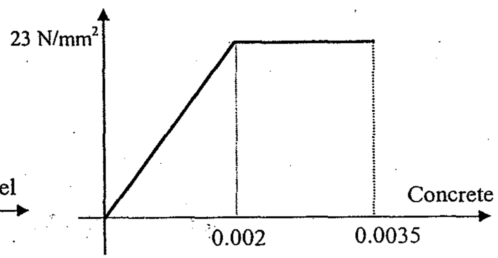


Case A Strain



Case B Strain





Solution

Case A

From the strain distribution, the neutral axis depth "c" is determined as follows:

$$\frac{c}{d}=\frac{0.0035}{0.0035+0.005}=0.411$$

c=0.411x 450= 185.29 mm

- **Force in the steel**

since $\varepsilon_s(0.005) > 0.001$ then from steel curve f_s=320 N/mm^2

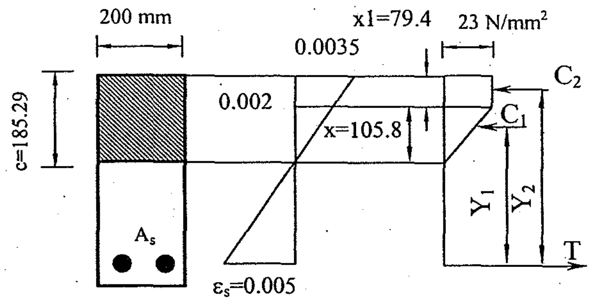
- **Force in the concrete**

The force in the concrete equals the stressed area multiplied by the width b. The concrete area can be divided into two parts as shown in the figure below

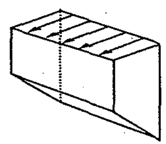
$$x=\frac{0.002}{0.0035}185.29=105.88\ mm$$

x1 =185.29-105.88 =79.41 mm

$C = C_1 + C_2$



Isometric for Stress distribution



$$C_1=\frac{23\times 105.88}{2}200=243524\,\mathrm{N}$$

$$C_2=23\times 79.41\times 200=365286\ N$$

$$A_s\ f_s=C_1+C_2$$

A_s (320) = 243524+365286

A_s=1902 mm^2

Note that the C.G. of force C_1 is at x/3

Y_1= 450-79.4-105.8/3=335.3 mm

Y_2=450-79.4/2= 410.3 mm

$M_u = C_1\ Y_1 + C_2\ Y_2$ = (243524 x 335.3 +365286 x 410.3)/10^6 =231.53 kN.m

Case B

$$\frac{c}{d}=\frac{0.001}{0.001+0.0008}=0.5556$$

c=0.5556 x 450 = 250 mm

- **Force in the steel**

since $\varepsilon_s(0.0008) < 0.001$ then find steel stress from graph

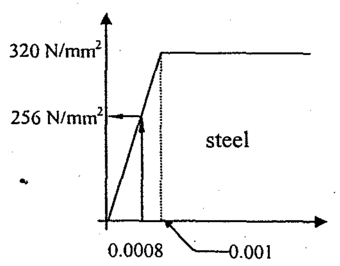
$$f_s=\frac{0.0008}{0.001}320=256\ N/mm^2$$

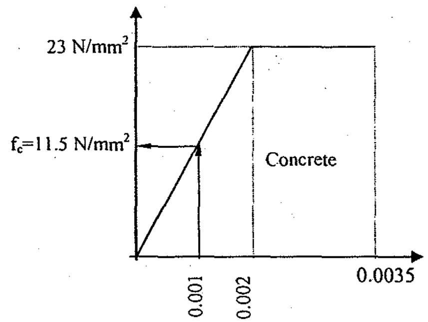
- **Force in the concrete**

The concrete force is equal to the compressed area of concrete multiplied by the width *b*.

The stress in the concrete is a triangular shape

from the concrete curve with strain=0.001 → f_c=11.5 N/mm²



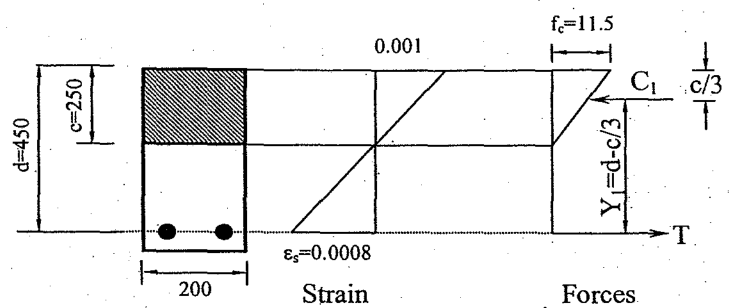


$$C_1=\frac{11.5\times 250}{2}200=287500\,\text{N} \qquad A_s\ f_s=C_1$$

A_s (256) = 287500 →→A_s=1123 mm²

Y_1= 450-250/3=366.667 mm

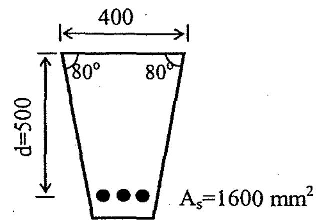
M_u= C_1 x Y_1=(287500x 366.667)/10^6 =105.41 kN.m



Example 2.7

Find the ultimate moment capacity for the cross-section shown in the figure below.

f_{cu}=30 N/mm² , and f_y=360 N/mm²



Solution

In this problem we have two unknowns a and M_u.

Step 1: Compute a.

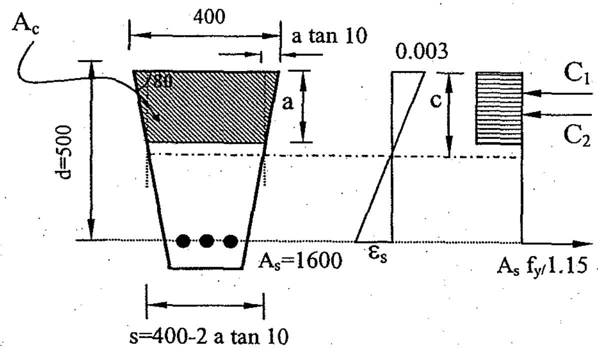
It should be noted that the code permits the use of the stress block for trapezoidal sections

The total compression force C equals to the concrete stress (0.67 f_{cu}/1.5) multiplied by the compressed area A_c. Assume that tension steel has yielded($f_s=f_y/1.15$)

$$\frac{0.67\ f_{cu}\ A_c}{1.5}=\frac{A_s\ f_y}{1.15}$$

$$\frac{0.67\times 30\times A_c}{1.5}=\frac{1600\times 360}{1.15}$$

A_c=37378 mm²



$$A_c = a\frac{400+s}{2} = a\,[400 - a\times\tan(10)]$$

$37378 = 400\ a - 0.176\ a^2$

Solving for a

a=97.65 mm

$$c = \frac{a}{0.8} = \frac{97.65}{0.8} = 122.06\ mm$$

$a/d = 0.195 > a/d)_{min}(0.1)$ ….ok

Step 2: Check f_s

Since c/d (0.244) < c_b/d (0.66), thus steel yields f_s=f_y/1.15

Since c/d (0.244)<c_{max}/d (0.44) then the beam satisfy code requirements

Step 3: Compute moment capacity, M_u

The concrete force is divided into two parts. The first is the two small triangles (C_1) and the second is rectangular C_2.

$$C_1 = 2\times\frac{1}{2}\times a\times a\ \tan(10^{\circ})\times\frac{0.67\times 30}{1.5} = 22530\ N$$

Taking moment about concrete force C2 (a/2 from the top)

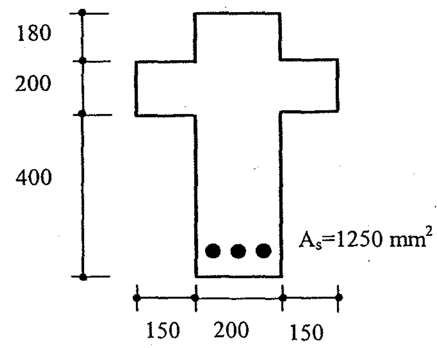
The distance between C_1 and C_2 $=\frac{79.65}{2}-\frac{79.65}{3}=\frac{79.65}{6}$

$$M_u = \frac{1600\times 360}{1.15}\left(500-\frac{97.65}{2}\right)+22530\times\frac{97.65}{6} = 226.3\times 10^6 = 226.35\ kN.m$$

Final Result: Mu=226.35 kN.m

Example 2.8

Find the ultimate moment capacity that this cross-section can resist. The material properties for the beam are
f_{cu}=20 N/mm^2, and f_y=400 N/mm^2



Solution

Step 1: Compute a.

Assume that tension steel has yielded. Since we have two unknowns M_u and (a), solving the equilibrium equations gives

$$\frac{0.67\times f_{cu}\times A_c}{1.5} = \frac{A_s\ f_y}{1.15}$$

$$\frac{0.67\times 20\times A_c}{1.5} = \frac{1250\times 400}{1.15}$$

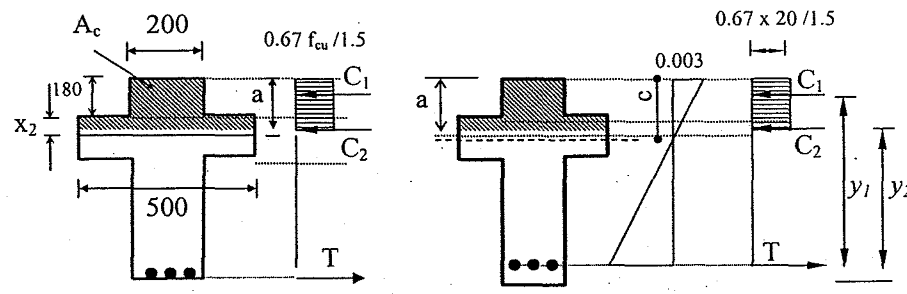
A_c=48670 mm^2

Since A_c is grater than (200 x 180), thus the distance a is bigger than 180 by the distance x_2 as follows

$$48670 = 200\times 180 + 500\ x_2$$

x_2 = 25.34 mm

a=180+ x_2 =205.34



Step 2: Check steel yield stress, f_s

c= a/0.8 =256.675 mm

d=180+200+400-50(cover)=730 mm

Since $c/d(0.35)<c_b/d\ (0.63)$, $f_s=f_y/1.15$

Since $c/d(0.35)<c_{max}/d\ (0.42)$ then the beam satisfy the code requirements

Step 3: Compute moment capacity

Taking moment about the tension force

$$M_u = C_1(y_1) + C_2(y_2)$$

$$C_1 = \frac{0.67\times 20}{1.5}\,\frac{200\times 180}{1000} = 321.60\ kN \qquad \rightarrow y_1 = 730 - \frac{180}{2} = 640\ mm$$

$$C_2 = \frac{0.67\times 20}{1.5}\,\frac{500\times 25.34}{1000} = 113.2\ KN \qquad \rightarrow y_2 = 730 - 180 - \frac{25.34}{2} = 537.33\ mm$$

$$M_u = 321.6\times\frac{640}{100} + 113.2\times\frac{537.33}{1000} = 266.65\ kN.m$$

Final Result: Mu=266.64 kN.m

2.9 Design of Singly Reinforced Sections by First Principles

To design a reinforced concrete section, the applied factored moment, concrete strength and steel yield strength must be given. It is then required to calculate the cross section unknowns including b, d and A_s.

If the beam supports a wall then its width is usually chosen equal to the wall width (either 120 mm or 250 mm). The width of the beams that do not support walls may be reasonably assumed to meet architectural requirements. The assumption of the beam width leaves the designer with two unknowns (d, A_s). In spite of having two equilibrium equations, one can not get these two unknowns. This is due to the fact that the stress block depth (a) is also an unknown.

Two alternative procedures can be followed:

1. The thickness of the beam is assumed as a function of the span as will be discussed in Chapter 6 in order to satisfy serviceability requirements such as deflection (span/10). This procedure is usually followed by practicing engineers. Apply the two equilibrium equations to obtain the remaining unknowns (a, As)
2. The area of steel A_s can be assumed. A reasonable assumption for such an area can be obtained by assuming that the lever arm equals to *0.8d*. Since concrete compressive strength has a limited effect on the ultimate capacity, a further simplification can be attained by assuming that f_{cu}=25 N/mm^2. Solving Eq. 2.9.A and 2.11.A for the area of steel A_s, one can get

$$A_s = 0.11\sqrt{\frac{M_u\, b}{f_y}} \qquad \text{.......................................(2.31)}$$

The assumed A_s is approximately 0.9-1% of the cross sectional area (μ=0.009-0.01). After assuming the area of steel, one can apply the two equilibrium equations to calculate the remaining unknowns (a, d). The procedure for using this approach is illustrated in example 2.10.

Procedure

- Step1: Make the necessary assumptions to keep only two unknowns
  - Assume the beam depth (d) or assume μ=0.01.
  - Or, assume area of steel, $A_s = (0.1-0.11)\sqrt{\frac{M_u\,b}{f_y}}$
  - If "b" is not given (assume b=120, 200, or 250 mm)
- Step2: Apply equilibrium equation T=C to find the depth of the stress block, "a" (Eq.2.9)
- Step3: Take the moment about the concrete force and calculate the area of steel or the beam depth (Eq. 2.11)
- Step 4: Check minimum area of steel A_{smin} (Eq. 2.30)
- Step 5: Check the code limits $M_{u,max}$, c_{max}/d, $A_{s,max}$(Eq. 2.21)

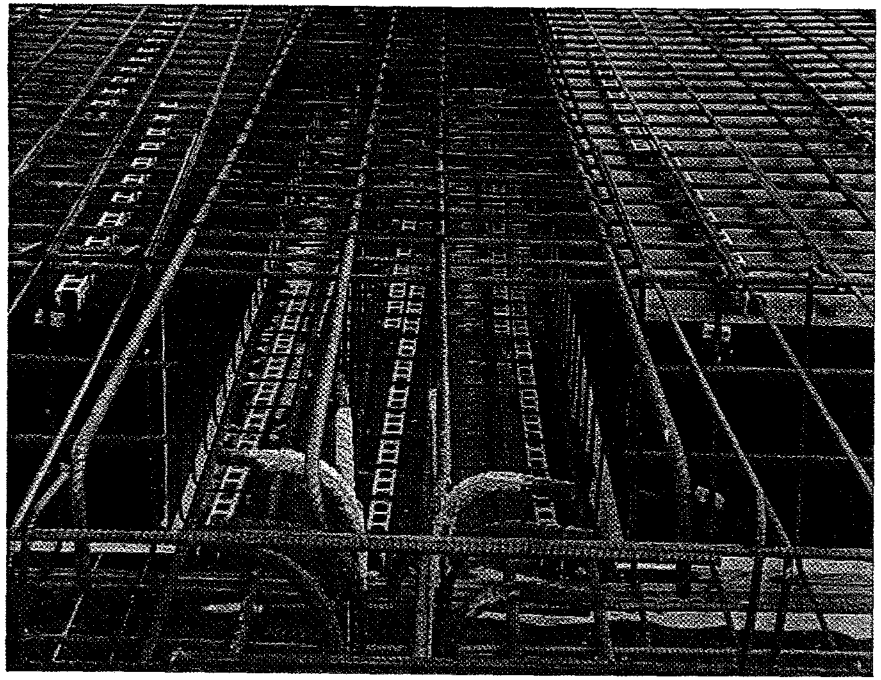


Photo 2.7 Reinforcement placement in a slab-beam roof

Example 2.9

A singly reinforced concrete beam with a width of 250 mm is subjected to an ultimate moment of 270 kN.m. Find the beam depth and area of steel, then Calculate A_{smax} and M_{umax}. f_{cu}=30 N/mm^2 and f_y=400 N/mm^2

Solution

In this example we have three unknowns a, d, As and we have only two equilibrium equations, thus we have to assume one of the unknowns.

Step 1: Assumptions

$\mu_{max} = 4.31\times10^{-4}\times30 = 0.0129$

Assume $\mu<\mu_{max}$ $\rightarrow\rightarrow$Assume μ=0.01

$A_s = \mu\, b\, d = 0.01 \times 250 \times d = 2.5\, d$ ....(1)

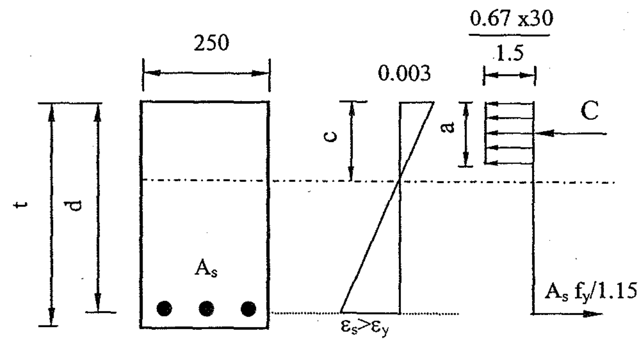
Step 2: Calculate a

$$\frac{0.67\, f_{cu}\, b\, a}{1.5} = \frac{A_s\, f_y}{1.15}$$

$$\frac{0.67\times30\times250\times a}{1.5} = \frac{2.5\, d\times400}{1.15}$$

a=0.2596 d

a/d = 0.2596 > a/d)$_{min}$(0.1) ....ok



Calculation of As, d

Step 3: Calculate d

$$M_u = \frac{A_s\, f_y}{1.15}\left(d-\frac{a}{2}\right)$$

$$270\times10^6=\frac{2.5\,d\,400}{1.15}\left(d-\frac{0.2596\,d}{2}\right) \quad \to\to d=597\ mm$$

From Eq. (1)............ $A_s = 2.5\ d = 1493\ mm^2$

Rounding d to the nearest 50mm, d=600 mm and A_s= 4ϕ22 (1520 mm^2)

Step 4: Check A_{smin}

$$A_{s\,min} = smaller\ \ of \begin{cases} \frac{0.225\sqrt{30}}{400}250\times600=460\ mm^2 \\ 1.3\times1493=1940\ mm^2 \end{cases} = 460\ mm^2 < A_s \cdots o.k$$

Step 5: Check A_{smax}, M_{umax}

From the code μ_{max}= $4.31\text{x}10^{-4}$ f_{cu} c_{max}/d =0.42 and R_{max}=0.187

μ_{max} = $4.31\text{x}10^{-4}$ (30) =0.01293

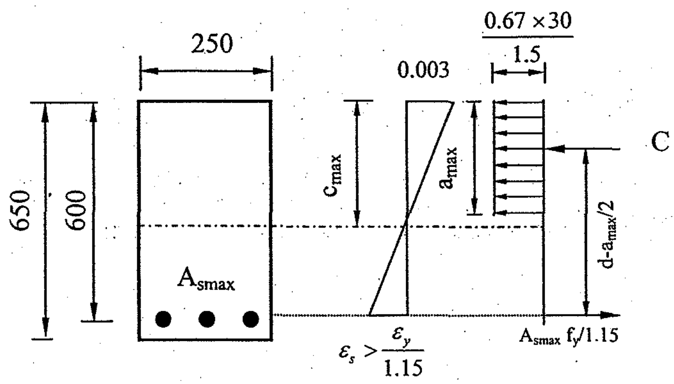
a_{max} =0.8 x 0.42x 600 = 201.6 mm

$A_{s,max}$ = μ_{max} b d =0.0129 x 250 x 600 =1939 mm^2 > A_s.......o.k

$$M_{u,max}=\frac{A_{s,max}\ f_y}{1.15}\left(d-\frac{a_{max}}{2}\right)=\frac{1939\times400}{1.15\times10^6}\left(600-\frac{201.6}{2}\right)=337\ kN.m$$

OR

$$M_{u,max}=\frac{R_{max}\ f_{cu}\ b\ d^2}{1.5}=\frac{0.187\times30\times250\times600^2}{1.5\times10^6}=337\ kN.m$$



Calculation of M_{umax}

Example 2.10

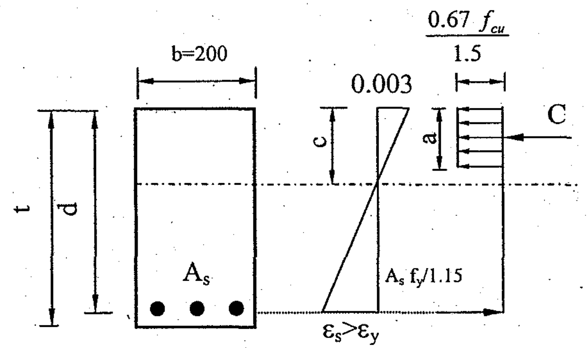
A singly reinforced concrete beam is subjected to an ultimate moment of 330 kN.m. Find the beam depth and area of steel.
The material properties are:
f_{cu}=25 N/mm^2 and f_y=280 N/mm^2

Solution

Step 1: Assumptions

In this example we have four unknowns b, a, d, A_s and we have only two equilibrium equations, thus we shall assume two (b, A_s)
Assume b=200 mm

$$A_s = 0.11\sqrt{\frac{M_u\ b}{f_y}} = 0.11\sqrt{\frac{330\times10^6\times200}{280}} = 1689\ mm^2$$



Step 2: Calculate a

$$\frac{0.67\ f_{cu}\ b\ a}{1.5}=\frac{A_s\ f_y}{1.15}$$

$$\frac{0.67\times25\times200\times a}{1.5}=\frac{1689\times280}{1.15}$$

a=184.12 mm

Step 3: Calculate d

$$M_u = \frac{A_s\, f_y}{1.15}\left(d - \frac{a}{2}\right)$$

$$330\times10^6 = \frac{1689\times 280}{1.15}\left(d - \frac{184.12}{2}\right) \rightarrow \text{d=895 mm}$$

a/d = 0.205 > a/d)$_{min}$(0.1) ….ok

Use d=900 mm t=950 mm and A_s= 1689 mm^2

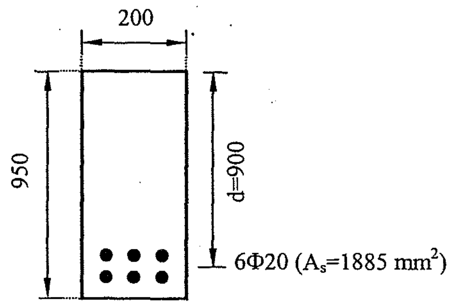
Step 4: Check As_{min}

$$A_{s\,min} = smaller\ \ of \begin{cases} \frac{0.225\sqrt{25}}{280}\,200\times 900 = 719\ mm^2 \\ \\ 1.3\times1689 = 2195\ mm^2 \end{cases} \qquad = 719\ mm^2 < A_s \cdots o.k$$

Step 5: Check A_{smax}, M_{umax}

Since c/d(0.26) < c_{max}/d (0.48) then A_s<A_{smax} and M_u<M_{umax}

$\boxed{\textbf{Final design d=900 mm, t=950 mm and } A_s\textbf{=1689 mm}^2}$



Final design

Note

1. The reinforcement is arranged in two rows.
2. The depth of the beam is measured from the c.g of the reinforcement.

2.10 Design of Singly Reinforced Sections Using Curves

Design aids are very useful tools in designing reinforced concrete sections. To prepare the design aids, equilibrium equations and compatibility of strains are utilized. There are several charts that can be used in the design process. We shall present several design charts followed by design examples to explain how to use such design aids.

2.10.1 Design Charts (R-μ)

Applying the equilibrium equation for the forces shown in Fig. 2.13

$$\frac{0.67\, f_{cu}\, b\, a}{1.5} = \frac{A_s\, f_y}{1.15}$$

Dividing by (b x d) and noting that μ=A_s/(b .d)

$$\frac{a}{d} = \frac{1.5\times\mu\times f_y}{0.67\times1.15\times f_{cu}} = 1.9468\,\frac{\mu\times f_y}{f_{cu}} \quad\text{.......(2.32)}$$

Taking moments about the concrete force C

$$M_u = \frac{A_s\, f_y}{1.15}\left(d - \frac{a}{2}\right) \quad\text{.......(2.33)}$$

Dividing Eq. 2.33 by b x d^2 gives

$$\frac{M_u}{b\,d^2} = \frac{\mu\, f_y}{1.15}\left(1 - \frac{1}{2}\frac{a}{d}\right) \quad\text{.......(2.34)}$$

Substituting with Eq. 2.32 in Eq. 2.34 gives

$$\boxed{R_u = \frac{M_u}{b\,d^2} = \frac{\mu\, f_y}{1.15}\left(1 - 0.9734\,\frac{\mu\times f_y}{f_{cu}}\right)} \quad\text{.......(2.35)}$$

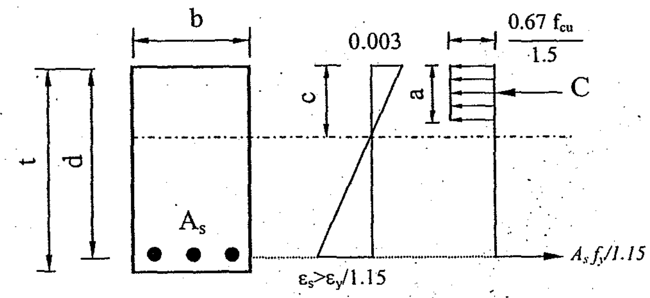


Fig. 2.13 Equilibrium of forces in rectangular sections

Substituting different values of μ in the Eq. 2.35, the relation between R_u, μ can be established. Fig. 2.14 shows an example of such curves. Appendix A contains R-μ design charts.

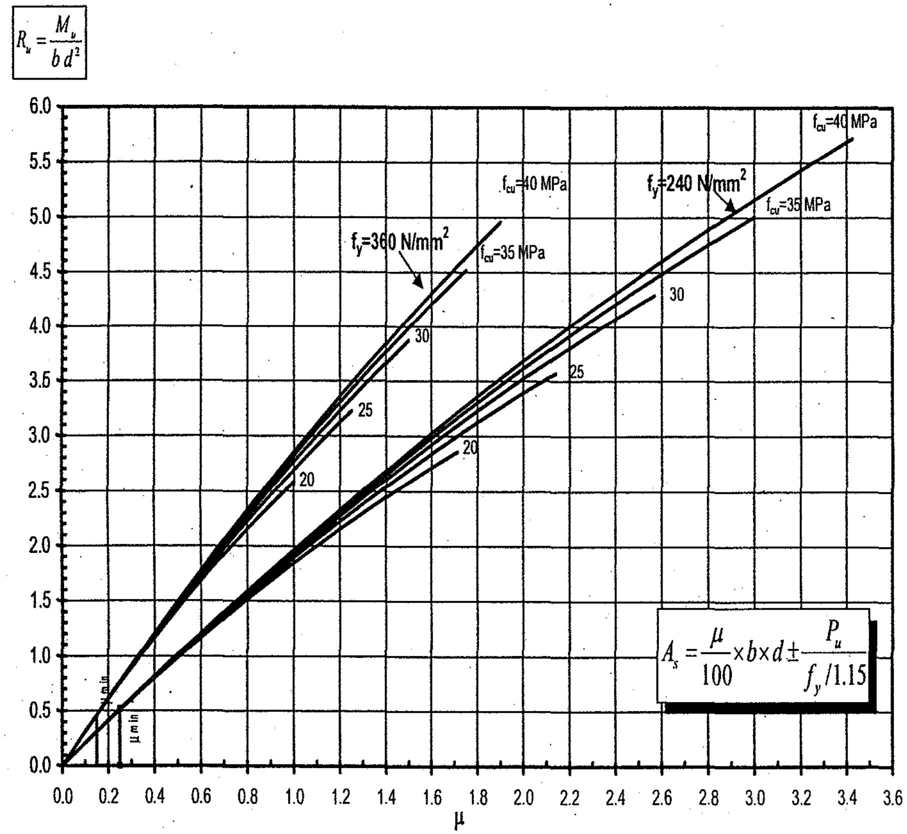


Fig. 2.14 Example of R_u-μ design curve

2.10.2 Design Chart (R-ω)

Defining ω as

$$\omega = \mu \frac{f_y}{f_{cu}}$$

Subsisting the value of ω in Eq. 2.32 gives

$$\frac{a}{d} = 1.9468\,\omega$$

Dividing Eq. 2.35 by f_{cu} gives

$$R1 = \frac{M_u}{f_{cu}\, b\, d^2} = \frac{\mu\, f_y}{1.15\, f_{cu}}(1 - 0.9734\,\frac{\mu\, f_y}{f_{cu}}) \quad \text{.......................} (2.37)$$

$$\boxed{R1 = \frac{\omega}{1.15}(1 - 0.9734\,\omega)} \quad \text{..................................} (2.38)$$

Substituting different values of ω in the Eq. 2.38, the relation between R1, ω can be established. The curve should be terminated at the value of ω_{max} listed in Table 2.1 and Eq. 2.23. Fig. 2.15 shows an example of such curves. Appendix A contains R1-ω design chart.

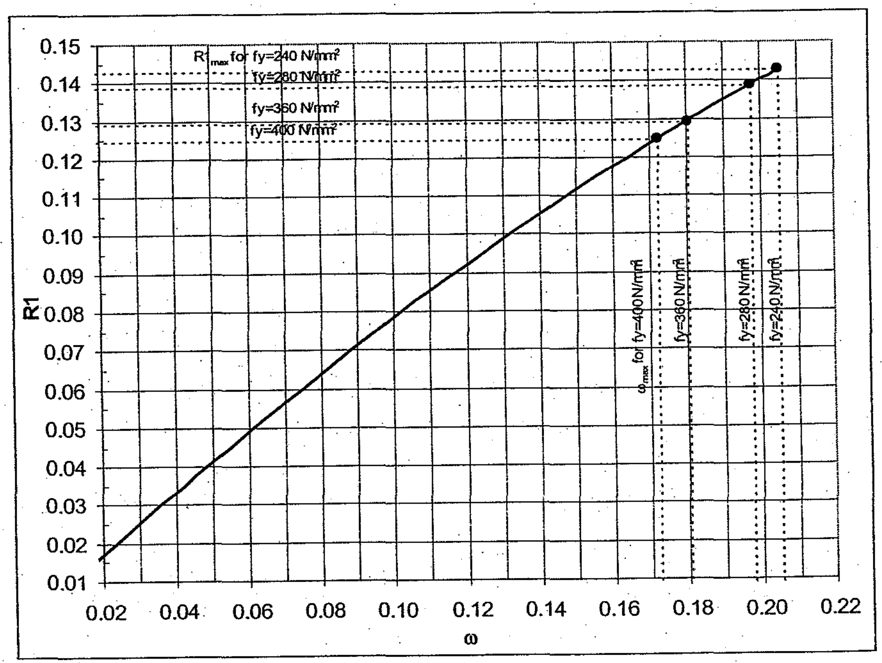


Fig. 2.15 Example of R1-ω design curve

Summary

The following table illustrates the use of the charts depending on the example information:

d is given, A_s required

1. Calculate R1 or R_u or K_u
2. Use charts (R-ω or R-μ or K_u-μ) to determine μ or ω
3. Calculate A_s
4. Check A_{smin} and A_{smax}

A_s is given, d required

1. Calculate μ or ω
2. Use the charts to determine R1 or R_u or K_u
3. Calculate *d*
4. Check A_{smin} and A_{smax}

A_s, d required

1. Assume $R1=1/2R_{max}$ ($R\approx0.07$)
2. Use the charts to determine ω
3. Calculate d, A_s

$$d^2=\frac{M_u}{f_{cu}\cdot b\ R1} \qquad A_s=\omega\, b\ d\,\frac{f_{cu}}{f}$$

4. Check A_{smin} and A_{smax}

Note 1: Each curve terminates at the value of the maximum reinforcement ratio μ_{max} or ω_{max}. Thus, there is ***no need*** to check the maximum moment or the maximum area of steel as long as the point is less than the maximum limit.

Note 2: It should be noted that beam depth needs to be increased if the point is located outside the curve as shown in Fig. 2.16.

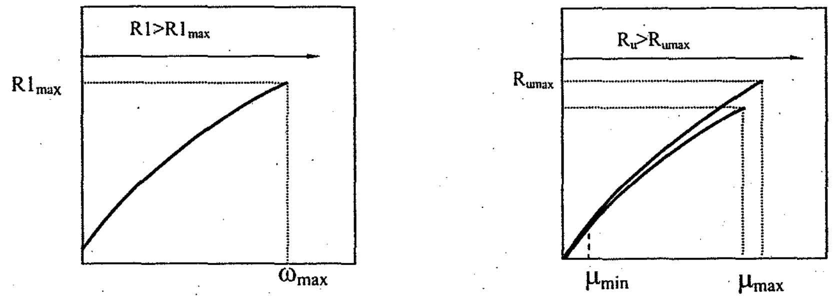


Fig. 2.16 Cases where the beam depth need to be increased

Note 3: For small values of R1(<0.04), ω can be approximated by ω=1.2 R1.

Note 4: If the both (A_s and d) are not given, consider assuming μ=0.008-0.01 and proceed as the previous procedure

Note 5: It should be noted that we have to check A_{smin} using Eq. 2.30 even if $\mu>\mu_{min}$ on the curve, because the curve tests only the value of $1.1/f_y$

Note 6: Since sometimes the beam depth is not known, a reasonable estimation for "d" can be concluded by assuming a=0.1 d and μ=0.01 and substituting in Eq. 2.34. gives:

$$d=11\sqrt{\frac{M_u}{b\,f_y}} \qquad (2.36)$$

Note 7: The design curves can be presented in a tabular form (R_u-μ) or (R_u-K_u) as given in appendix A

Photo 2.8 Trammell Crow Center (209m ,50 stories)

Example 2.11

A reinforced concrete cross-section is subjected to a bending moment of a factored value of 400 kN.m. The beam has a width of 200mm. It is required to design the cross section using the (R1-ω) curve, knowing that f_{cu}=30 N/mm^2 and f_y=280 N/mm^2 .

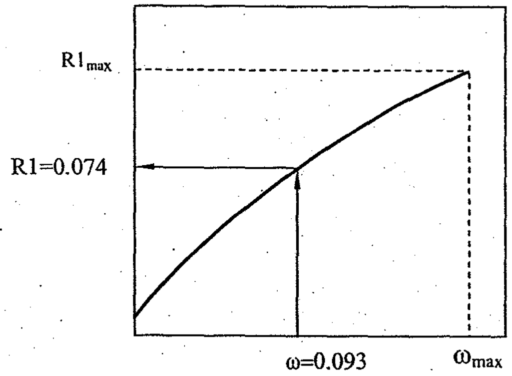
Solution

Step 1: Assume μ and get R1

Since both (A_s and d) are not given, then →→ assume μ=0.01

$$\omega = \mu \frac{f_y}{f_{cu}} = 0.01 \frac{280}{30} = 0.0933$$

From the chart with ω=0.0933, R1=0.074



Step 2: Compute d, A_s

$$R1 = \frac{M_u}{f_{cu}\ b\ d^2}$$

$$0.074 = \frac{400 \times 10^6}{30 \times 200 \times d^2}$$

d=949 mm

$$A_s = 0.0933 \times 200 \times 949 \frac{30}{280} = 1899\ mm^2$$

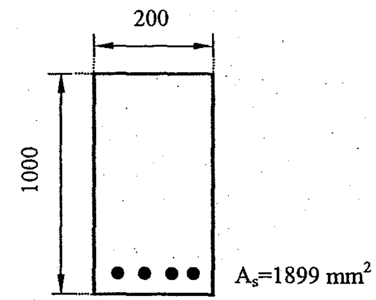
Take d=950 mm, t=1000mm

Step 3: Check A_{smin}, A_{smax}

$$A_{s\,min} = smaller\ \ of \begin{cases} \dfrac{0.225\sqrt{f_{cu}}}{f_y} bd = \dfrac{0.225\sqrt{30}}{280} \times 200 \times 950 = 836 \lrcorner \\ 1.3A_s = 1.3 \times 1899 = 2469 \end{cases} = 836\ mm^2 < A_s ....o.k$$

Since $R_1 < R_{1,max}$ thus $A_s < A_{smax}$ ....o.k.

$\boxed{\textbf{Final Result: d=950 mm, t=1000 mm, } A_s\textbf{=1899 mm}^2}$



Final design

Example 2.12

A reinforced concrete cross-section is subjected to a bending moment of a factored value of 350 kN.m. The beam has a width of 250mm. It is required to design the cross section using the (Ru-μ) curve, knowing that f_{cu}=25 N/mm^2 and f_y=400 N/mm^2.

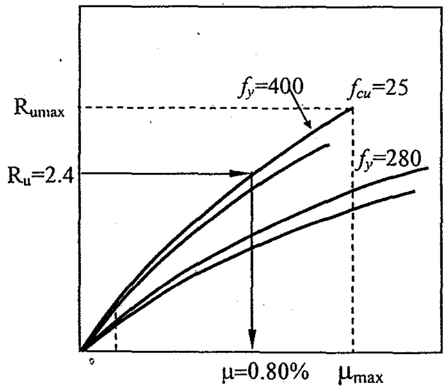
Solution

Step 1: Assume μ and get R_u

Since both (A_s and d) is not given, then,

Assume R=1/2 R_{max} →→→ R_u=2.4

From the curve μ= 0.8% (0.008)



Step 2: Compute d, A_s

$$R_u=\frac{M_u}{b\,d^2}$$

$$2.4=\frac{350\times10^6}{250\times d^2} \qquad \rightarrow\rightarrow d=763\ mm$$

$$A_s=\mu\,b\,d=0.008\times250\times763=1526\ mm^2=15.26\ cm^2$$

Take d=800 mm, t=850 mm

Note: it is more economical to use the calculated depth (763 mm) not the chosen depth (800 mm) to compute A_s

Step 3: Check A_{smin}, A_{smax}

$$A_{s\min}=smaller\ \ of\begin{cases}\dfrac{0.225\sqrt{f_{cu}}}{f_y}bd=\dfrac{0.225\sqrt{25}}{400}\times250\times800=563\\ 1.3A_s=1.3\times1526=1983\end{cases}=563\ mm^2<A_s.........o.k$$

Since $\mu<\mu_{max}$, thus $A_s<A_{smax}$.........o.k

Example 2.13

A reinforced concrete cross-section is subjected to a bending moment of a factored value of 290 kN.m. The beam has a width of 150mm. It is required to design the cross section using the (R_u-k_u) table, knowing that the material properties are f_{cu}=40 N/mm^2 and f_y=360 N/mm^2.

Solution

Step1: Assume μ and get K_u

Assume μ= 0.8% (0.008)

From the table (R_u-k_u) with f_y=360 N/mm^2 determine K_u=0.655

Step 2: Compute d, A_s

$$d=K_u\sqrt{\frac{M_u}{b}}=0.655\sqrt{\frac{290\times10^6}{150}}=910.74\ mm$$

$$A_s=\mu\,b\,d=0.008\times150\times910.74=1093\ mm^2=10.93\ cm^2$$

d=950 mm t=1000 mm

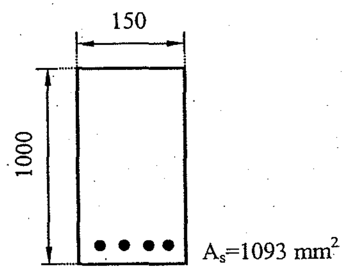
Note: use the calculated depth (910 mm) to calculate A_s

Step 3: Check A_{smin}, A_{smax}

$$A_{s\min}=smaller\ \ of\begin{cases}\dfrac{0.225\sqrt{f_{cu}}}{f_y}bd=\dfrac{0.225\sqrt{f_{cu}}}{360}\times150\times950=563\\ 1.3A_s=1.3\times1093=1421\end{cases}=563\ cm^2<A_s.........o.k$$

Since $\mu<\mu_{max}$, thus $A_s<A_{smax}$.........o.k

Final Result: t=1000 mm, A_s=1093 mm^2



Final design

Example 2.14

Redesign the beam in example 2.9 using the design aids (R_u-μ) and assuming that d=600 mm

Solution

Step 1: Estimate A_s

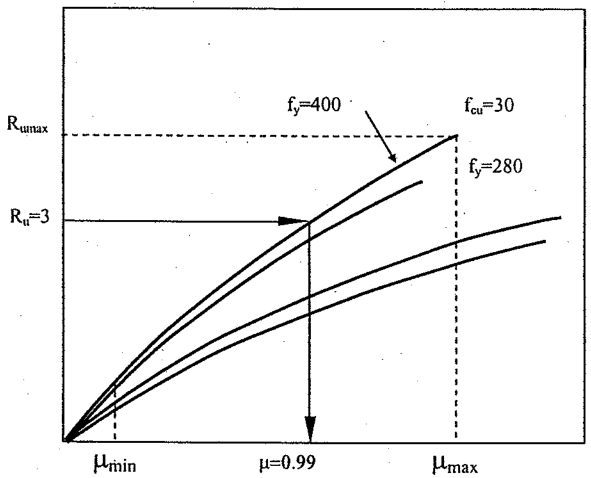
Calculate R_u

$$R_u = \frac{M_u}{b\,d^2} = \frac{270\times10^6}{250\times600^2} = 3$$

From the chart (f_y=400 N/mm^2 and f_{cu} =30 N/mm^2)with R_u=3→μ=0.99

$$A_s = \mu\, b\, d = \frac{0.99}{100}\,250\times600 = 1485\ mm^2$$

Compare the previous value with A_s obtained from example 2.9 (1493 mm^2)



Step 2: Minimum and Maximum, A_s

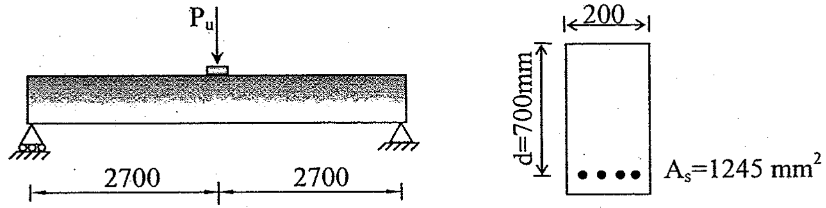
Since R_u is less than the R_{umax} value in the curve, thus A_s<A_{smax}

$$A_{s\,min} = smaller\ \ of \begin{cases} \dfrac{0.225\sqrt{f_{cu}}}{f_y}bd = \dfrac{0.225\sqrt{30}}{400}\times250\times600 = 462 \\ 1.3A_s = 1.3\times1485 = 1930 \end{cases} = 462\ mm^2 < A_s ....o.k$$

Example 2.15

Determine the value of the ultimate load (P_u) that can be applied to the beam shown in the figure using design aids.

f_{cu}=25 N/mm^2 and f_y=360 N/mm^2 (Neglect own weight of the beam)



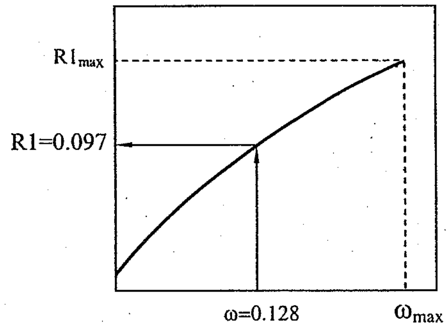
Solution

$$A_s = \omega\,\frac{f_{cu}}{f_y}\,b\,d$$

$$1245 = \omega\,\frac{25}{360}\,200\times700$$

ω=0.128

From the chart (R1-ω) with ω=0.128 get R1=0.097



$$R1 = \frac{M_u}{f_{cu}\,b\,d^2}$$

$$0.097 = \frac{M_u\times10^6}{25\times200\times700^2}$$

M_u= 237.65 kN.m.

The maximum moment occurs at midspan and equals to $M_u = \dfrac{P_u\times L}{4}$.

$$237.65 = \frac{P_u\times5.4}{4}$$

P_u=176.04 k.N.

3

DOUBLY REINFORCED BEAMS AND T-BEAMS

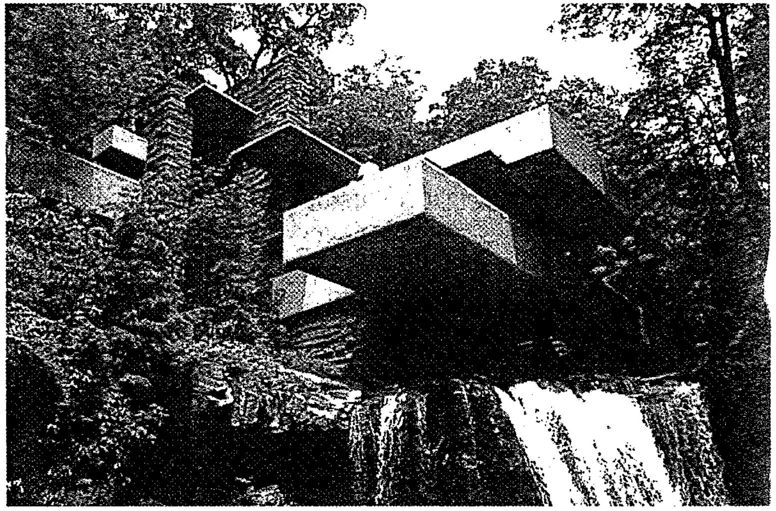


Photo 3.1 Edgar J. Kaufmann House (Falling-water), Frank Lloyd Wright 1936, (6 m cantilever)

3.1 Doubly Reinforced Sections

3.1.1 Introduction

Doubly reinforced sections are those that include both tension and compression steel reinforcement. In most cases, they become necessary when architectural requirements restrict the beam depth.

From the economic point of view, it is recommended to design the member as a singly reinforced section with tension reinforcement only. If the required area of the tension steel exceeds the maximum area of steel recommended by the code,

compression steel should be added. Adding compression steel reinforcement may change the mode of failure from compression failure to tension failure or may change the section status from over-reinforced to under-reinforced section. Compression steel also reduces long-term deflection and increases beam ductility. For economic considerations, the Egyptian code recommends limiting the compression reinforcement amount to only 40% percent of the tension steel.

The compression area of steel A'_s is usually expressed as a ratio from the tension area of steel A_s as follows:

$$A'_s = \alpha\ A_s \quad \text{...}(3.1)$$

where α usually ranges from 0.1 to 0.4.

Despite all of the aforementioned benefits, adding compression steel in reinforced concrete beams **will not** increase the section moment capacity significantly. This is because the tension force is constant ($T = A_s f_y/1.15$) and the lever arm between the tension force and the resultant of the two compressive forces (in concrete and in steel) is slightly affected by adding compression reinforcement. This can be noticed by examining Fig. 3.1. In this figure, the vertical axis gives the percentage increase in the capacity of a doubly reinforced section as compared to an identical one without compression steel.
The use of compression steel is more beneficial when the tension steel provided is near the maximum allowed percentage of steel μ_{max}. Adding compression reinforcement with α=0.6 will increase the beam capacity by 6 to 13 percent for beams with μ=0.8% and $\mu = \mu_{max}$ respectively.

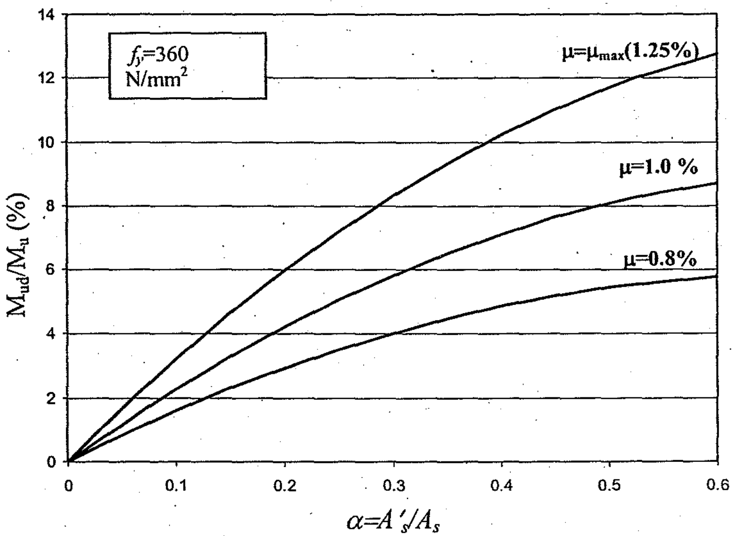


Fig. 3.1 Effect of compression reinforcement on moment capacity

3.1.2 Analysis of Doubly Reinforced Sections

Case A: compression steel yields

The equilibrium of forces and strain compatibility shall be applied to analyze the section. The strain distributions and the internal forces in beams with compression reinforcement are shown in Fig. 3.2. The compressive force is the sum of two parts; i) concrete force C_c, and ii) compression steel force C_s.

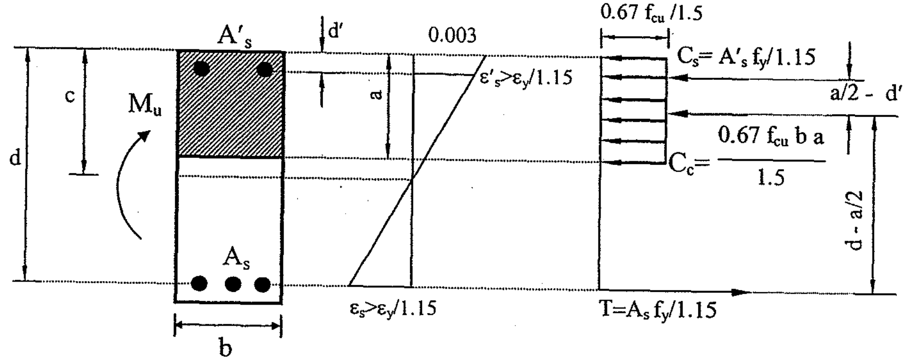


Fig. 3.2 Analysis of sections with compression reinforcement (steel yields)

It will be assumed that both the compression and the tension steel have yielded. The stress block distance "a" is calculated utilizing the equilibrium of forces as follows

$$\text{Compression force} = \text{Tension force} \quad \text{.............................}(3.2)$$

$$C_c + C_s = T \quad \text{...}(3.3)$$

$$\frac{0.67\ f_{cu}\ b\ a}{1.5} + \frac{A'_s\ f_y}{1.15} = \frac{A_s\ f_y}{1.15} \quad \text{.................................}(3.4)$$

The compression steel stress f'_s is checked using compatibility of strains as follows:

$$\varepsilon'_s = 0.003\ \frac{c-d'}{c} = 0.003\frac{a-0.8\ d'}{a} \quad \text{...........................}(3.5)$$

$$f_s' = \varepsilon'_s \times E_s = 600\ \frac{c-d'}{c} = 600\ \frac{a-0.8\,d'}{a} \quad \text{.............................}(3.6)$$

If the value of f_s' in Eq. 3.6 is less than $f_y/1.15$, the analysis should be carried out according to the procedure outlined in case B. On the other hand, if f_s' in Eq. 3.6 is larger than $f_y/$ 1.15, the assumption of yielding of compression steel is valid and the moment capacity can be determined by taking the moment of the forces around the concrete force as follows:

$$M_u = \frac{A_s\ f_y}{1.15}\left(d-\frac{a}{2}\right) + \frac{A'_s\ f_y}{1.15}\left(\frac{a}{2}-d'\right) \quad \text{.............................}(3.7)$$

Case B: compression steel does not yield

Applying the equilibrium condition and referring to Fig. 3.3, one gets:

$$\frac{0.67\, f_{cu}\, b\, a}{1.5} + A'_s\, f'_s = \frac{A_s\, f_y}{1.15} \quad \text{.........(3.8)}$$

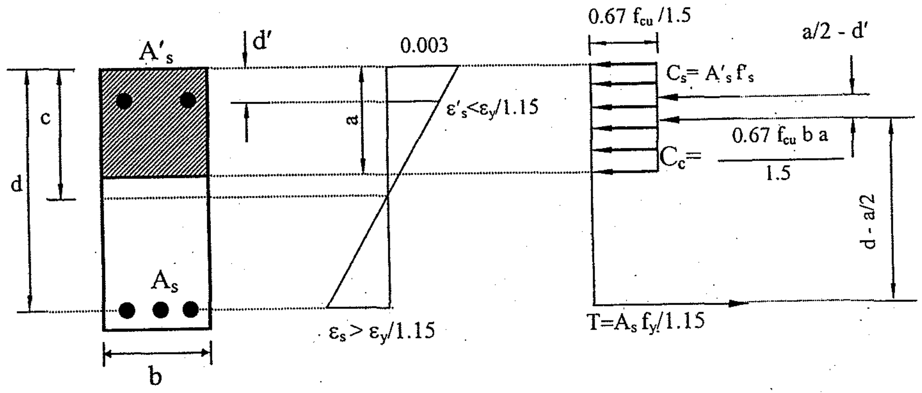


Fig. 3.3 Analysis of sections with compression reinforcement (steel does not yield)

Substituting the expression of f'_s from Eq. 3.6 into Eq. 3.8 gives:

$$0.4466 f_{cu}\, b\, a + A'_s\, 600\, \frac{(a - 0.8d')}{a} = \frac{A_s\, f_y}{1.15} \quad \text{.........(3.9)}$$

This can be reduced to a second order equation in terms of the stress block distance (a), given by

$$0.4466\, f_{cu}\, b\, a^2 - (A_s\, f_y / 1.15 - 600 \times A'_s)\, a - 480 A'_s\, d' = 0 \quad \text{.........(3.10)}$$

Solution of Eq. 3.10 gives the value of "a". The moment capacity can be determined by taking the moment around the concrete force as follows:

$$M_n = \frac{A_s\, f_y}{1.15}\left(d - \frac{a}{2}\right) + A'_s\, f'_s \left(\frac{a}{2} - d'\right) \quad \text{.........(3.11)}$$

Note that the positive sign indicates that the compression steel force C_s is assumed to be located above the concrete compression force C_c.

Simplified Approach for the Analysis of Doubly Reinforced Sections

The previous procedure indicates that the compression steel strain ε'_s is affected by the distance d' (refer to Fig. 3.3). The compression steel yields if the distance d' is small compared to the neutral axis distance as presented by Eq. 3.6.

Setting $f'_s = f_y/1.15$, one can solve Eq. 3.6 for the maximum d' that ensures yielding of the compression reinforcement.

$$\frac{d'_{max}}{a} = 1.25\left(1 - \frac{f_y}{690}\right) \quad \text{.........(3.12)}$$

If the value of the actual d'/a is less than the value d'_{max}/a, the compression steel will yield and f'_s equals to $f_y/1.15$. Table 3.1 lists the values of d'_{max}/a that ensures yielding.

The ECP 203 presents a simplified approach for such an analysis. It permits assuming that the compression steel yields if the ratio of the compression steel depth d' to tension steel depth d is less than the values given in Table 3.1, otherwise a compatibility of strains (Eq.3.6) has to be utilized.

Table 3.1 Values of (d') to ensure yielding of compression steel

f_y(N/mm^2)	240	280	360	400
d'/d at $c<c_{max}$(code values)	≤ 0.20	≤ 0.20	≤ 0.15	≤ 0.10
d'/d at $c=c_{max}$ (max. values)	≤ 0.326	≤ 0.285	≤ 0.210	≤ 0.176
d'/a at $c\le c_{max}$	≤ 0.815	≤ 0.74	≤ 0.60	≤ 0.525

The simplified approach for the analysis of a doubly reinforced section can be summarized in the following steps

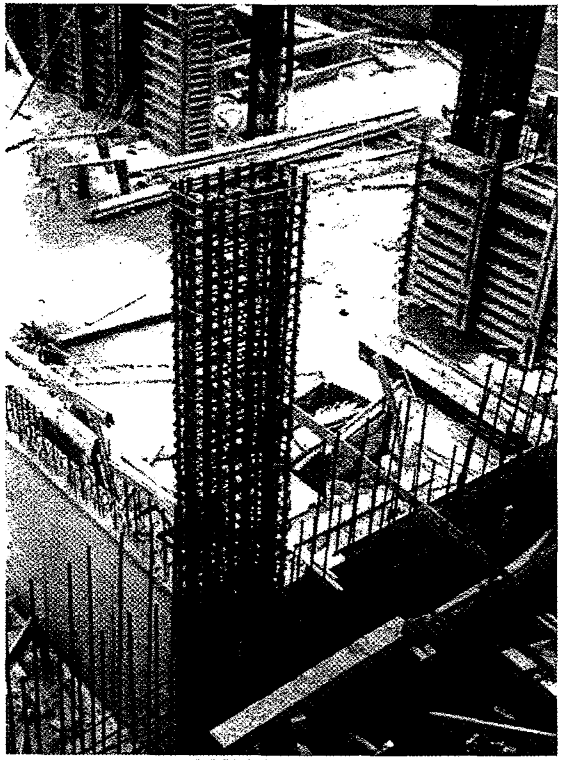
Given : f_{cu}, f_y, b, d', d, A_s and A'_s
Required : M_u

Case A: check if d'/d< code limits (Table 3.1), then compression steel yields.

- **Step 1** calculate "a" using Eq. 3.4.
- **Step 2** calculate M_u using Eq. 3.7.

Case B: check if d'/d> code limits (Table 3.1), then compression steel does not yield

- **Step 1** calculate "a" using Eq. 3.8 or Eq. 3.10.
- **Step 2** calculate M_u using Eq. 3.11.



3.1.3 Maximum Area of Steel for Doubly Reinforced Sections

To ensure ductile failure of a doubly reinforced section, the neutral axis distance c_{max} is limited to that of the singly reinforced section as given in Table 4-1 in the code or Table 2.1 given in Chapter 2. Thus, increasing the tension reinforcement above $A_{s,max}$ is allowed by the code only by adding compression reinforcement that keeps the same neutral axis distance as shown in Fig. 3.4.I.

A doubly reinforced section can be looked at as composed of a singly reinforced concrete section and a steel section. The singly reinforced section (Fig. 3.4.II) has an area of steel equal to $A_{s,max}$ obtained from Table 2.1, and the steel section has the same amount of top and bottom steel of area A'_s (Fig 3.4.III). Thus, the maximum area of steel for a doubly reinforced section $A_{sd,max}$ is given by

$$A_{sd,\max} = A_{s,\max} + A'_s \quad \text{...(3.13)}$$

$$A'_s = \alpha\, A_{sd,\max}$$

$$A_{sd,\max} = \frac{A_{s,\max}}{1-\alpha} \quad \text{...(3.14a)}$$

$$\mu_{d,\max} = \frac{\mu_{\max}\ \text{(for singly rft section (Table 2.1)}}{1-\alpha} \quad \text{......................(3.14b)}$$

$$\boxed{A_{sd,\max} = \mu_{\max}\, b\, d + A'_s} \quad \text{...(3.15)}$$

where μ_{max} is obtained from Table 2.1.

The yielding of the compression reinforcement can be verified by comparing the ratio d'/d and with the maximum allowed value given in Table 3.1. If the compression steel does not yield, the maximum area of steel can be obtained from:

$$A_{sd,\max} = \mu_{\max}\, b\, d + A'_s \frac{f'_s}{f_y/1.15} \quad \text{...............................(3.16)}$$

Maximum Moment Calculation

Referring to Fig. 3.4, the maximum moment for doubly reinforced sections $M_{ud,max}$ can be calculated using the following procedure

$$M_{ud,\max} = M_{u,\max} + M' \quad \text{...(3.17)}$$

$$M_{ud,\max} = M_{u,\max} + \frac{A'_s\, f_y}{1.15}(d-d') \quad \text{...............................(3.18)}$$

$$\boxed{M_{ud,max} = \frac{R_{max}\, f_{cu}\, b\, d^2}{1.5} + \frac{A'_s\, f_y}{1.15}(d - d')} \quad \text{.............................(3.19)}$$

where R_{max} is obtained from Table 2.1

An alternative procedure to obtain the maximum moment is to take the moment around the concrete compression force. Referring to Fig.3.4.I, the maximum moment is given by:

$$M_{ud,max} = \frac{A_{sd,max}\, f_y}{1.15}\left(d - \frac{a_{max}}{2}\right) + \frac{A'_s\, f_y}{1.15}\left(\frac{a_{max}}{2} - d'\right) \quad \text{...................(3.20)}$$

Note that if the calculated neutral axis location "c" is less than the maximum value allowed by the code "c_{max}" then the following rule applies:

$$\boxed{If\ \ \frac{c}{d} \le \frac{c_{max}}{d} \quad then \quad \left(\begin{array}{l} f_s = f_y/1.15 \\ \mu < \mu_{d\,max} \\ A_s < A_{sd,max} \\ M_u < M_{ud,max} \end{array}\right.}$$

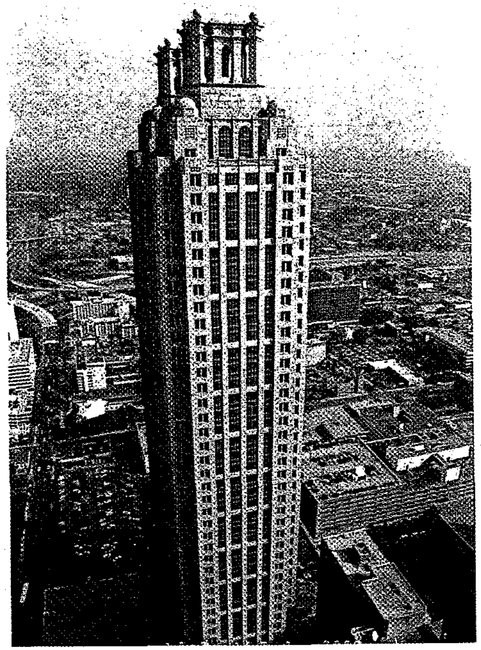


Photo 3.3 Peachtree Tower (1990), Atlanta USA (235m, 50 stories)

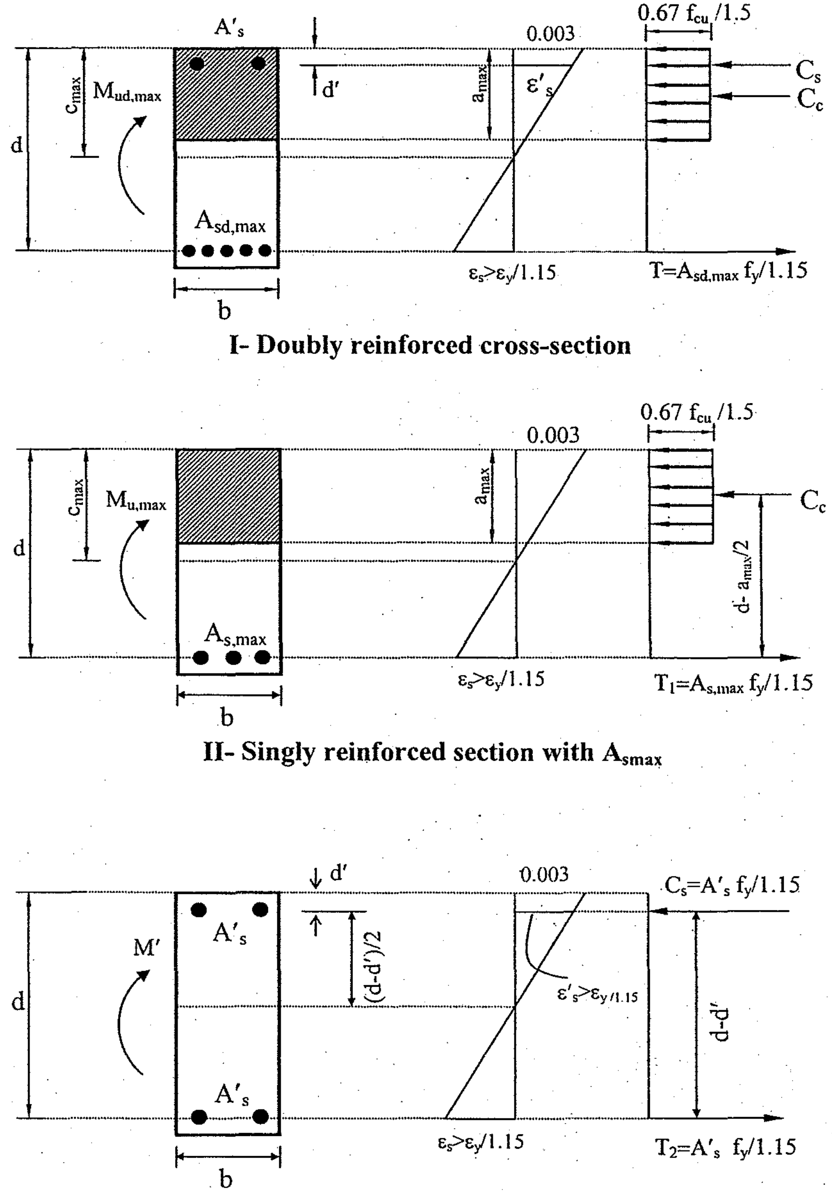


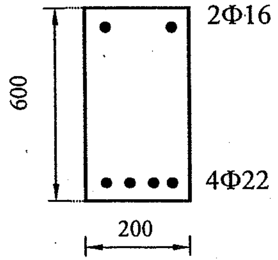
Fig. 3.4 Maximum moment and area of steel for doubly reinforced sections

Example 3.1(compression steel yields)

Find the moment capacity of the cross-section shown in Figure. Assume that d′ = 50 mm and the material properties are:

$f_{cu} = 25\ N/mm^2$

$f_y = 400\ N/mm^2$



Solution

$A_s = 4\Phi22 = 15.2\ cm^2 = 1520\ mm^2$

$A'_s = 2\Phi16 = 4.02\ cm^2 = 402\ mm^2$

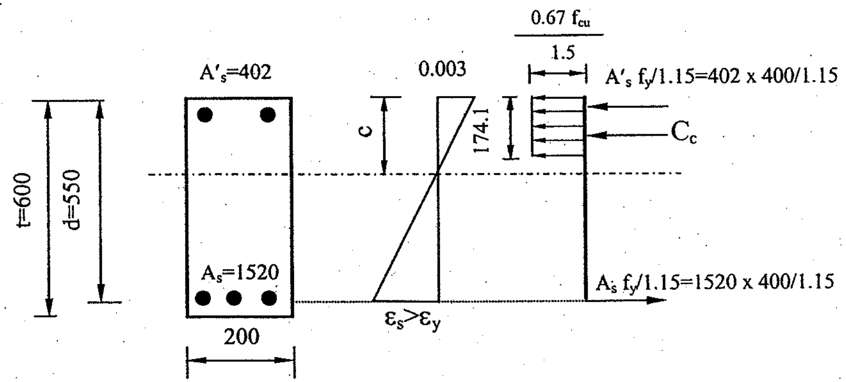
$d = 600 - 50 = 550$ mm

Step 1: Compute a.

$d'/d = 50/550 = 0.09$

Since d′/d (0.09) <0.1 (Table 3.1), one can assume that compression steel yields. This assumption will be checked in step 2. Applying Eq. 3. 4 gives:

$$\frac{0.67\, f_{cu}\, b\, a}{1.5} + \frac{A'_s\, f_y}{1.15} = \frac{A_s\, f_y}{1.15}$$



$$\frac{0.67\times25\times200\times a}{1.5} + \frac{402\times400}{1.15} = \frac{1520\times400}{1.15}$$

a = 174.1 mm

c = a/0.8 = 217.6 mm

$a/d = 0.317 > a/d)_{min}(0.1)$ ....ok

c/d = 217.6/550 = 0.396

Step 2: Check f_s and f'_s

$$f'_s = 600\,\frac{c-d'}{c} = 600\,\frac{217.6-50}{217.6} = 462 > \frac{400}{1.15} \ldots ok\ldots.(\textit{compression steel yields})$$

$$f_s = 600\,\frac{d-c}{c} = 600\,\frac{550-217.6}{217.6} = 917 > \frac{400}{1.15} \ldots ok\ldots.(\textit{tension steel yields})$$

OR

- Since d′/d(0.09) <0.1 or d′/a (0.287)<(0.52, Table 3.1)then compression steel yields
- Since c/d(0.396) < (c_b/d = 0.63, Table 2.1) then tension steel yields

Step3: Compute moment capacity, M_u

Taking moment around concrete force C, and applying Eq. 3.7.

$$M_u = \frac{A_s\, f_y}{1.15}\left(d - \frac{a}{2}\right) + \frac{A'_s\, f_y}{1.15}\left(\frac{a}{2} - d'\right)$$

Thus

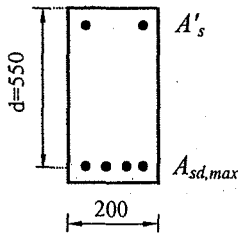
$$M_u = \frac{1520\times400}{1.15}\left(550 - \frac{174.1}{2}\right) + \frac{402\times400}{1.15}\left(\frac{174.1}{2} - 50\right)$$

$$M_u = 249.94x10^6 = 249.94\ kN.m$$

Final Result: M_u = 249.94 kN.m

Example 3.2

Calculate the maximum area of steel and the maximum moment capacity that is allowed by the Egyptian Code (ECP 203) for the doubly reinforced section shown in Example 3.1. The material properties are: f_{cu} = 25 N/mm^2 and f_y = 400 N/mm^2



Solution

Step 1: Calculate maximum area of steel

From Table 2.1 and for $f_y = 400$ N/mm^2 : $\mu_{max} = 4.31\text{x}\ 10^{-4}\ f_{cu}$,

since d'/d (0.09) < 0.176 (at $c = c_{max}$), table 3.1, then compression steel has yielded

$A_{sd,max} = \mu_{max}\ b\ d + A'_s$

$A_{sd,max} = (4.31 \times 10^{-4} \times 25)\ 200 \times 550 + 402 = 1587\ \text{mm}^2 \quad > A_s(1520)$ ....o.k

Step 2: Calculate maximum moment capacity

From Table 2-1: $R_{max} = 0.187$, $c_{max}/d = 0.42$ for $f_y = 400$ N/mm^2

Using Eq.3.19 to find $M_{ud,max}$

$$M_{ud,max} = \frac{R_{max}\ f_{cu}\ b\ d^2}{1.5} + \frac{A'_s\ f_y}{1.15}(d - d')$$

$$M_{ud,max} = \left[\frac{0.187 \times 25 \times 200 \times 550^2}{1.5} + \frac{402 \times 400}{1.15}(550 - 50)\right]/10^6 = 258.5\ kN.m$$

The same result can be obtained using Eq.3.20 as follows:

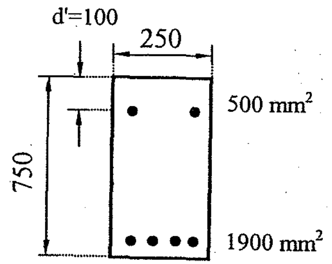
$a_{max} = 0.8\ c_{max} = 0.8 \times 0.42 \times 550 = 184.8$ mm

$$M_{ud,max} = \frac{A_{sd,max}\ f_y}{1.15}\left(d - \frac{a_{max}}{2}\right) + \frac{A'_s\ f_y}{1.15}\left(\frac{a_{max}}{2} - d'\right)$$

$$M_{ud,max} = \frac{1587 \times 400}{1.15}\left(550 - \frac{184.8}{2}\right) + \frac{402 \times 400}{1.15}\left(\frac{184.8}{2} - 50\right) = 258.5\ kN.m$$

Example 3.3 (compression steel does not yield)

Find the moment capacity of the cross-section shown in figure.
$f_{cu} = 30$ N/mm^2, and $f_y = 400$ N/mm^2



Solution

d= 750 – 50 = 700 mm

Step 1: Compute a.

$$\frac{d'}{d} = \frac{100}{700} = 0.143 > 0.10\ (\textit{see table 3.1})\ldots\textit{compression steel does not yield}$$

We can use Eq 3.8 or Eq 3.10 to calculate a

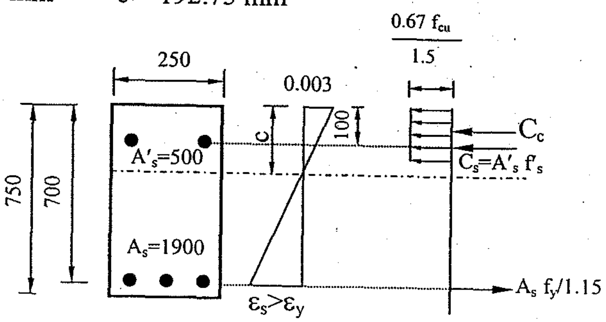
$$\frac{0.67\ f_{cu}\ b\ a}{1.5} + A'_s\ f'_s = \frac{A_s\ f_y}{1.15}$$

$$0.4466\ f_{cu}\ b\ a^2 - (A_s\ f_y/1.15 - 600 \times A'_s\ )\ a - 480 A'_s\ d' = 0$$

$$3350\ a^2 - 360869\ a - 24 \times 10^6 = 0$$

Solving for the only unknown "a"

a =154.2 mm→→ c = 192.73 mm



Step 2: Check f_s and f'_s.

$$f'_s = 600\,\frac{192.73-100}{192.73} = 288.6\ N/mm^2 < \frac{400}{1.15}\ \textit{compression steel does not yield}$$

Since c/d(0.275) < c_b/d = 0.63 then tension steel yields $f_s = f_y/1.15$

Step 3: Compute M_u

Taking moment around concrete force C_c

$$M_n = \frac{A_s f_y}{1.15}\left(d-\frac{a}{2}\right) - A'_s f'_s\left(d'-\frac{a}{2}\right) = \frac{A_s f_y}{1.15}\left(d-\frac{a}{2}\right) + A'_s f'_s\left(\frac{a}{2}-d'\right)$$

$$M_u = \frac{1900\times 400}{1.15}\left(700-\frac{154.2}{2}\right) + 500\times 288.6\left(\frac{154.2}{2}-100\right) = 408.3x10^6 = 408.3\ kN.m$$

Final Result: M_u = 408.3 kN.m

3.1.4 Design of Doubly Reinforced Sections Using First Principles

The same procedure used in designing singly reinforced sections is used for the design of doubly reinforced sections. The unknowns in these types of problems are the beam depth, area of steel, neutral axis position and the ratio of the compression steel α.

Given : f_{cu}, f_y, M_u, b, d'
Required : d, A_s and A'_s
Unknowns: a, d, A_s and A'_s

Since we have only two equilibrium equations, we have to limit the unknowns to only two. If not given, the depth of the compression steel will be assumed 0.05-0.1 of the beam depth to ensure yielding of compressed bars for all steel grades. The design procedure can be summarized in the following steps:

1. Make the necessary assumptions
   d' =0.05-0.10 d (compression steel yields for all f_y)
   Assume $A_s = \mu_{max}\ b\ d$ →(μ_{max} for singly reinforced section (Table 4.1))
   Assume α=0.2-0.4 and
   Equilibrium of forces gives

$$\frac{0.67\ f_{cu}\ b\ a}{1.5} + \frac{(\alpha\times A_s)\ f_y}{1.15} = \frac{A_s\ f_y}{1.15}$$

Get $a = \lambda\ d$

Taking moment around the concrete force gives

$$M_u = \frac{A_s\ f_y}{1.15}\left(d-\frac{a}{2}\right) + \frac{A'_s\ f_y}{1.15}\left(\frac{a}{2}-d'\right)$$

Solve the above equation to determine (a, d), then calculate

$$A_s = \mu_{max}\ b\ d$$

$$A'_s = \alpha\ A_s$$

2. Check the minimum area of steel

$$A_s > A_{smin}$$

3. Check the maximum area steel and the maximum moment by ensuring that

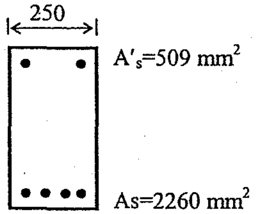
$$(c/d < c_{max}/d)$$

Example 3.4

The doubly reinforced section shown in figure is subjected to a bending moment of a factored value of 200 kN.m.

$d' = 50$ mm, $f_{cu} = 27$ N/mm^2, and $f_y = 280$ N/mm^2

Use the first principles to determine the required beam depth.



Solution

Step 1: Compute a.

Assume that both compression and tension steel has yielded.

This assumption will be checked later

Given : $f_{cu}, f_y, M_u, b, d', A_s$ and A'_s

Required : d

Unknowns : a, d

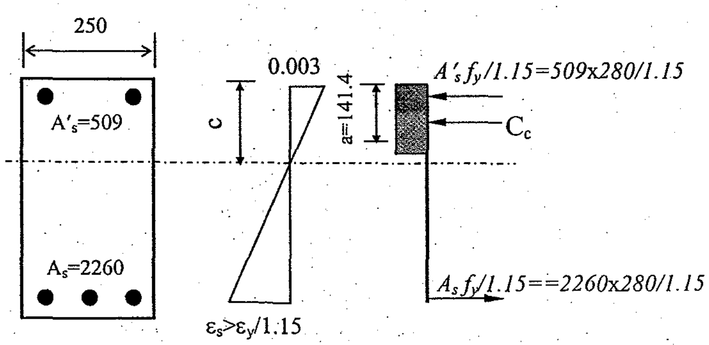
Since we have two unknowns only in this example, nothing needs to be assumed. Apply the first equilibrium equation:

$$\frac{0.67\, f_{cu}\, b\, a}{1.5} + \frac{A'_s\, f_y}{1.15} = \frac{A_s\, f_y}{1.15}$$

$$\frac{0.67 \times 27 \times 250 \times a}{1.5} + \frac{509 \times 280}{1.15} = \frac{2260 \times 280}{1.15}$$

a = 141.4 mm

c = a/0.8 = 176.75 mm



Step 2: Compute beam depth d

Taking moment around concrete force C_c

$$M_u = \frac{A_s\, f_y}{1.15}\left(d - \frac{a}{2}\right) + \frac{A'_s\, f_y}{1.15}\left(\frac{a}{2} - d'\right)$$

$$200 \times 10^6 = \frac{2260 \times 280}{1.15}\left(d - \frac{141.4}{2}\right) + \frac{509 \times 280}{1.15}\left(\frac{141.4}{2} - 50\right)$$

d = 429.5 mm

Step 3: check f_s and f'_s

$d'/d = 50/429.5 = 0.116$

Since $d'/d = 0.116 < 0.2$ (code limit for mild steel see table 3.1), thus

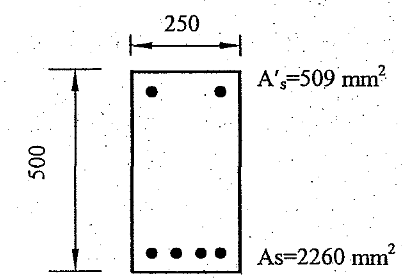
$f'_s = f_y/1.15$

c/d = 176.75/429.5 = 0.4115

From Table 2.1 $c_{max}/d = 0.48$ for $f_y = 280$ N/mm^2

$$\boxed{\because \frac{c}{d}(0.411) \le \frac{c_{max}}{d}(0.48) \quad then \left\{ \begin{array}{c} f_s = f_y/1.15 \\ \mu < \mu_{d\,max} \\ A_s(22.6\ cm^2) < A_{sd,max} \\ M_u(200\ kN.m) < M_{ud,max} \end{array} \right.}$$

Final Result: d = 450 mm and t = 500 mm



Example 3.5

A reinforced concrete cross-section is subjected to 265 kN.m. Architectural considerations require limiting the thickness of the section as much as possible. Economic considerations limited the value of α to 0.3. According to these constraints, design the cross section. Check the maximum area of steel and the maximum moments allowed by the code knowing that:
b = 250 mm, f_{cu} = 30 N/mm^2, and f_y = 360 N/mm^2

Solution

Step 1: Assumptions

Given : $f_{cu}, f_y, M_u, b, \alpha\,(A'_s)$
Required : $d,\ d',\ A_s$
Unknowns : $a,\ d,\ d',\ A_s$

We have four unknowns, thus we shall assume two (d' and As(μ))

1. Assume d' = 0.10 d
2. Assume $\mu = \mu_{max}$ → (for singly reinforced section)

$\mu = 5 \times 10^{-4} f_{cu} = 5 \times 10^{-4} \times 30 = 0.015$

$$A_s = \mu\, b\, d = 0.015 \times 250 \times d = 3.75\, d$$

$$A'_s = \alpha\, A_s = 0.3 \times 3.75\, d = 1.125\, d$$

Step 2: Apply the equilibrium equation T=C Eq. 3.4

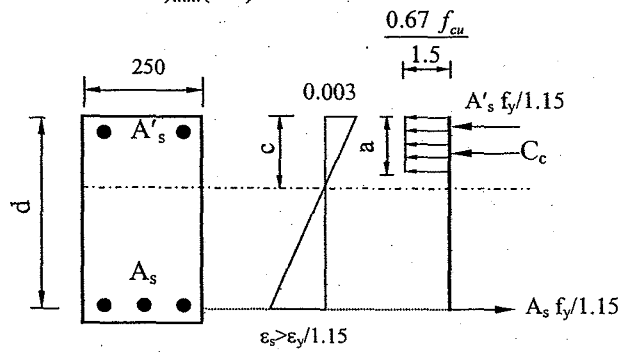
Assume that compression and tension steel has yielded.

$$\frac{0.67\, f_{cu}\, b\, a}{1.5} + \frac{A'_s\, f_y}{1.15} = \frac{A_s\, f_y}{1.15}$$

$$\frac{0.67 \times 30 \times 250 \times a}{1.5} + \frac{1.125\, d \times 360}{1.15} = \frac{3.75\, d \times 360}{1.15}$$

a = 0.2453 d

a/d = 0.2453 > a/d)$_{min}$(0.1) ….ok



Step 3: Apply the second equilibrium equation, Eq. 3.7

Taking moment around concrete force C_c

$$M_u = \frac{A_s\, f_y}{1.15}\left(d - \frac{a}{2}\right) + \frac{A'_s\, f_y}{1.15}\left(\frac{a}{2} - d'\right)$$

$$265 \times 10^6 = \frac{3.75\, d \times 360}{1.15}\left(d - \frac{0.2453\, d}{2}\right) + \frac{1.125\, d \times 360}{1.15}\left(\frac{0.2453\, d}{2} - 0.1\, d\right)$$

$$265 \times 10^6 = 1037.9\, d^2$$

d = 505.29 mm

$A_s = 3.75\, d = 3.75 \times 505.29 = 1894.8\ mm^2$

$A'_s = 1.125\, d = 568.45\ mm^2$

Take d = 550 mm, d' = 0.1 d = 55mm

and t = 600mm

Step 4: Calculate minimum area of steel

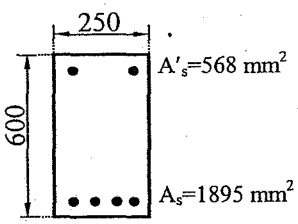
$$A_{s\,min} = smaller\ of \begin{cases} \dfrac{0.225\sqrt{f_{cu}}}{f_y} bd = \dfrac{0.225\sqrt{30}}{360} \times 250 \times 550 = 470 \\ 1.3 A_s = 1.3 \times 1895 = 2462 \end{cases} = 470 cm^2 < A_s .....o.k$$

Step 5.1: Calculate maximum area of steel

Since d'/d (0.1) < 0.21 (from Table 3.1) and even less than the code value of 0.15, we can assume that compression steel yields.

$A_{sd,max} = \mu_{max}\, b\, d + A'_s$ = 5x10^{-4} x 30 x 250 (550) + 568.45

$\mathbf{A_{sd,max}}$ = 2631 mm^2 > A$_s$(1895) . . . . o.k.



Final design

Step 5.2: Calculate maximum moment

$$M_{ud,max} = \frac{R_{max}\, f_{cu}\, b\, d^2}{1.5} + \frac{A'_s\, f_y}{1.15}(d - d')$$

From Table 2.1 R_{max}=0.194

$$M_{ud,max} = \frac{0.194 \times 30 \times 250 \times 550^2}{1.5} + \frac{568 \times 360}{1.15}(550 - 55) = 381\, kN.m \quad > M_u(265)$$

Final Result: d= 550 mm, A$_s$ =1895 mm^2 and A'$_s$ =568 mm^2

Example 3.6

Design a doubly reinforced concrete cross-section to withstand an ultimate moment of 265 kN.m by assuming area of steel. Check the maximum area of steel and the maximum moments allowed by the code.
b = 250 mm, f_{cu} = 30 N/mm², and f_y = 360 N/mm²

Solution

Step 1: Assumptions

Given : f_{cu}, f_y, M_u, b
Required : d, d', A_s, A'_s
Unknowns : a, d, d', A_s, A'_s
We have five unknowns, thus we shall assume three (d' and A_s, $A'_s(\alpha)$)

1. Assume α=0.3, d'=50 mm
2. use the approximate relation to assume A_s,

$$A_s = 0.11\sqrt{\frac{M_u\, b}{f_y}} = 0.11\sqrt{\frac{265\times10^6\times250}{360}} = 1492\ mm^2$$

Take A_s = 1500 mm²

$$A'_s = \alpha\ A_s = 0.3\times1500 = 450\ mm^2$$

Step 2: Apply first equilibrium equation T=C

Assume that compression and tension steel has yielded.

$$\frac{0.67\times30\times250\times a}{1.5} + \frac{450\times360}{1.15} = \frac{1500\times360}{1.15}$$

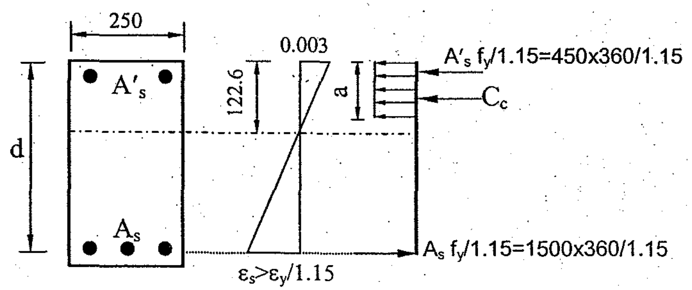
a = 98.12 mm and c = a/0.8 = 122.65 mm

Step 3: Apply second equilibrium equation ΣM=0

Taking moment around concrete force C_c

$$M_u = \frac{A_s\, f_y}{1.15}\left(d-\frac{a}{2}\right)+\frac{A'_s\, f_y}{1.15}\left(\frac{a}{2}-d'\right)$$

$$265\times10^6 = \frac{1500\times360}{1.15}\left(d-\frac{98.12}{2}\right)+\frac{450\times360}{1.15}\left(\frac{98.12}{2}-50\right)$$



d = 614 mm. c/d = 122.65/614 = 0.2

Since d'/d (50/614 = 0.081)< 0.15 (code limit for f_y = 360 N/mm²), then our assumption that compression steel has yielded is correct.

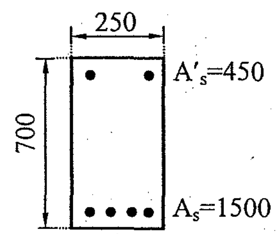
Take d = 650 mm and t = 700mm

Step 4: Calculate minimum area of steel

$$A_{s\min} = smaller\ \ of \begin{cases} \dfrac{0.225\sqrt{f_{cu}}}{f_y}bd = \dfrac{0.225\sqrt{30}}{360}\times250\times650 = 556 \\ 1.3A_s = 1.3\times1500 = 1950 \end{cases} = 556\ cm^2 < A_s ....o.k$$

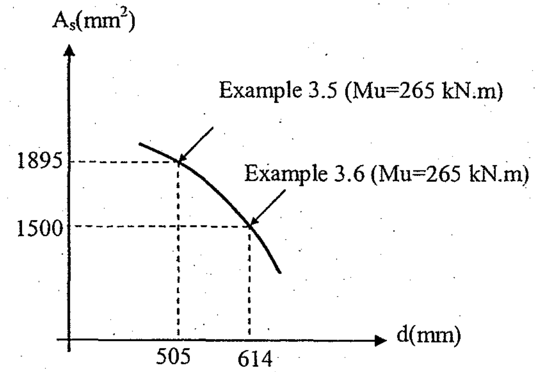
Step 5: Check maximum area of steel

Since c/d(0.2) <0.44 (code limit for f_y = 360 N/mm²), then our assumption is correct and $A_s<A_{sd,max}$, $M_u<M_{ud,max}$.



Final design

> **NOTE:** The cross sections in Example 3.5 and 3.6 are subjected to the same bending moment, however, the same capacity was obtained using different depth and area of steel as shown in figure. For the same moment capacity, area of steel may be reduced but the beam depth has to be increased.



Example 3.7

Design a doubly reinforced concrete cross-section to withstand an ultimate moment of 360 kN.m knowing that the beam thickness equals to 650 mm.
b = 250 mm, f_{cu} = 35 N/mm^2, and f_y = 400 N/mm^2

Solution

Step 1: Assumptions

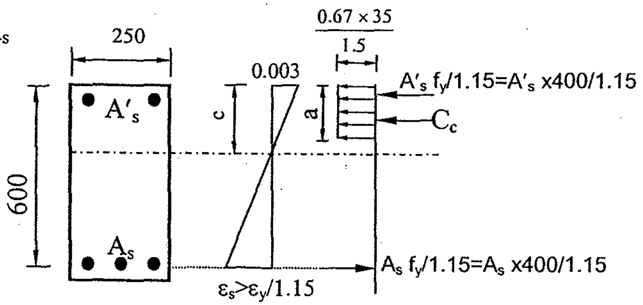
Assume α=0.3, d′=50 mm, d = 650 −50 = 600 mm. Since we have two unknowns, (*a* and *A$_s$*), the equilibrium equations are used directly as follows:

Step 2: Apply first equilibrium equation T=C

$$d'/d = 50/600 = 0.0833 < 0.10 \ \textit{compression steel yields}$$

$$\frac{0.67 \times 35 \times 250 \times a}{1.5} + \frac{0.3 \times A_s \times 400}{1.15} = \frac{A_s \times 400}{1.15}$$

a = 0.0623 A_s



Step 3: Apply second equilibrium equation ΣM=0

Taking moment around concrete force C_c

$$M_u = \frac{A_s\, f_y}{1.15}\left(d - \frac{a}{2}\right) + \frac{A'_s\, f_y}{1.15}\left(\frac{a}{2} - d'\right)$$

$$360 \times 10^6 = \frac{A_s \times 400}{1.15}\left(600 - \frac{0.0623\ A_s}{2}\right) + \frac{0.3 \times A_s \times 400}{1.15}\left(\frac{0.0623 A_s}{2} - 50\right)$$

Solving the above equation gives A_s=1904 mm^2, $A'_s = 0.3 \times A_s = 571\ mm^2$

Thus a=0.0623 x A_s=118.61 mm

Step 4: Check A_{sdmax} and A_{smin}

c=a/.8= 148.267 mm

c/d=0.24 < c/d$_{max}$(0.42)….ok (A_s<A_{sdmax})

$$A_{s\,\min} = smaller\ \ of \begin{cases} \dfrac{0.225\sqrt{35}}{400} \times 250 \times 600 = 499 \\ 1.3 \times 1904 = 2475 \end{cases} = 499\ cm^2 < A_s .....o.k$$

3.1.5 Design of Doubly Reinforced Sections Using Curves

The design of doubly reinforced sections from the first principles is complicated. Therefore, design curves were prepared to facilitate their design.
In developing such curves, both compression steel and tension steel were assumed to reach yield. In addition, two values for d′/d were used in developing these curves and tables namely (0.05,0.1). These selected values were chosen to satisfy code requirements and to ensure yielding of the compression reinforcement for all types of steel.
Recalling the first equilibrium equation 3.4 and referring to Fig. 3.5

$$\frac{0.67\, f_{cu}\, b\, a}{1.5} + \frac{A'_s\, f_y}{1.15} = \frac{A_s\, f_y}{1.15}$$

Dividing by (f_{cu} b x d) and rearranging

$$\frac{0.67}{1.5}\frac{a}{d} = \frac{\mu\, f_y}{1.15\, f_{cu}}(1-\alpha)$$

where $\alpha = A'_s/A_s$ and $\mu = A_s/(b\ .\ d)$

$$\text{Define } \ \omega = \mu \frac{f_y}{f_{cu}} \ \ldots\ldots\ldots\ldots\ldots\ldots (3.21)$$

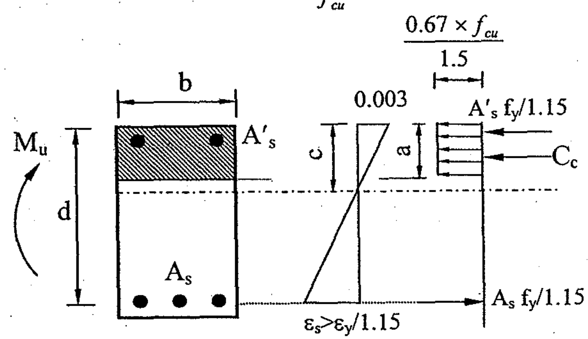


Fig 3.5 Location of the neutral axis for doubly reinforced sections

$$\frac{a}{d} = 1.9468\, \omega (1-\alpha) \ \ldots\ldots\ldots\ldots\ldots\ldots (3.22)$$

Recalling the moment equation around the concrete compression force

$$M_u = \frac{A_s\, f_y}{1.15}\left(d - \frac{a}{2}\right) + \frac{A'_s\, f_y}{1.15}\left(\frac{a}{2} - d'\right)$$

dividing by (f_{cu} b x d^2) and rearranging

$$R1 = \frac{M_u}{f_{cu}\, b\, d^2} = \frac{A_s\, f_y}{1.15\, f_{cu}\, b\, d^2}\left(d - \frac{a}{2}\right) + \frac{A_s'\, f_y}{1.15\, f_{cu}\, b\, d^2}\left(\frac{a}{2} - d'\right)$$

$$R1 = \frac{\omega}{1.15}\left(1 - \frac{a}{2\,d}\right) + \frac{\alpha\,\omega}{1.15}\left(\frac{a}{2\,d} - \frac{d'}{d}\right) \qquad \text{.................................(3.23)}$$

To find ω_{max} in case of doubly reinforced sections, recall equation 3.14

$A_{sd,max} = \frac{A_{s,max}}{1-\alpha}$, dividing by (b d f_y /f_{cu})

$$\boxed{\omega_{d\,max} = \frac{\omega_{max}}{1-\alpha}} \qquad \text{...(3.24)}$$

where ω_{max} is given in Table 2.1

Figure 3.6 shows an example of such curves. Appendix A contains R1-ω design charts. Table C in the appendix gives the maximum moment index R_{1max} for each values of α, ω_{dmax}. To use the table enter with α and f_y and either (R_{1max} to find ω_{dmax}) or (ω_{dmax} to find R_{1max}). The design procedure for using Table C to design doubly reinforced section is illustrated in Example 3.8

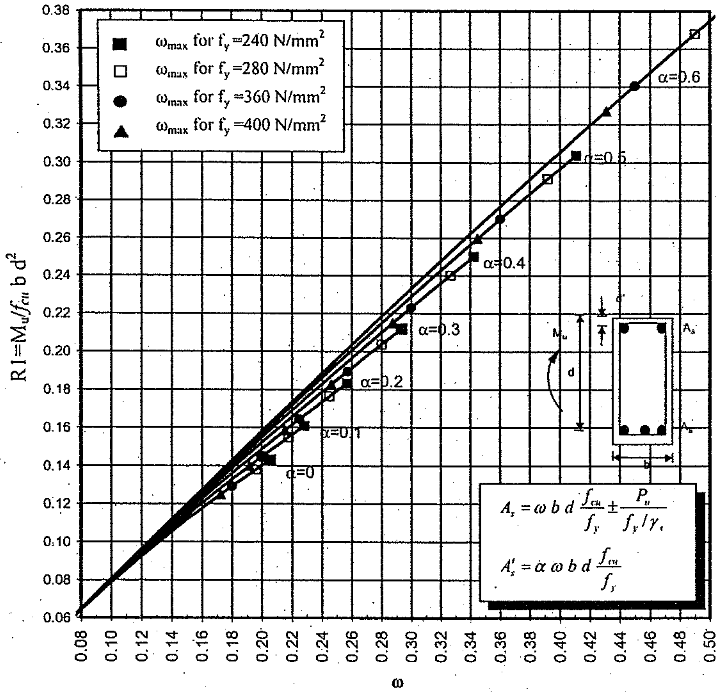


Fig. 3.6 Design chart for doubly reinforced sections

Procedure for Using Charts and Tables

The general steps for using the charts or tables can be summarized in the following:

- Assume d′/d=0.05-0.1
- Assume α=0.2-0.4
- Choose the highest *R1* for the chosen α to get the smallest possible depth
- Compute the depth "*d*" from the following relation
  $R1 = \frac{M_u}{f_{cu}\, b\, d^2}$
- From the tables or the charts get ω
- Compute the area of steel from the following relation
  $A_s = \omega \frac{f_{cu}}{f_y} b\ \ d \quad , \quad A_s' = \alpha \times A_s$
- **Note1:** No need to check $A_{sd,max}$ because the limits are already in the charts
  Note2: No need to check $A_{s,min}$ because we usually use μ near μ_{max}

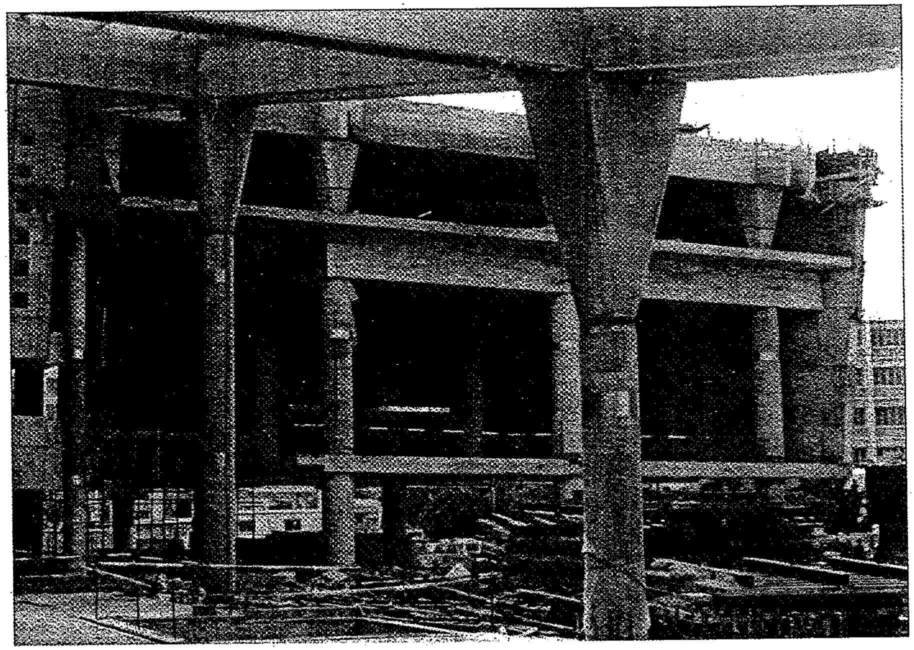


Photo 3.4 A reinforced concrete building during construction

Example 3.8

Design a doubly reinforced concrete section subjected to M_u = 480 kN.m using design aids.

b = 250 mm, f_{cu} = 20 N/mm^2, and f_y = 240 N/mm^2

Solution

Assume d'/d = 0.10

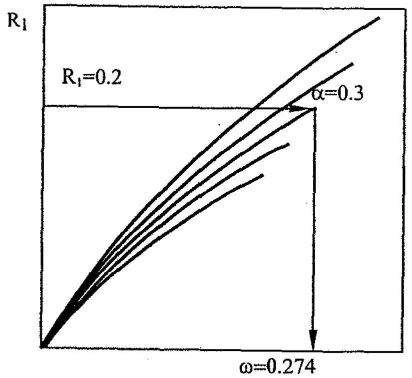
Assume α = 0.3

Enter the table or the chart with α = 0.3 and get the value of R1 and ω very close to the maximum allowable for f_y = 240 N/mm^2 to get the smallest possible depth. Thus get R1 = 0.2 and ω = 0.274

$$R1 = \frac{M_u}{f_{cu}\, b\, d^2}$$

$$0.2 = \frac{480\times10^6}{20\times 250\times d^2}$$

d = 693 mm

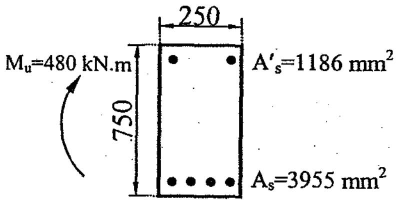


$$A_s = \omega \frac{f_{cu}}{f_y} b\ \ d$$

$$A_s = 0.274\times\frac{20}{240}\times 250\times 693 = 3955\ mm^2$$

$$A_s' = \alpha\ A_s = 0.3\times 3955 = 1186\ mm^2$$

Take d = 700 and t = 750 and A_s = 3955 mm^2



Final design

Example 3.9

Design a doubly reinforced section subjected to M_u = 320 kN.m using Table C.
b = 200 mm, d = 600, f_{cu} = 25 N/mm^2, and f_y = 280 N/mm^2

Solution

Assume d'/d = 0.10

In using Table C use $M_u = M_{ud,max}$

$$R1_{d\max} = \frac{M_{ud,max}}{f_{cu}\ b\ d^2} = \frac{320\times10^6}{25\times 200\times 600^2} = 0.177$$

From the table C with d'/d = 0.10 and R_{1damx} = 0.177, we can notice that the moment capacity exceeds singly reinforced section and we must use compression
reinforcement. Thus, by interpolation get

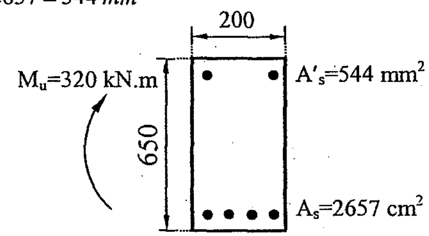
α = 0.205 and ω_{dmax} = 0.248

fy	ω_{dmax}	α	$R1_{d\max} = \frac{M_{ud,max}}{f_{cu}\, b\, d^2}$			
			d'/d=0.05	d'/d=0.1	d'/d=0.15	d'/d=0.2
280	0.196	0.00	0.138	0.138	0.138	0.138
	0.206	0.05	0.147	0.146	0.146	0.145
	0.218	0.10	0.156	0.155	0.154	0.153
	0.231	0.15	0.167	0.165	0.164	0.162
	0.245	0.20	0.179	0.176	0.174	0.172
	0.261	0.25	0.192	0.189	0.186	0.184
	0.280	0.30	0.207	0.204	0.200	0.197

$$A_s = \omega_{d\max}\frac{f_{cu}}{f_y} b\ \ d$$

$$A_s = 0.248\times\frac{25}{280} 200\times 600 = 2657\ mm^2$$

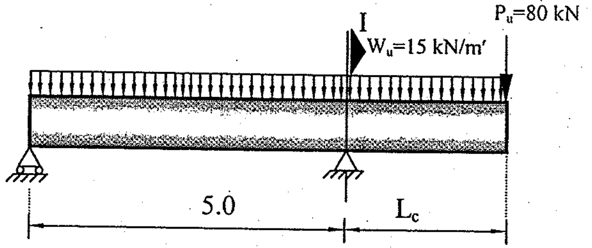
$$A_s' = \alpha\ A_s = 0.205\times 2657 = 544\ mm^2$$

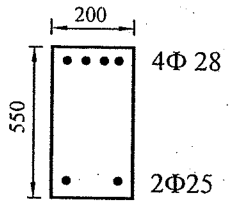


Final design

Example 3.10

Find the maximum cantilever span L_c for the beam shown in the figure at section I using R1-w design tables. The material properties are
$f_y = 360$ N/mm^2, $f_{cu} = 40$ N/mm^2





Section I

Solution

Assume cover = 50mm

d' = 50 mm

d = 550-50 = 500mm

Since section I is subjected to –ve moment, the tension steel is at the top and the compression steel is at the bottom of the beam.

$A_s = 2463$ mm^2, $A'_s = 982$ mm^2

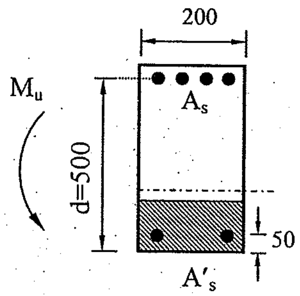
$$\alpha = \frac{982}{2463} = 0.398 \approx 0.4$$

d'/d = 50/500 = 0.1

$$A_s = \omega \frac{f_{cu}}{f_y} b \;\; d$$

$$2463 = \omega \times \frac{40}{360} \times 200 \times 500$$

ω = 0.2217



From Table R-w design tables (d'/d=0.10) with (ω = 0.2217 and α = 0.4) get R1 = 0.17

R1	ω											
	α=0.0		α=0.1		α=0.2		α=0.3		α=0.4		α=0.5	
0.01		0.012		0.012		0.012		0.012		0.012		0.012
0.02		0.023		0.024		0.024		0.024		0.024		0.024
0.03		0.036		0.036		0.036		0.036		0.036		0.036
0.04		0.048		0.048		0.048		0.048		0.049		0.049
0.05		0.061		0.061		0.061		0.061		0.061		0.061
0.06		0.074		0.074		0.074		0.074		0.074		0.074
0.07		0.088		0.087		0.087		0.087		0.087		0.087
0.08		0.102		0.101		0.100		0.100		0.099		0.099
0.09		0.117		0.115		0.114		0.113		0.112		0.112
0.10		0.132		0.129		0.128		0.126		0.125		0.125
0.11		0.147		0.144		0.142		0.140		0.139		0.138
0.12	400	0.164		0.159		0.156		0.154		0.152		0.151
0.13	360	0.181		0.175		0.171		0.168		0.166		0.164
0.14	240/280	0.199	360/400	0.192		0.186		0.182		0.179		0.177
0.15			280	0.209		0.202		0.197		0.193		0.191
0.16			240	0.226	400/360	0.218		0.211		0.207		0.204
0.17					280	0.234		0.227		0.22		0.218
0.18					240	0.251	400	0.242		0.236		0.231

$$R1 = \frac{M_u}{f_{cu}\, b\, d^2}$$

$$0.170 = \frac{M_u}{40 \times 200 \times 500^2}$$

$M_u = 340 \times 10^6 = 340$ kN.m (internal moment)

$$M_u = \frac{W_u\, L_c^2}{2} + P_u\, L_c \quad \text{(external moment)}$$

External moment =Internal moment

$$\frac{15\, L_c^2}{2} + 80\, L_c - 340.0 = 0$$

Solving the second order equation gives:

$\boxed{L_c = 3.256 \text{ m.}}$

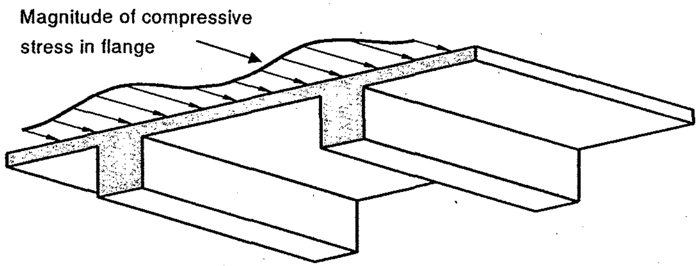
3.2 T-Beams

3.2.1 Application of T-Beams

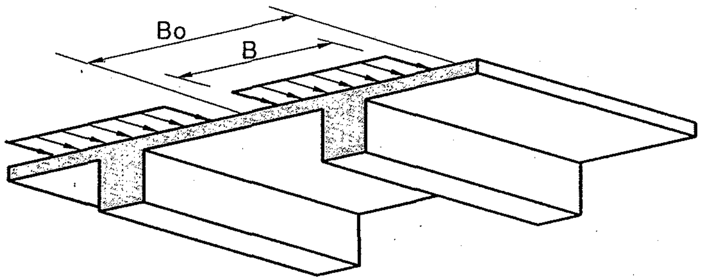
Reinforced concrete buildings usually consist of beams and slabs that were cast monolithically. Thus, slabs and beams act together in resisting the applied loads. As a result, the beam will have an extension concrete part at the top called "*flange*", and the beam is called a T-beam. The portion of the beam below the slab is called the "*web*". The stress distribution in the slab will vary according to the ratio between the thickness of the slab and the overall thickness.

3.2.2 Effective Flange Width

The distribution of the flexural compressive stresses in the flange of the slab is shown in Fig. 3.7. The compressive stress is a maximum at the beam locations, and minimum between the beams. The concept of replacing the non-uniform stresses over the width B_o to uniform stresses over a width B is called *the effective width*. The compression force developed in the reduced width B equals the compressive force in the real compression zone of width B_o.



a) Distribution of maximum flexural compressive stresses



b) Flexural compressive stress distribution assumed in design

Fig. 3.7 Distribution of compressive stresses across the flange

To simplify section capacity calculations, most codes assume uniform distribution of the stress and specify a limited width of the slab to be considered when analyzing the beam capacity called the "*effective width*" as shown in Fig. 3.8. For T-sections, the Egyptian code ECP 203 section (6-3-1-9) requires that the effective width does not exceed the following

$$B \le \begin{array}{l} 16\,t_s + b \\ \dfrac{L_2}{5} + b \\ C.L \text{ to } C.L \end{array} \qquad \text{for T-sections} \quad \ldots\ldots(3.25.a)$$

$$B \le \begin{array}{l} 6\,t_s + b \\ \dfrac{L_2}{10} + b \\ C.L \text{ to edge} \end{array} \qquad \text{for L-sections} \quad \ldots\ldots(3.25.b)$$

where

$L_2 = L$ *simple beam*

$L_2 = 0.8\,L$ *one end continuous*

$L_2 = 0.7\,L$ *continuous beam*

in which L is the effective span (explained clearly in Chapter Six)

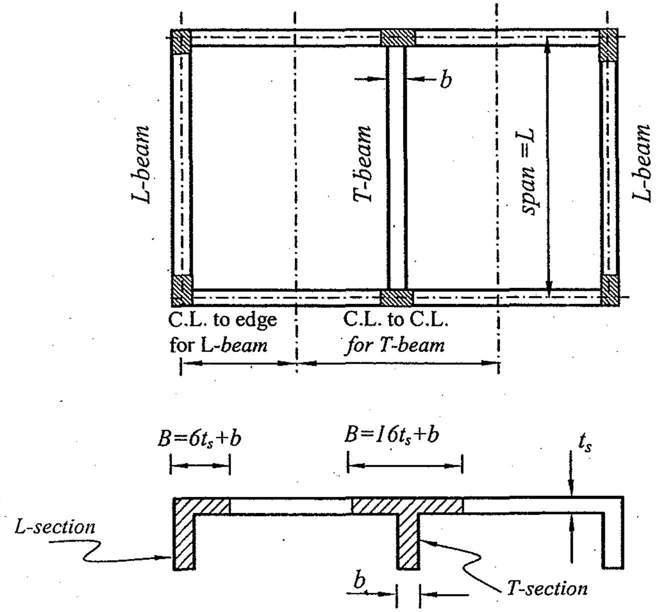


Fig. 3.8 Definition of T and L-beams in the ECP 203

The direction of the bending moment distinguishes between rectangular and T-sections. If the flange participates in resisting the compression stresses resulting from the bending moment, then the section acts as a T-section. On the other hand, if only the web of the beam resists the compression stresses, then the section acts as a rectangular section.

Consider for example the simply supported beam with cantilever shown in Case I of Fig. 3.9. According to the given schematic bending moment diagram, section A-A acts as a T-Sec., while section B-B acts as a rectangular-section. Case II, on the other hand, shows a simply supported beam with cantilever, in which the slab is located at the bottom part of the beam (called an inverted beam). In such a case, section C-C acts as a rectangular section while section D-D acts as a T-Sec.

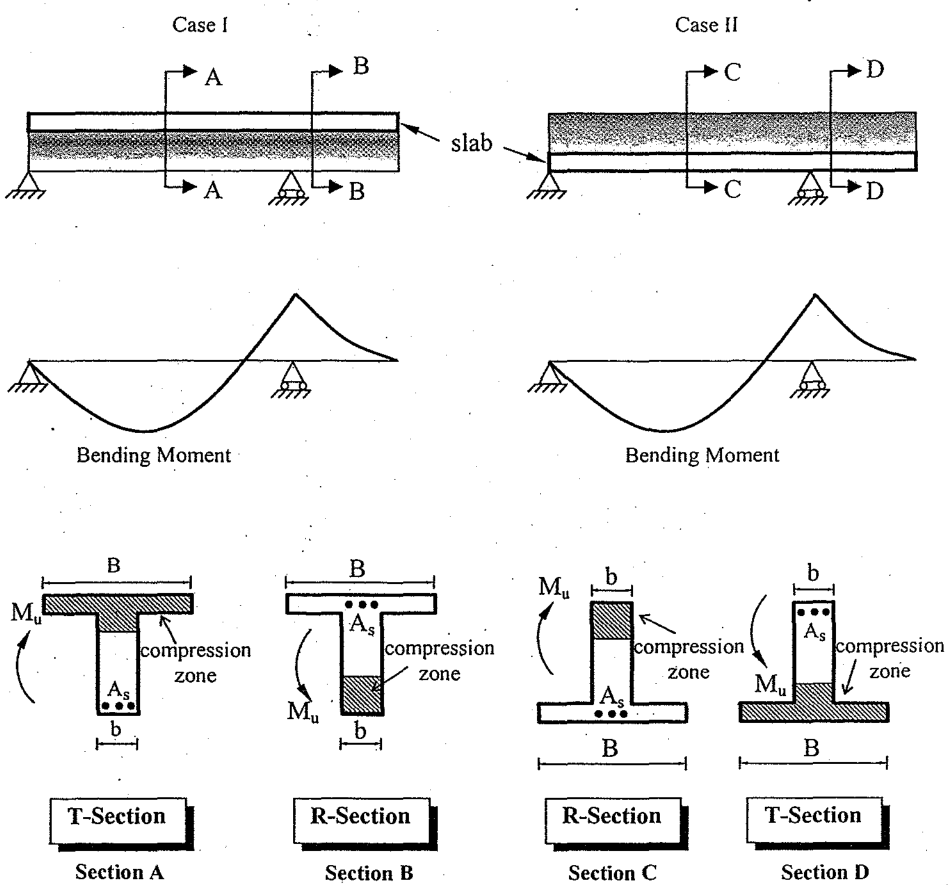
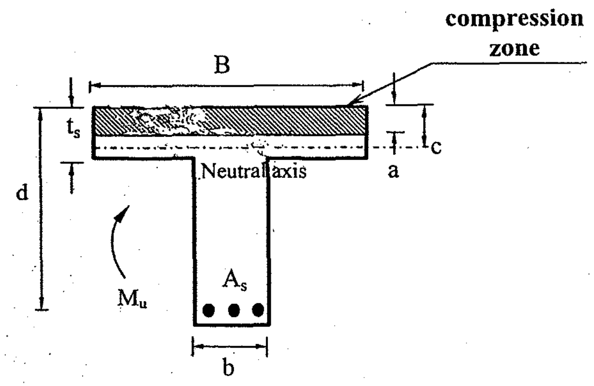
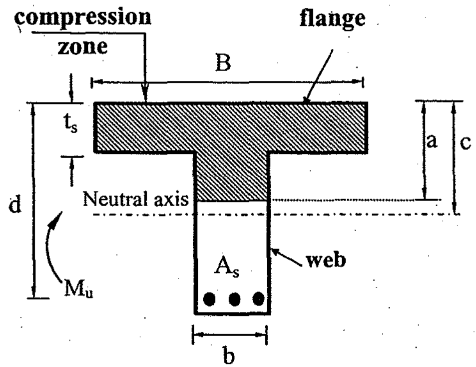


Fig. 3.9 Compression zone for T-beams

When designing a T-Sec., the neutral axis could be located inside the flange (Fig. 3.10.A) or outside the flange (Fig. 3.10.B). Each case shall be analyzed in detail in section 3. 2.3



A-Neutral axis inside the flange,



B-Neutral axis in the web

Fig. 3.10 Location of the neutral axis for T-sections.

3.2.3 Analysis of T-Beams

Case A: Neutral axis inside the flange ($a \leq t_s$)

The details of this case are shown in Fig. 3.11. Applying the equilibrium equation gives:

$$T = C_c$$

$$\frac{0.67\, f_{cu}\, B\, a}{1.5} = \frac{A_s\, f_y}{1.15} \quad \text{.......................................} (3.26)$$

After calculating the stress block distance "a" the nominal moment can be computed by taking the moment around the compression force C_c:

$$M_u = \frac{A_s\, f_y}{1.15}\left(d - \frac{a}{2}\right) \quad \text{.......................................}(3.27.A)$$

In some cases the depth of the neutral axis (a) is very small and less than the minimum required by the code (0.1 d). In such a case, the stress block distance is assumed to be $a=0.1\,d$. Thus Eq. 3.27.A becomes

$$M_u = A_s\, f_y\, (0.826\, d) \quad \text{.......................................}(3.27.B)$$

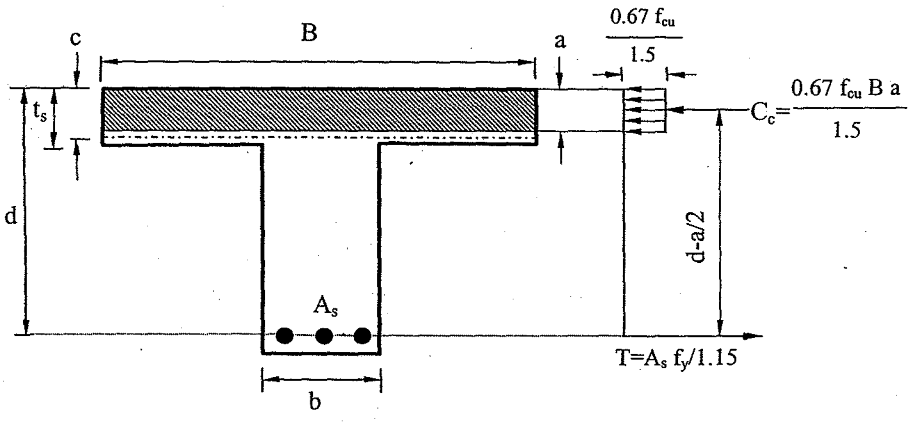


Fig. 3.11 Neutral axis located in the flange ($a<t_s$)

Case B: Neutral axis outside the flange (a >ts)

If the external moment is large enough, the neutral axis will be located outside the flange(Fig. 3.12.A). The ECP 203 permits the use of the stress block in the case of T-beams as stated in clause 4.2.1.1. For the sake of simplicity and for comparison with rectangular sections, the compression zone will be divided in

two parts. The first part is in the flange with a width (B-b) and thickness of t_s (Fig. 3.12.C). The second part is in the web with a width "b" and depth "a" (Fig. 3.12. B).

The force in the flange C_f equals:

$$C_f = \frac{0.67 f_{cu}\,(B-b)\, t_s}{1.5} \quad \text{.......................................} (3.28)$$

The force in the web C_w equals:

$$C_w = \frac{0.67\, f_{cu}\, b\, a}{1.5} \quad \text{.......................................} (3.29)$$

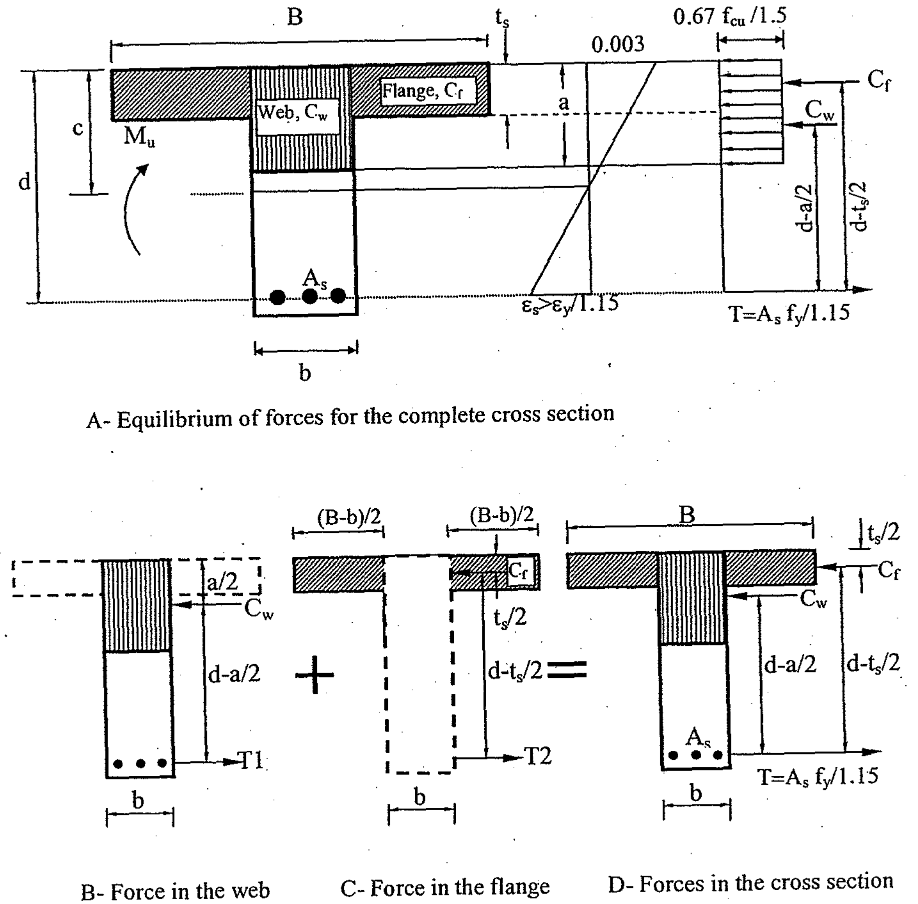


Fig. 3.12 Analysis of T-beam for the case of neutral axis outside the flange

Applying the equilibrium equation

$$T = C_f + C_w \quad \text{.......................................} \quad (3.30)$$

$$\frac{A_s\, f_y}{1.15} = \frac{0.67 f_{cu}\,(B-b)\, t_s}{1.5} + \frac{0.67 f_{cu}\, b\, a}{1.5} \quad \text{.......................} \quad (3.31)$$

Solving Eq. 3.31 gives the equivalent stress block distance *a*.

The moment of the internal forces may be taken around any point such as the location of the tension steel as follows:

$$M_u = C_f\left(d - \frac{t_s}{2}\right) + C_w\left(d - \frac{a}{2}\right) \quad \text{.......................} \quad (3.32)$$

For the sake of simplicity, the code (Section 4-2-1-2-__) permits neglecting the compression part in the web and calculating the compression force as if it is in the flange only as shown in Fig. 3.13. In this case, the ultimate moment is taken the smaller of the following two equations:

$$M_u = \frac{A_s\, f_y}{1.15}\left(d - \frac{t_s}{2}\right) \quad \text{.......................} \quad (3.33a)$$

$$M_u = \frac{0.67 f_{cu}\, B\, t_s}{1.5}\left(d - \frac{t_s}{2}\right) \quad \text{.......................} \quad (3.33b)$$

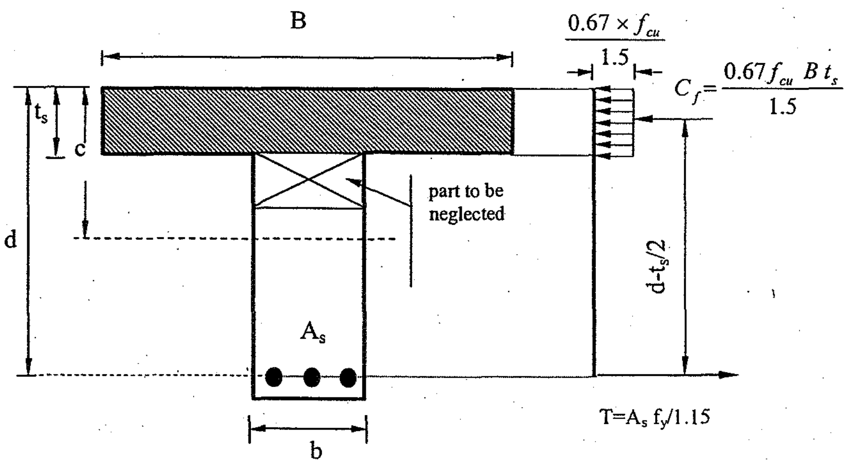


Fig. 3.13 Calculation of the ultimate moment using Code simplifications

3.2.4 Minimum Area of Steel for T-sections

The minimum area for T-sections is the same as the rectangular sections as stated in the ECP 203 section (4-2-1-2-g). The minimum area is related to the web width "*b*", not to the flange width "*B*" as shown in Fig. 3.14.

$$A_{s\,min} = smaller\ of \begin{cases} \dfrac{0.225\sqrt{f_{cu}}}{f_y}\, b\, d > \dfrac{1.1}{f_y}\, b\, d \\ 1.3\, A_s \end{cases} \quad \text{.......................} \quad (3.34)$$

$$but\ not\ less\ than \begin{Bmatrix} \dfrac{0.25}{100}\, b\, d\ (mild\ steel) \\ \dfrac{0.15}{100}\, b\, d\ (high\ grade) \end{Bmatrix} \quad \text{.......................} \quad (3.35)$$

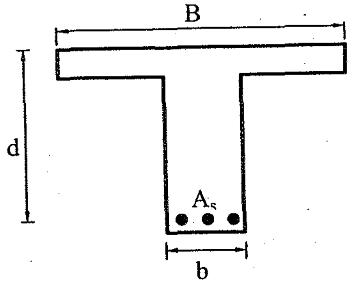


Fig. 3.14 Calculation of A_{smin}

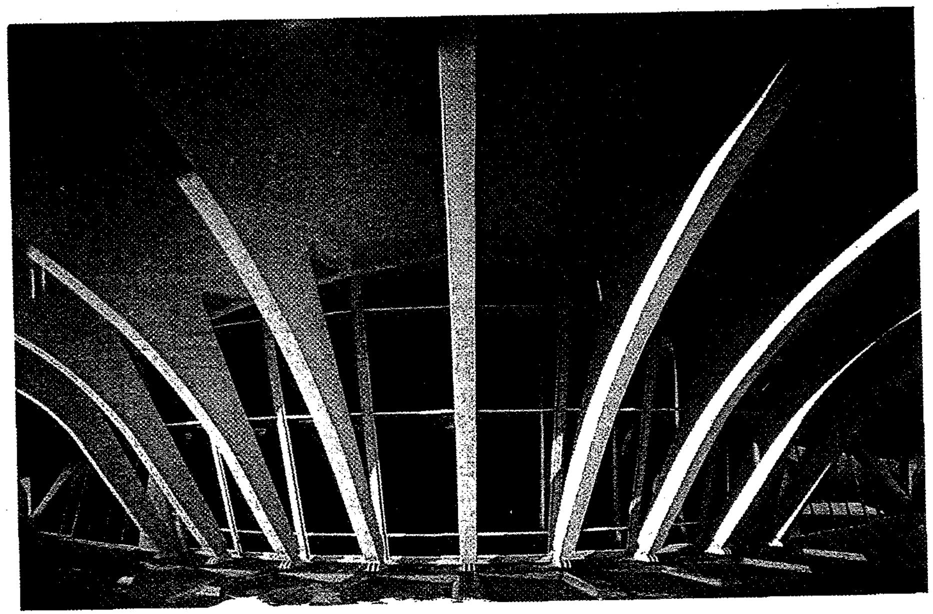


Photo 3.5 A part of an art museum (USA)

3.2.5 Maximum Area of Steel for T-sections

The maximum area of steel allowed for T-sections is usually several times the limits for rectangular sections. Thus, it is rare that a T-beam can exceed the maximum area of steel. It should be mentioned that, whether the neutral axis is located inside the flange (case A) or outside the flange (case B), the ECP 203 uses the same procedure for calculating the maximum area of steel.

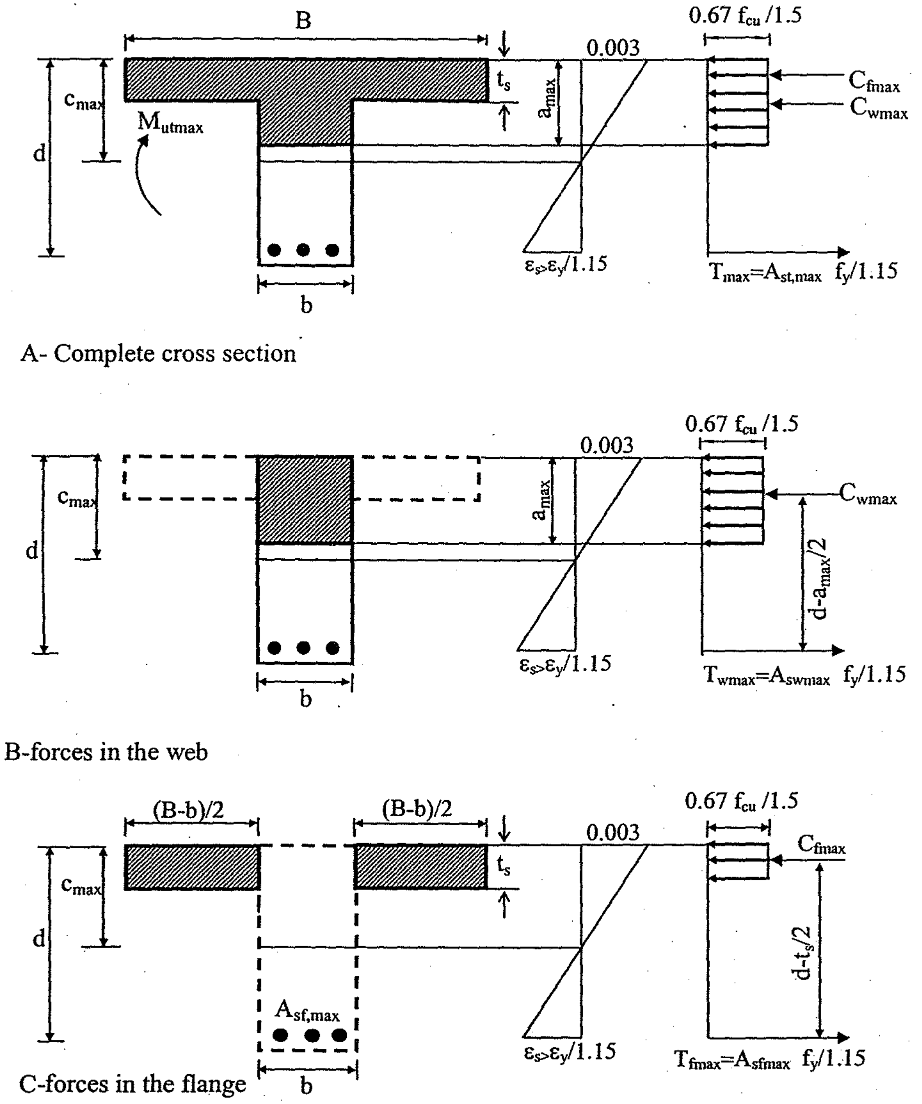


Fig. 3.15 Calculation of the maximum area of steel and moment for T-sections

The maximum area of steel can be calculated by applying the same principle used in rectangular sections. The c_{max}/d values listed in table 2.1 are used to determine the maximum area of steel. Referring to Fig. 3.15 and observing the notations used in that figure, one can drive the following equation:

$$C_{w\max} = \frac{0.67\, f_{cu}\, b\, a_{\max}}{1.5} \quad \text{.......... (3.36)}$$

Applying the equilibrium equation $T_{wmax} = C_{wmax}$ gives

$$A_{sw\max} \frac{f_y}{1.15} = \frac{0.67 f_{cu}\, b\, a_{\max}}{1.5} \quad \text{.......... (3.37)}$$

The procedure for calculating the maximum area of steel for section B is equivalent to that of rectangular section, thus

$$A_{sw\max} = \mu_{\max}\, b \times d \quad \text{.......... (3.38)}$$

where μ_{max} is determined from Table 4-1 in the code or Table 2.1 in Chapter 2.

$$C_{f\max} = \frac{0.67 f_{cu}\,(B-b)\, t_s}{1.5} \quad \text{.......... (3.39)}$$

Applying the equilibrium equation $T_{fmax} = C_{fmax}$, gives

$$A_{f\max} \frac{f_y}{1.15} = \frac{0.67 f_{cu}\,(B-b)\, t_s}{1.5} \quad \text{.......... (3.40)}$$

$$A_{f\max} = 0.5136 \frac{f_{cu}\,(B-b)\, t_s}{f_y} \approx \frac{1}{2} \times \frac{f_{cu}\,(B-b)\, t_s}{f_y} \quad \text{.......... (3.41)}$$

$$A_{st,max} = A_{sw,max} + A_{sf,max} \quad \text{.......... (3.42)}$$

$$\boxed{A_{st,\max} = \mu_{\max}\, b\, d + \frac{f_{cu}\,(B-b)\, t_s}{2 \times f_y}} \quad \text{.......... (3.43)}$$

OR

$$\boxed{A_{st,\max} = \frac{C_{w\max} + C_{f\max}}{f_y/1.15}} \quad \text{.......... (3.44)}$$

The maximum area of steel allowed for a T-section is much bigger than that for a rectangular section especially when the section has a wide flange. Figure 3.16 presents the maximum area of steel for T-sections. It is clear from the figure that the maximum area of steel can be as high as 6-8%(about five-six times more than the maximum allowed for rectangular sections).

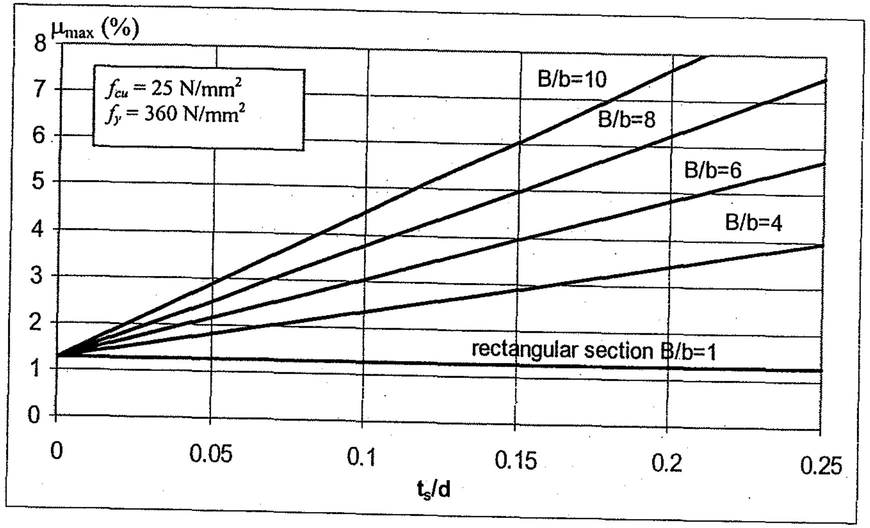


Fig. 3.16 Maximum area of steel for T-sections

Maximum Moment For T-sections

Referring to Fig. 3.15 and summing the moment of the forces around the tension steel

$$M_{ur,max} = C_{w\max}\left(d - \frac{a_{\max}}{2}\right) + C_{f\max}\left(d - \frac{t_s}{2}\right) \quad \text{.......................... (3.45)}$$

The part in the web is equivalent to a singly reinforced section, thus

$$M_{ur,max} = \frac{R_{\max} f_{cu}\, b\, d^2}{1.5} + C_{f\max}\left(d - \frac{t_s}{2}\right) \quad \text{.......................... (3.46)}$$

Note: It should be clear that if the calculated neutral axis depth c is less than the maximum allowed value c_{max}, then there is no need to check the maximum area of steel or maximum moments

$$\boxed{If\ \frac{c}{d} \le \frac{c_{max}}{d} \quad then \begin{cases} f_s = f_y/1.15 \\ \mu < \mu_{t\max} \\ A_s < A_{st,\max} \\ M_u < M_{ur,\max} \end{cases}} \quad \text{.......................... (3.47)}$$

3.2.6 Design of T-sections Using First Principles

Given : f_{cu}, f_y, M_u, b, B

Required : A_s and d

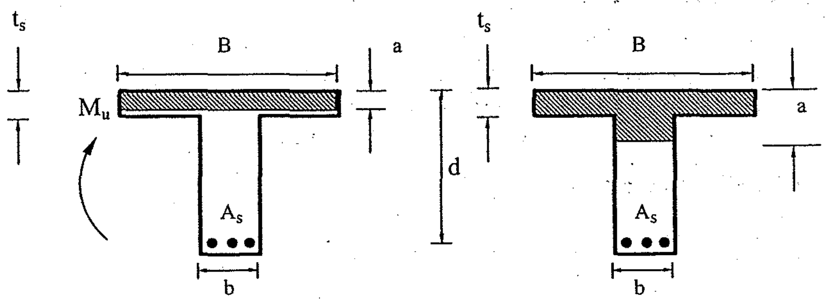
Unknowns: a, A_s, d

The design procedure can be summarized in the following steps:

1. Make the necessary assumptions (either)
   i. Assume d
   ii. Or, estimate μ or A_s as discussed in Chapter 2(see example 3.13)
2. Assume that the neutral axis is inside the flange then determine the depth of the stress block a using the equilibrium equation:

$$\frac{0.67\, f_{cu}\, B\, a}{1.5} = \frac{A_s\, f_y}{1.15}$$

3.Calculate the beam depth using the moment equation as follows:

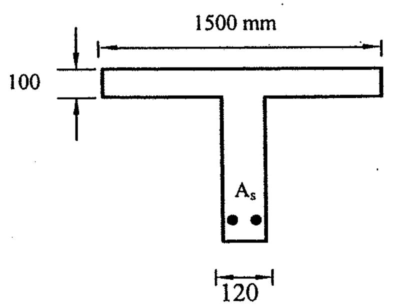


if a < t_s then calculate the depth using Eq. 3.27.A or Eq. 3.27.B	If a> t_s calculate the depth using the simplified Eq. 3.33
$M_u = \frac{A_s\, f_y}{1.15}\left(d - \frac{a}{2}\right)$ a>0.1d (3.27.A) $M_u = 0.826 A_s f_y\, d$ a<0.1d (3.27.B)	$M_u = \frac{A_s\, f_y}{1.15}\left(d - \frac{t_s}{2}\right)$

3. Determine A_s (only if you used 1-i, skip this step if you used 1-ii)
   $A_s = \mu\, b\, d \quad , \quad A_s' = 0$
4. Check minimum area of steel
5. Check maximum area steel and maximum moment ($c/d < c_{max}/d$)

Example 3.11

The figure shows a T-section that is subjected to an ultimate moment of a value of 220 kN.m. Using the first principles, calculate the required depth and area of steel. The material properties are $f_{cu} = 25$ N/mm^2 and $f_y = 360$ N/mm^2



Solution

Step 1: Assumptions

In this example we have three unknowns a, d, A_s and we have only two equilibrium equations, thus we shall assume one unknown (A_s)
Assume $A_s = 0.01$ b d = 0.01 x 120 x d= 1.2 d
Assume that the neutral axis inside the flange (a<ts)

Step 2: Determine a

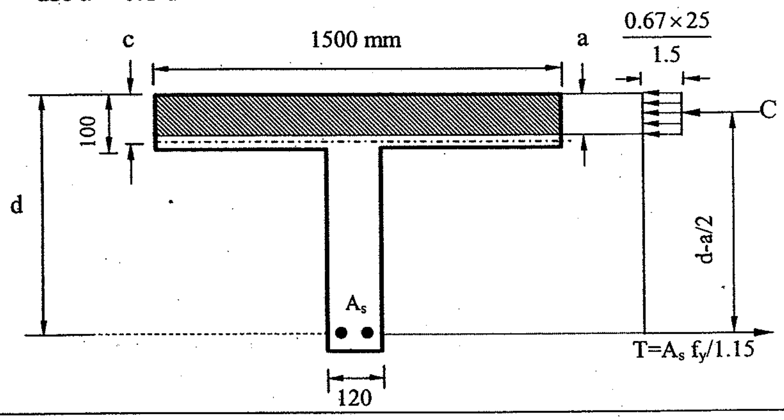
$$\frac{0.67\, f_{cu}\, B\, a}{1.5} = \frac{A_s\, f_y}{1.15}$$

$$\frac{0.67 \times 25 \times 1500 \times a}{1.5} = \frac{1.2\, d \times 360}{1.15}$$

a = 0.0224 d

a/d = 0.0224 < $a/d)_{min}$(0.1)

use a = 0.1 d



Step 3: Calculate d

$$M_u = \frac{A_s\, f_y}{1.15}\left(d - \frac{a}{2}\right) = 0.826\, A_s\, f_y\, d$$

$$220 \times 10^6 = (0.826) \times 1.2\, d \times 360 \times d = 356.86\, d^2$$

d = 785.2 mm

a = 0.1 d = 78.5 mm < t_s (our assumption is correct a<t_s)

A_s = 1.2 d =942.2 mm^2

Use d = 800 mm , As = 982 mm^2 (2Φ25)

Step 4: Check A_{smin}

$$A_{s\min} = smaller\ of \begin{cases} \dfrac{0.225\sqrt{25}}{360} 120 \times 800 = 300\ mm^2 \\ 1.3 \times 942.2 = 1224.8\ mm^2 \end{cases} = 293.33\ mm^2 < A_s \cdots o.k$$

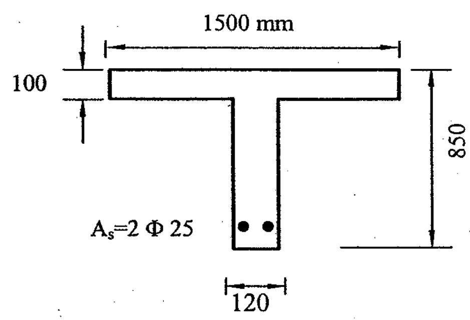
Step 5: Check $A_{st,max}$, $M_{ut,max}$

From the code c_{max}/d = 0.44

a/d= 0.1 → c/d=0.125

Since c/d (0.125) <c_{max}/d(0.44) then M_u<M_{umax} and A_s<A_{smax}

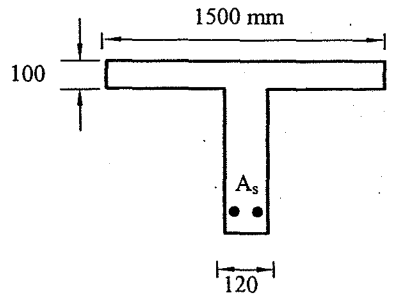
Final design d = 800 mm, t = 850 mm and A_s = 2Φ25 (9.82 cm^2)



Final Design

Example 3.12

Calculate the maximum area of steel and maximum moment for the section given in example 3.11. $f_{cu} = 25\ N/mm^2$ and $f_y = 360\ N/mm^2$



Solution

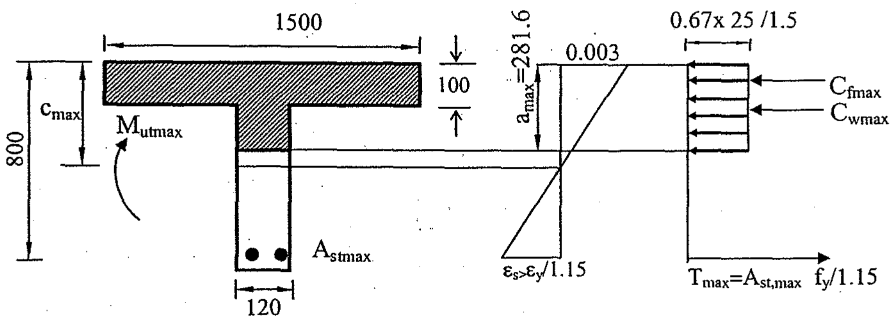
Step 1: Calculation of Maximum area of steel

From Table 2.1 $c_{max}/d=0.44$ for $f_y = 360\ N/mm^2$

$$a_{max} = 0.8\ c_{max} = 0.8 \times 0.44 \times 800 = 281.6\ mm$$

$$C_{w\max} = \frac{0.67\ f_{cu}\ b\ a_{\max}}{1.5} = \frac{0.67\times25\times120\times281.6}{1.5} = 377344\ N$$

$$C_{f\max} = \frac{0.67 f_{cu}\ (B-b)\ t_s}{1.5} = \frac{0.67\times25\times\ (1500-120)\ 100}{1.5} = 1541000\ N$$



To find the maximum area of steel, apply the equilibrium

$$A_{st,\max} = \frac{C_{w\max} + C_{f\max}}{f_y/1.15}$$

$$A_{st,\max} = \frac{377344+1541000}{360/1.15} = 6128\ mm^2 > A_s \cdots ok$$

OR use Eq. 3.43

From Table 2.1, $\mu_{max} = 5 \times 10^{-4} f_{cu}$

$\mu_{max} = 5 \times 10^{-4} f_{cu} = 5 \times 10^{-4} \times 25 = 0.0125$

$$A_{st,\max} = \mu_{\max} b\ d + \frac{f_{cu}(B-b)t_s}{2\times f_y} = 0.0125\times120\times800 + \frac{25(1500-120)100}{2\times360} \approx 6000\ mm^2$$

Step 2: Calculation of Maximum Moments

To find the maximum moment take the moment around tension steel

$$M_{ut,\max} = C_{w\max}\ (d - \frac{a_{\max}}{2}) + C_{f\max}(d - \frac{t_s}{2})$$

$$M_{ut,\max} = 377344\,(800 - \frac{281.6}{2}) + 1541000\,(800 - \frac{100}{2}) = 1404.5\ kN.m \quad > M_u \ldots ok$$

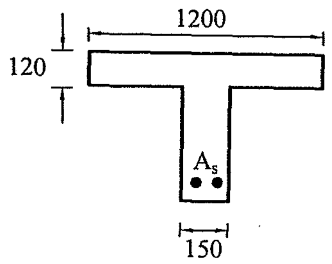
OR use Eq. 3.46

$$M_{ut,\max} = \frac{R_{\max} f_{cu}\ b\ d^2}{1.5} + C_{f\max}(d - \frac{t_s}{2})$$

$$M_{ut,\max} = \frac{0.194\times25\times120\times800^2}{1.5} + 1541000\times(800 - \frac{100}{2}) = 1404\ kN.m$$

Example 3.13 T Sections ($a<t_s$)

The figure shows a T-section that is subjected to an ultimate moment of a value of 280 kN.m. Using the first principles, calculate the required depth and area of steel. Check the code limits for maximum and minimum area of steel and M_{umax}
$f_{cu} = 30$ N/mm^2 and $f_y = 400$ N/mm^2



Solution

Step 1: Assumptions

In this example we have three unknowns a, d, A_s and we have only two equilibrium equations, thus we shall assume one (A_s)

1. Assume that $a<t_s$
2. The area of steel may be assumed as

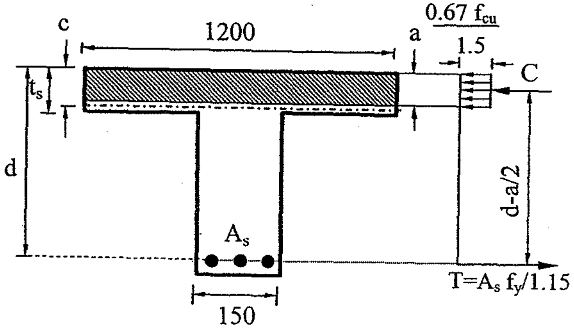
$$A_s = 0.11\sqrt{\frac{M_u\, b}{f_y}} = 0.11\sqrt{\frac{280\times10^6\times150}{400}} = 1127\ mm^2$$

Step 2: Calculate a

$$\frac{0.67\, f_{cu}\, B\, a}{1.5} = \frac{A_s\, f_y}{1.15}$$

$$\frac{0.67\times30\times1200\times a}{1.5} = \frac{1127\times400}{1.15}$$

a = 24.38 mm $\quad\longrightarrow$ Our assumption is correct $a<t_s$



Step 3: Calculate d

Assume $a<0.1d$, thus use $a = 0.1d$

use Eq. (3.27.B)

$$M_u = A_s\, f_y\, (0.826\, d)$$

$$280\times10^6 = 0.826\times1127\times400\times d$$

d = 751.8 mm

a (24.38 mm)< 0.1 d (75.18 mm) our assumption is correct $a<0.1d$

Use d = 800 mm , $A_s = 11.27$ cm^2

Step 4: Check A_{smin}

$$A_{s\,min} = smaller\ \ of \begin{cases} \dfrac{0.225\sqrt{30}}{400}\,150\times800 = 370\ mm^2 \\ \\ 1.3\times1127 = 1465\ mm^2 \end{cases} = 370\ mm^2 < A_s \cdots o.k$$

Step 5: Check A_{stmax}, $M_{ut,max}$

From the code $c_{max}/d = 0.42$

$a/d = 0.1 \rightarrow c/d = 0.125$

Since c/d (0.125) $<c_{max}/d(0.42)$ then $M_u<M_{ut,max}$ and $A_s<A_{st,max}$

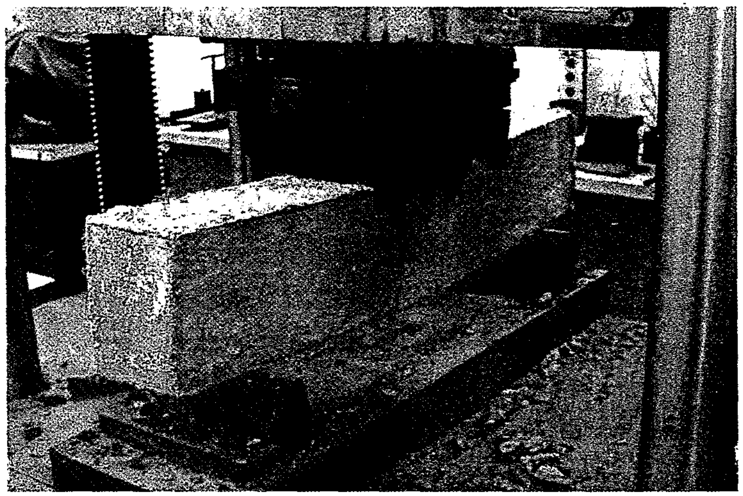
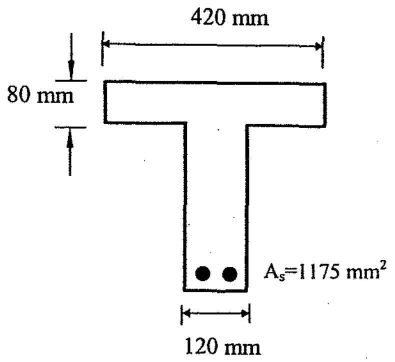


Photo 3.6 Testing of simply supported reinforced concrete beam under flexure

Example 3.14 ($a>t_s$)

Compute the depth for the T-section shown in figure if it is subjected to an ultimate moment of 380 kN.m using first principles.

$f_{cu} = 22.5\ N/mm^2$ and $f_y = 400\ N/mm^2$



Solution

Step 1: Assumptions

In this example we have two unknowns a, d (A_s is given). Thus the calculation of a can proceed immediately

Step 2: Calculate a

Assume that a<ts

$$\frac{0.67\, f_{cu}\, a\, B}{1.5} = \frac{A_s\, f_y}{1.15}$$

$$\frac{0.67 \times 22.5 \times 420 \times a}{1.5} = \frac{1175 \times 400}{1.15}$$

a = 96.82 mm

Since a > t_s we can use the approximate equation (Eq.3.33) to find "d"

Step 3: Calculate d

$$M_u = \frac{A_s\, f_y}{1.15}\left(d - \frac{t_s}{2}\right) \qquad \text{.......(3.33a)}$$

$$380 \times 10^6 = \frac{1175 \times 400}{1.15}\left(d - \frac{80}{2}\right)$$

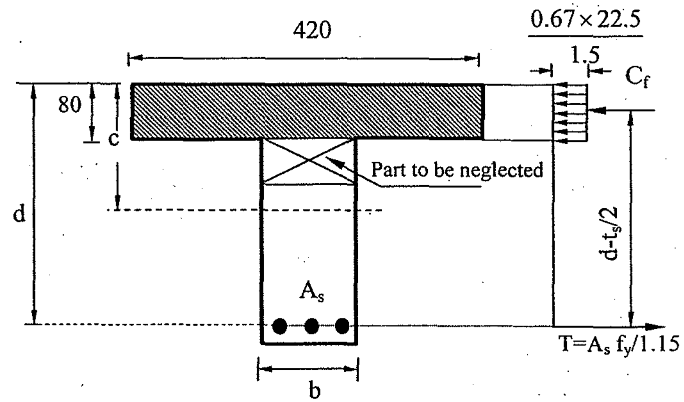
d = 969.8 mm →→→(1)

$$M_u = \frac{0.67 f_{cu}\, B\, t_s}{1.5}\left(d - \frac{t_s}{2}\right) \text{.......(3.34a)}$$

$$380 \times 10^6 = \frac{0.67 \times 22.5 \times 420 \times 80}{1.5}\left(d - \frac{80}{2}\right)$$

d = 1165 mm →→→(2)

Take the largest d from (1) and (2) →→d = 1200 mm



Step 4: Calculate A_{smin}

$$A_{s\min} = smaller\ of \begin{cases} \dfrac{1.1^*}{400} 120 \times 1200 = 396\ mm^2 \\ \\ 1.3 \times 1175 = 1526\ mm^2 \end{cases} = 396\ mm^2 < A_s \ldots o.k$$

* 1.1 is bigger than $0.225\sqrt{f_{cu}}$

Step 5: Check A_{stmax}, $M_{ut,max}$

To determine the exact position of the neutral axis, assume ($a>t_s$).

$$\frac{A_s\, f_y}{1.15} = \frac{0.67 f_{cu}\,(B-b)\, t_s}{1.5} + \frac{0.67 f_{cu}\, b\, a}{1.5}$$

$$\frac{1175 \times 400}{1.15} = \frac{0.67 \times 22.5 \times (420-120)\, 80}{1.5} + \frac{0.67 \times 22.5 \times 120\, a}{1.5}$$

a = 138.9 mm

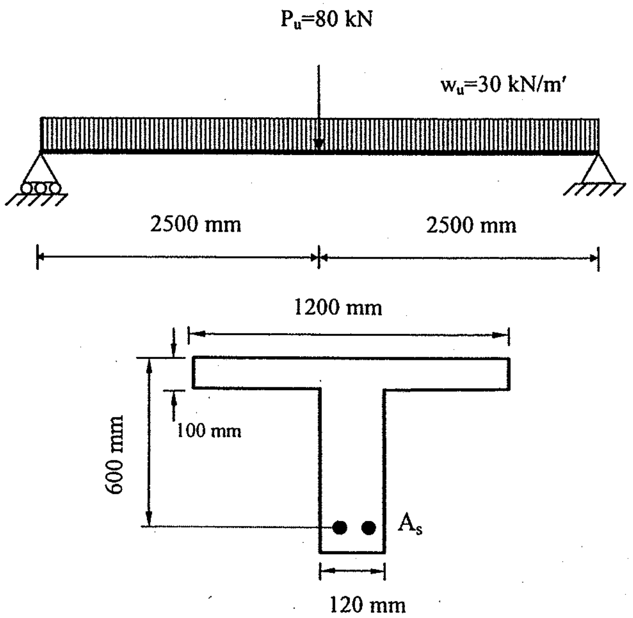
$$\frac{c}{d} = \frac{138.9/0.8}{969.8} = 0.179$$

For $f_y = 400\ N/mm^2$ it can determined from Table 2.1 that $c_{max}/d = 0.42$

Since $c/d < c_{max}/d$, thus steel yields, $A_s < A_{st,max}$ and $M_u < M_{ut,max}$ ....o.k

Example 3.15

The figure below shows a simply supported in which the midspan section has a T-shape. Compute the area of steel for the T-section shown in figure. f_{cu} = 30 N/mm^2 and f_y = 360 N/mm^2



Solution

Step 1: Assumptions

Assume that the neutral axis inside the flange (a<ts)

Step 2: Compute a

$$\frac{0.67\, f_{cu}\, B\, a}{1.5} = \frac{A_s\, f_y}{1.15}$$

$$\frac{0.67 \times 30 \times 1200 \times a}{1.5} = \frac{A_s \times 360}{1.15}$$

a = 0.019 A_s

Step 3: Compute A_s

The critical section is at mid span

$$M_u = \frac{30 \times 5^2}{8} + 80 \times \frac{5}{4} = 193.75\ kN.m$$

Assume a<0.1d, thus use a = 0.1d, use Eq. (3.27.B)

$$M_u = A_s\, f_y\, (0.826\, d)$$

$$193.75 \times 10^6 = A_s \times 360 \times 0.826 \times 600$$

A_s = 1086 mm^2

a = 0.019 (1086) = 20.6 mm

a (20.6) < t_s (100) our assumption is correct a<ts

a (20.6) < 0.1d (60) our assumption is correct a<0.1d

Step 3: Check A_{smin}

$$A_{s\min} = smaller\ \ of \begin{cases} \dfrac{0.225\sqrt{30}}{360} 120 \times 600 = 246\ mm^2 \\ \\ 1.3 \times 1085 = 1410\ mm^2 \end{cases} = 246\ mm^2 < A_s \cdots o.k$$

Step 5: Check $A_{st,max}$, $M_{ut,max}$

From Table 2.1 c_{max}/d = 0.44

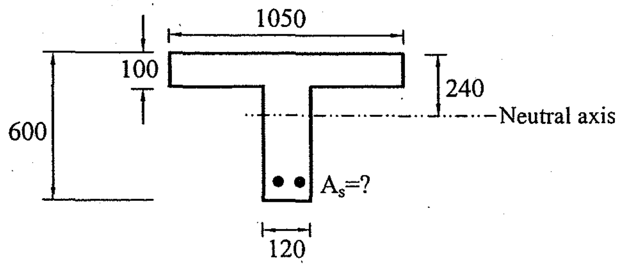
a/d=0.1 → c/d= 0.125

Since c/d (0.125) <c_{max}/d(0.44) then M_u<$M_{ut,max}$ and A_s<$A_{st,max}$

Example 3.16 T Sections

For the neutral axis position given in the figure below, calculate the required area of steel and the ultimate moment. Check the code limits for maximum and minimum area of steel and M_{umax}

$f_{cu} = 25$ N/mm^2 and $f_y = 360$ N/mm^2



Solution

Step 1: Assumptions

Given : $f_{cu}, f_y, M_u, B, b, d, a$
Required : A_s, M_u
Unknowns : A_s, M_u

Since we have only two equilibrium equations and two unknowns (A_s, M_u), we can proceed directly without any further assumptions

Step 2: Calculate A_s

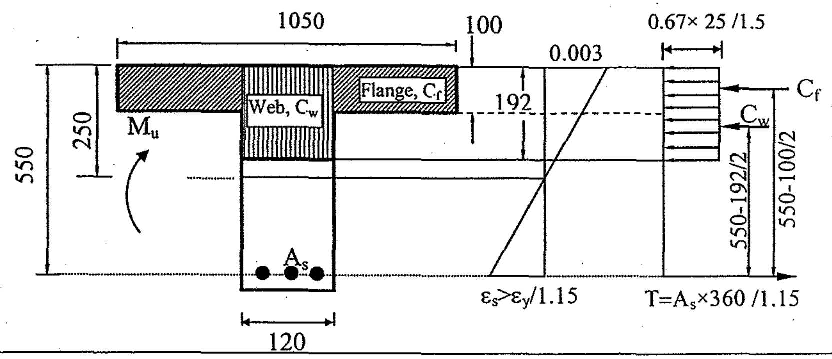
$d = t -$ cover=600-50=550 mm

$a = 0.8\,c$=0.8×240=192 mm

$$\frac{A_s\,f_y}{1.15} = \frac{0.67\,f_{cu}\,(B-b)\,t_s}{1.5} + \frac{0.67\,f_{cu}\,b\,a}{1.5}$$

$$\frac{A_s \times 360}{1.15} = \frac{0.67\times 25\times(1050-120)\,100}{1.5} + \frac{0.67\times 25\times 120\times 192}{1.5}$$

$A_s = 4139.30$ mm^2



Step 3: Calculate Mu

$$M_u = C_f\left(d - \frac{t_s}{2}\right) + C_w\left(d - \frac{a}{2}\right)$$

$$M_u = \frac{0.67\times 25\times(1050-120)\,100}{1.5}\left(550 - \frac{100}{2}\right) + \frac{0.67\times 25\times 120\times 192}{1.5}\left(550 - \frac{192}{2}\right) = 636.05 \text{ kN.m}$$

Step 4: Check A_{smin}

$$A_{s\,min} = smaller\ \ of \begin{cases} \dfrac{0.225\sqrt{25}}{360}\,120\times 550 = 206\ mm^2 \\[2ex] 1.3\times 4139.30 = 5381\ mm^2 \end{cases} = 206\ mm^2 < A_s \cdots o.k$$

Step 5: Check A_{stmax}, $M_{ut,max}$

$$\frac{c}{d} = \frac{240}{550} = 0.436$$

From the code $c_{max}/d = 0.44$

Since c/d (0.436) <c_{max}/d(0.44) then $M_u < M_{ut,max}$ and $A_s < A_{st,max}$

3.2.7 Design of T-sections Using Curves

3.2.7.1 Development of the Curves

Equilibrium equations are used to generate design aids for T-sections. Assume that the neutral axis is inside the flange ($a<t_s$)
Taking moment around the concrete force and referring to Fig. 3.17

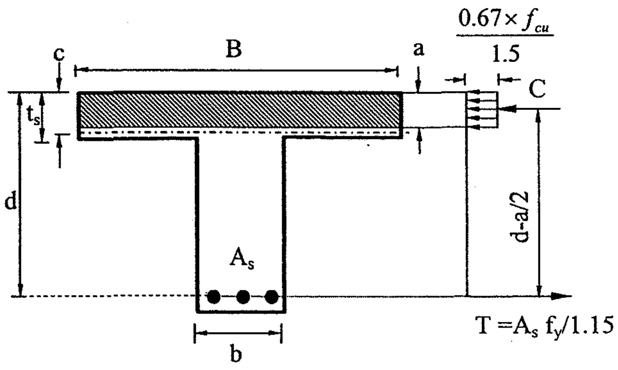


Fig. 3.17 Equilibrium of force for T-section

$$M_u=\frac{A_s\,f_y}{1.15}\left(d-\frac{a}{2}\right)=\frac{A_s\,f_y\,d}{1.15}\left(1-\frac{a}{2\,d}\right) \quad \text{.......................} (3.48)$$

we can also express the moment as the tension force (A_s f_y) multiplied by the lever arm (jd). Note that the factor 1.15 is included in the coefficient "*J*"

$$M_u=A_s\,f_y\,j\,d \quad \text{...} (3.49)$$

Comparing Eq. 3.48 and Eq. 3.49, one can determine J as

$$\boxed{j=\frac{1}{1.15}\left(1-\frac{1}{2}\times\frac{a}{d}\right)=\frac{1}{1.15}\left(1-0.4\frac{c}{d}\right)} \quad \text{...........................} (3.50)$$

Dividing Eq. 3.48 by ($f_{cu}\,B\,d^2$) and noting that in case of T-section $A_s=\mu$ B d

$$\frac{M_u}{f_{cu}\,B\,d^2}=\frac{\mu\,f_y}{1.15\,f_{cu}}\left(1-0.4\,\frac{c}{d}\right) \quad \text{.................................} (3.51)$$

From the equilibrium of forces (C = T) Fig. 3.17 we can determine that:

$$\frac{0.67\,f_{cu}\,B\,a}{1.5}=\frac{A_s\,f_y}{1.15}$$

$$\frac{a}{d}=1.9468\,\frac{\mu\times f_y}{f_{cu}} \quad \text{..} (3.52)$$

Substitution in equation 3.51 gives

$$\frac{M_u}{f_{cu}\,B\,d^2}=\frac{0.4109}{1.15}\,\frac{c}{d}\left(1-0.4\frac{c}{d}\right) \quad \text{.....................................} (3.53)$$

Define $R_T=\dfrac{M_u}{f_{cu}\,B\,d^2}$

$$R_T=\frac{0.4109}{1.15}\,\frac{c}{d}\left(1-0.4\frac{c}{d}\right) \quad \text{.....................................} (3.54)$$

$$d=C1\sqrt{\frac{M_u}{f_{cu}\,B}} \quad \text{..} (3.55)$$

Comparing Eq. 3.53 with Eq. 3.55 gives

$$C1=\sqrt{\frac{1}{R_T}}=\sqrt{\frac{2.8}{\frac{c}{d}\left(1-0.4\,\frac{c}{d}\right)}} \quad \text{.....................................} (3.56)$$

It can be determined from Eq. 3.49 that the area of steel equals

$$A_s=\frac{M_u}{f_y\,j\,d} \quad \text{..} (3.57)$$

Equations 3.50 and 3.53 are the basis of design aids (C1-J) and (R_T-J)
Figure 3.18 shows an example of (C1-J). Appendix A contains (C1-J) and (R_T-J) design charts.

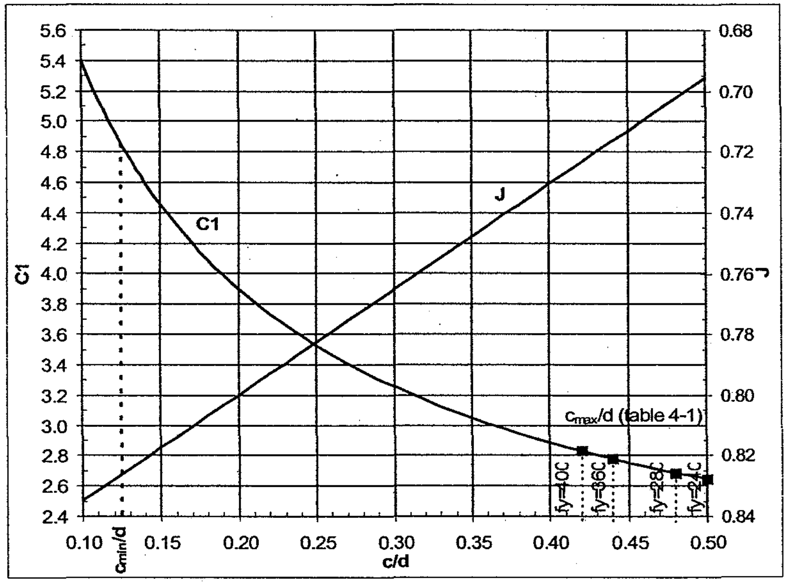


Fig. 3.18 C1-J design chart

Substituting with the minimum a_{min}/d allowed by the code (c_{min}/d=0.125) in Eq. 3.56 gives C1=4.85. Thus any value for C1 above 4.85 will lead to c/d below the minimum code value. In this case use c_{min}/d=0.125 and J=0.826.

Fig 3.19 presents different design cases when using (C1-J) curve. In normal design situations, case I is the most frequently used. When using chart C, the same rules apply, if R_T<0.042 then; use J = J_{max}, if R_T> 0.042 then determine J from the curve, and if $R_T > R_{Tmax}$ increase d.

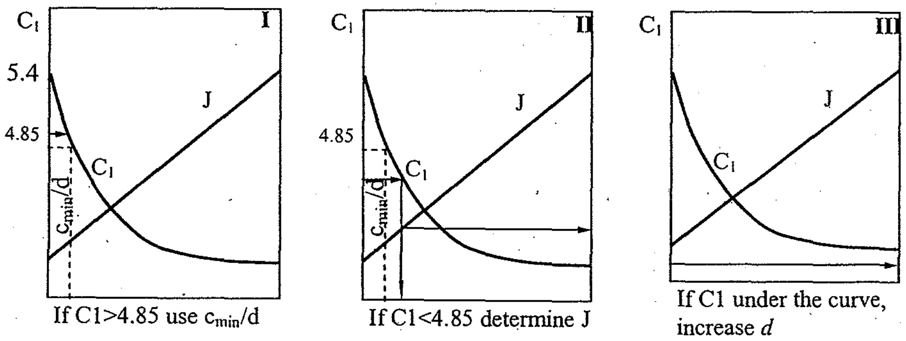


Fig 3.19 Different cases for Using C1-J curve

Note1: The code minimum value for the depth of the stress block ratio "*a*" is 0.1d. Thus substituting with a/d = 0.1 in Eq. 3.50 gives j_{max}

$$\boxed{j_{max} = \frac{1}{1.15}\left(1-\frac{1}{2}\times 0.1\right) = 0.826}$$

Note 2 : To estimate the depth of the beam, assume C1 equals

C1=3-4 for Rectangular beams
C1=4-5 for slabs
C1=5-10 for T- beams

3.2.7.2 Using the Design Aids (charts C1-J and R_T-J)

Step 1: If "*d*" is not given, assume C1=5-10 for T-sections and C1=3-4 for R-sections and determine *d* from the relation

$$d = C_1\sqrt{\frac{M_u}{f_{cu}\,B}}$$

If C_1 in *(C1-J)* > 4.85 or R_T in *(R_T-J)*<0.042 then, use J=J_{max}=0.826 and goto step 4

Step 2: From the curve determine J value

If c/d < c_{min}/d, then use J=J_{max}=0.826

If c/d > c_{max}/d change the cross section (increase *d*)

Step 3: calculate a=0.8 c

check whether a<t_s or a>t_s

Step 4: Determine A_s

$$A_s = \begin{cases} if\ a \le t_s \cdots A_s = \dfrac{M_u}{f_y . J\, d} \\ if\ a > t_s \cdots A_s = \dfrac{M_u}{f_y/1.15\ (d - t_s/2)} \end{cases}$$

Step 5: Check $A_{s,min}$

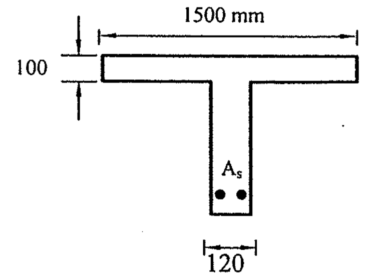
$$\boxed{A_{s\,min} = smaller\ \ of \begin{cases} \dfrac{0.225\sqrt{f_{cu}}}{f_y}b\,d \\ 1.3A_s \end{cases}}$$

(use *b* not *B* in this relation)

Note: Both (C1-J) and (R_T-J) can be used for designing a rectangular section by replacing B with b

Example 3.17 (a<ts)

The figure shows a T-section that is subjected to an ultimate moment of a value of 220 kN.m. Using C1-J, calculate the required depth and area of steel.
$f_{cu} = 25$ N/mm^2 and $f_y = 360$ N/mm^2



Solution

Step 1: Determine d

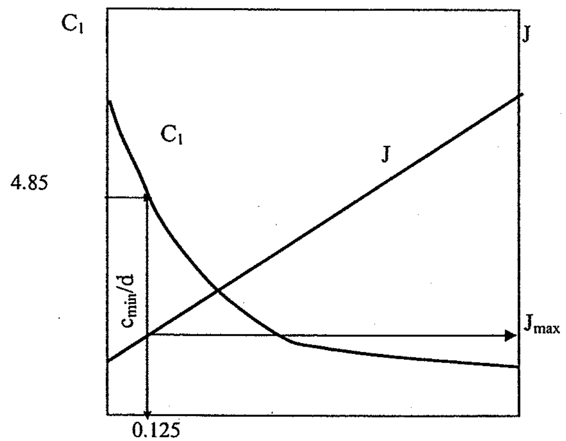
Assume C1 = 10 mm as a first trial

$$d = C_1 \sqrt{\frac{M_u}{f_{cu} B}}$$

$$d = 10\sqrt{\frac{220\times10^6}{25\times1500}} = 765\ mm \quad say\ d{=}800\ mm \quad and\ t{=}850\ mm$$

Using C1-J, one can find that

$C_1 = 10.0 > 4.85$ (out side the curve)



Step 2: Determine J and check c/d limits

Use c_{min}/d and $J = J_{max} = 0.826$
because $c/d < c_{max}/d$, the condition that $A_s < A_{smax}$ is satisfied

Step 3: Compute "a"

$a = 0.8\ (0.125)\ 800 = 80$ mm $< t_s(100mm)$ $a<t_s$

Step 4: Calculate area of steel, A_s

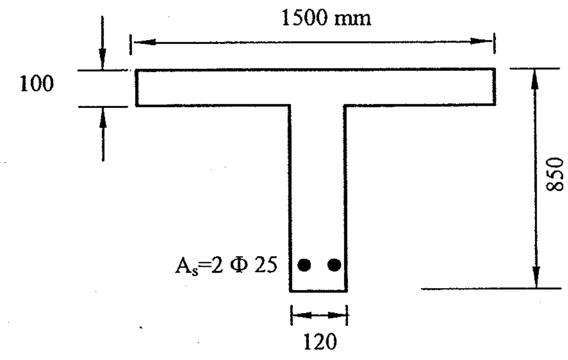
The stress block is located inside the slab, continue using the following relation

$$A_s = \frac{M_u}{f_y\ J\ d}$$

$$A_s = \frac{220\times10^6}{360\times0.826\times800} = 924.8\ mm^2\ (2\Phi\ 25)$$

Step 5: Check A_{smin}

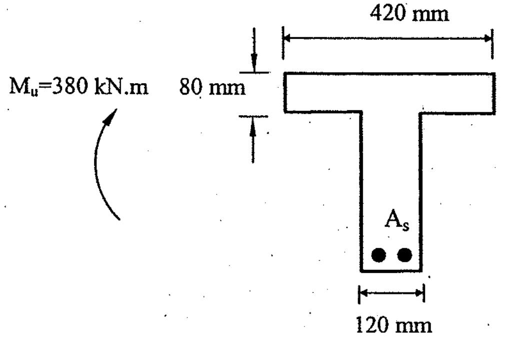
$$A_{s\,min} = smaller\ of \begin{cases} \dfrac{0.225\sqrt{25}}{360}120\times800 = 300\ mm^2 \\ 1.3\times924.8 = 1202\ mm^2 \end{cases} = 300\ mm^2 < A_s \cdots o.k$$



Final Design

Example 3.18 (a>ts)

Compute the depth and area of steel for the T-section shown in figure if it is subjected to an ultimate moment of 380 kN.m using C1-J curve.
$f_{cu} = 22.5$ N/mm^2 and $f_y = 400$ N/mm^2



Solution

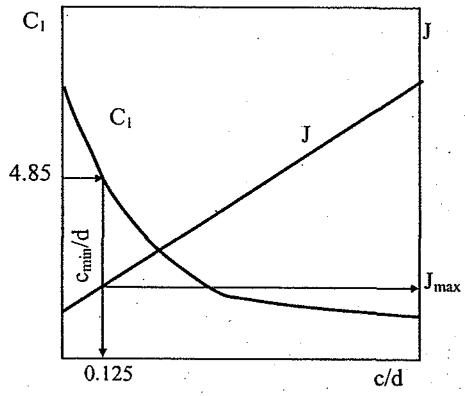
Step 1: Determine d

Assume that C1=5.0

$$d = C1\sqrt{\frac{M_u}{f_{cu}\,B}}$$

$$d = 5\times\sqrt{\frac{380\times10^6}{22.5\times420}} = 1002\ mm \quad say\ d = 1000\ mm\ and\ t = 1050\ mm$$

C1 = 5 > 4.85 (outside the curve), use c/d_{min}



Step 2: Determine J and check c/d limit

Use c_{min}/d

$c/d = 0.125$

Because $c/d < c_{max}/d$, then $A_s < A_{smax}$ and $M_u < M_{utmax}$

Step 3: Compute "a"

$a = 0.8\ (0.125)\ 1000 = 100\ mm > t_s(80mm)$ ....($a>t_s$, outside the flange)

Step 4: Calculate area of steel, A_s

Since the neutral axis is outside the flange, the part of the compression force in the web will be neglected and code approximate equation as follows.

$$A_s = \frac{M_u}{f_y/1.15\times(d-t_s/2)} = \frac{380\times10^6}{400/1.15\times(1000-80/2)} = 1138\ mm^2$$

Step 5: Check A_{smin}

$$A_{s\,min} = smaller\ of \begin{cases} \frac{1.1^*}{400}120\times1000 = 330\ mm^2 \\ 1.3\times1138 = 1479\ mm^2 \end{cases} = 330\ mm^2 < A_s \cdots o.k$$

*Note that 1.1 is greater than $0.225\sqrt{22.5}$

3.3 Design of L-Sections

L-sections are often encountered in external beams of reinforced concrete structures. If such a beam is connected to a slab it will be only allowed to deflect in the vertical axis and the neutral axis will be very close to horizontal as shown Fig 3.20A. The analysis in this case is the same as T-beams except with smaller width Eq. 3.25.B. However, if the beam is allowed to deflect in both vertical and horizontal directions (isolated beam), the neutral axis will be inclined as indicated in (Fig. 3.20B). Since the applied loads do not cause any moments laterally, the compression and tension forces must be in a vertical plane as the applied loads as shown in Fig. 3.20B.

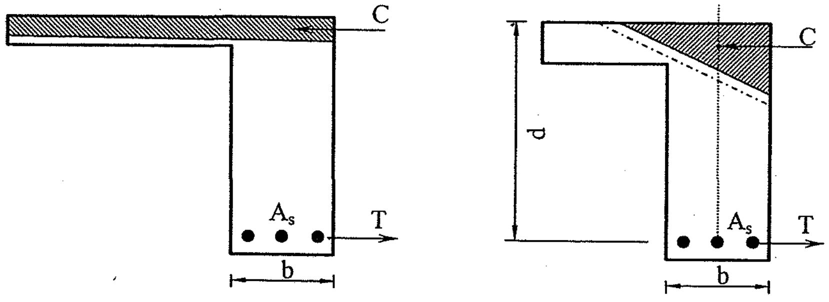


Fig. 3.20 Neutral axis position in L-sections

Fig. 3.21 presents the forces and strain for a reinforced concrete isolated L-section. It can be easily determined that the distance X1 equals 1.5 b. The force in the compression zone equals the area of the compressed zone multiplied by the concrete stress as follows

$$C = \frac{0.67\, f_{cu}}{1.5}\left(\frac{X_1\, Y_1}{2}\right) \quad \text{.......} (3.58)$$

$$T = A_s\,\frac{f_y}{1.15} \quad \text{.......} (3.59)$$

The second equilibrium equation can be computed by taking the moment around the concrete force. The lever arm in this case is the vertical distance between the tensile force and the center of gravity of the compressed triangle. Thus, the internal moment equals

$$M_u = \frac{A_s\, f_y}{1.15}\left(d - \frac{Y_1}{3}\right) \quad \text{.......} (3.60)$$

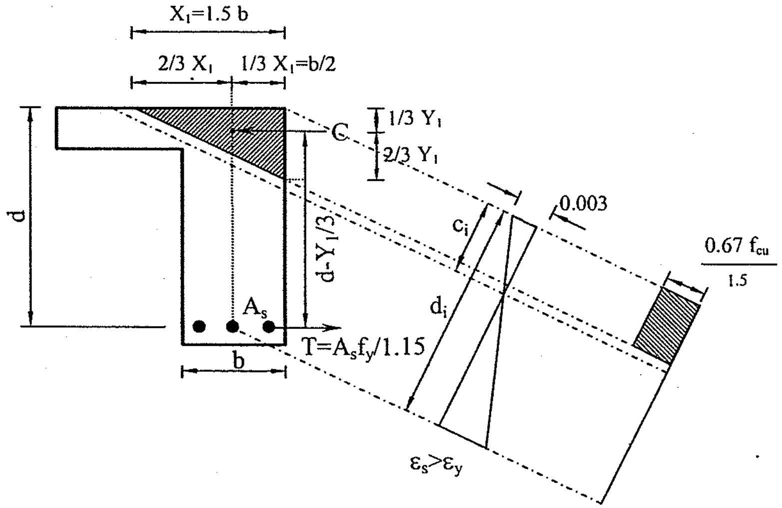


Fig. 3.21 Stresses and strain for isolated L-section

Example 3.19 illustrates the calculation of the ultimate moment capacity of L-section using the first principle, while example 3.20 shows the simplified procedure using design curves.

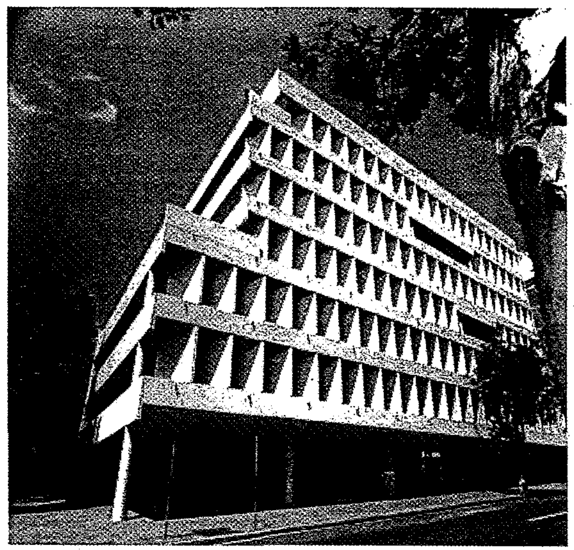
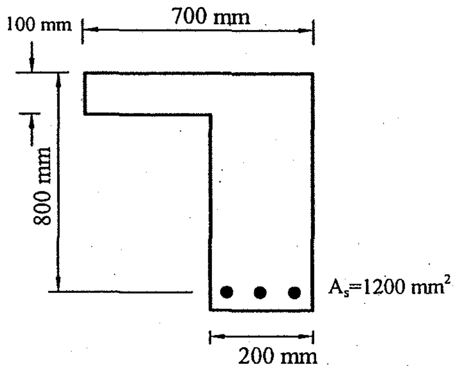


Photo 3.7 Beam-column structural system

Example 3.19

For the L-section shown in figure determine the capacity of the section knowing that the beam is not laterally supported (isolated), knowing that $f_{cu} = 25$ N/mm^2 , $f_y = 360$ N/mm^2



Solution

Step 1: Compute Y_1

From the geometry $X_1 = 1.5$ b $= 300$ mm

$$C = \frac{0.67\ f_{cu}}{1.5}\left(\frac{X_1\ Y_1}{2}\right)$$

$$T = A_s \frac{f_y}{1.15} = 1200 \frac{360}{1.15} = 375652\ N$$

Since C = T

$$375652 = \frac{0.67 \times 25}{1.5}\left(\frac{300 \times Y_1}{2}\right)$$

$Y_1 = 224$ mm

Step 2: Compute M_u

Take the moment around the compression force C

$$M_u = \frac{A_s\ f_y}{1.15}\left(d - \frac{Y_1}{3}\right)$$

$$M_u = \frac{1200 \times 360}{1.15}\left(800 - \frac{224}{3}\right) = 272.5 \times 10^6\ N.mm = 272.5\ kN.m$$

Step 3: Check steel stress

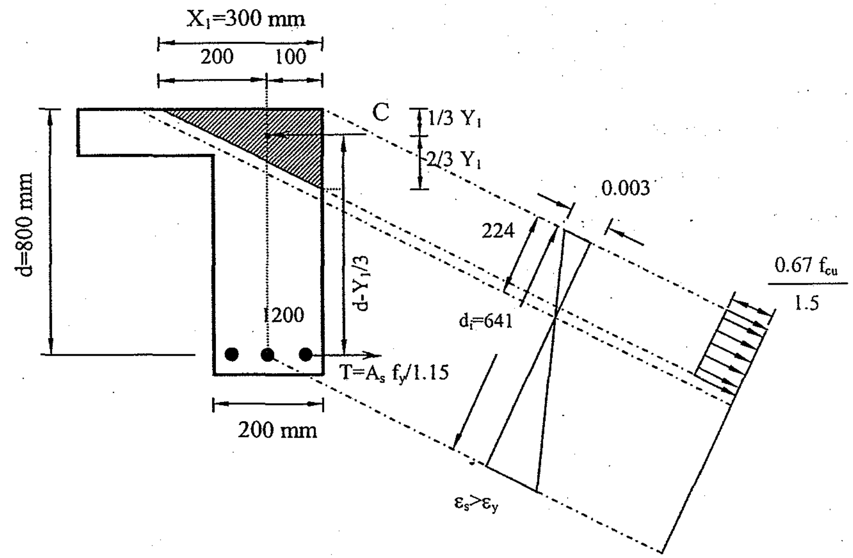
$$\theta = \tan^{-1}\left(\frac{224}{300}\right) = 36.74^{\circ}$$

$$c_i = \frac{Y_1}{0.8} = \frac{224 \times \cos\theta}{0.8} = 224.4\ mm$$

$$d_i = d\ \cos\theta = 800 \times \cos\theta = 641\ mm \qquad c_i/d_i = 0.35$$

or directly $c/d = Y_1/d = 280/800 = 0.35$

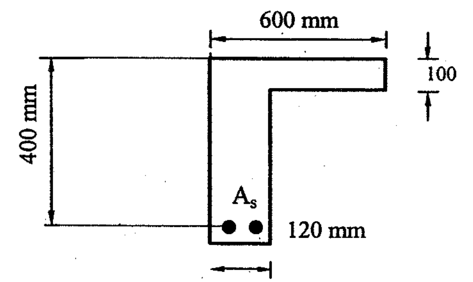
Since $c_i/d_i(0.35) < c_{max}/d(0.44)$, $f_s = f_y/1.15$ and $A_s < A_{smax}$ ...o.k



Forces and strain distributions

Example 3.20

Compute the area of steel for the L-section shown in the figure if it is subjected to $M_u = 120$ kN.m using R_T-J curve.
$f_{cu} = 35\ N/mm^2$ and $f_y = 400\ N/mm^2$



Solution

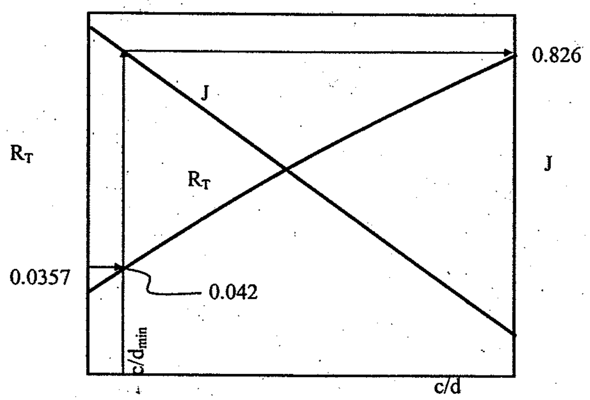
Step 1: Assumptions

Assume that the neutral axis inside the flange (a<ts)

Step 2: Determine R_T

$$R_T = \frac{M_u}{f_{cu}\ B\ d^2} = \frac{120\times10^6}{35\times600\times400^2} = 0.0357$$

$\because R_T < 0.042$ use c/d_{min}



Step 2:Determine J and check c/d limits

Use $c_{min}/d = 0.125$ and $J = J_{max} = 0.826$

because $c/d < c_{max}/d$, the condition that $A_s < A_{smax}$ is satisfied

Step 3: compute "a"

$a = 0.8\ (0.125)\ 400 = 40\ mm < t_s(100mm)\ \ a<t_s$

Step 4: Calculate area of steel, A_s

The stress block is located inside the slab, thus

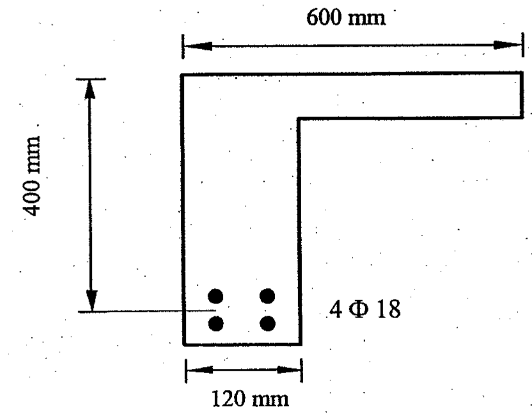
$$A_s = \frac{M_u}{f_y\ J\ d}$$

$$A_s = \frac{120\times10^6}{400\times0.826\times400} = 908\ mm^2$$

Choose 4 Φ 18 =1018 mm^2

Step 5: Check A_{smin}

$$A_{s\,min} = smaller\ \ of \begin{cases} \frac{0.225\sqrt{35}}{400} 120\times400 = 160\ mm^2 \\ 1.3\times908 = 1180\ mm^2 \end{cases} = 160\ mm^2 < A_s \cdots o.k$$



Final Design

4

SHEAR IN R/C BEAMS

Photo 4.1 Yokohama Landmark Tower, Japan.

4.1 Introduction

Reinforced concrete beams resist loads by means of internal moments and shears. In the design of reinforced concrete members, moment is usually considered first, leading to the dimensions of the cross-section and the arrangement. The beam is then proportioned for shear.

Because a shear failure is sudden and brittle, the design for shear must ensure that the shear strength equals or exceeds the flexural strength at all points of the beam. This chapter presents the shear behavior and design of relatively slender (shallow) beams. More advanced topics related to the strength and behavior of slender beams can be found in Chapter (7). The behavior of deep beams is presented in Volume (3) of this text.

4.2 Shear stresses in Elastic Beams

The beam shown in Fig. (4.1) is acted upon by a system of loads which lie in a plane of symmetry. The infinitesimal slice of length *dx* is bounded by the two sections 1-1 and 2-2 which are subjected to bending moments M and M+dM and shearing forces Q and Q+dQ (Fig.4.1c), respectively.

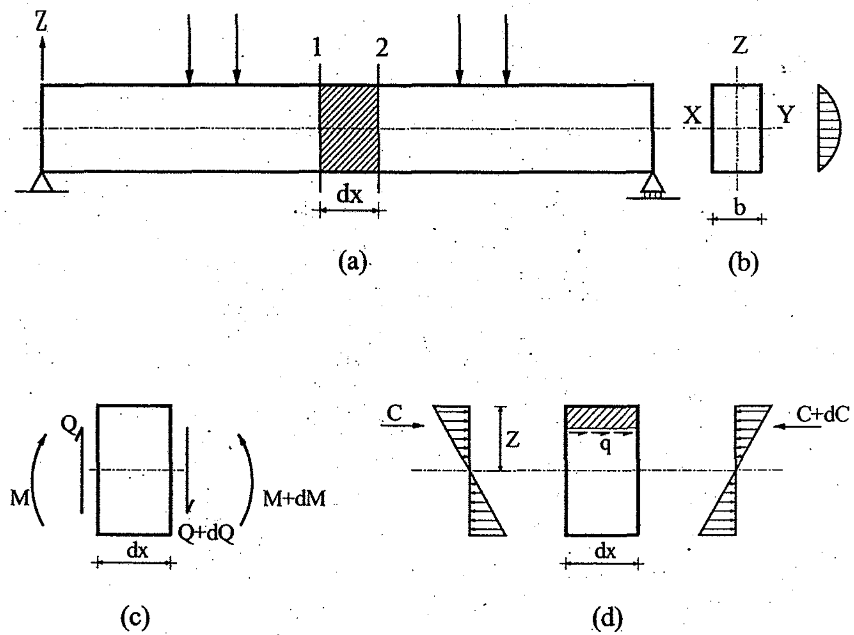


Fig. 4.1 Shear stresses in an elastic beam

The compressive forces on the hatched area in Fig. (4.1d), which are the resultants of the normal stresses induced by the bending moments, are indicated by the two forces C and C+dC. Investigating the equilibrium of the upper portion of the infinitesimal slice, it is evident that there should be horizontal shear stresses in order to equilibrate the force dC.

$$dC = q\,b\,dx \quad \text{.......(4.1)}$$

$$\frac{dC}{dx} = qb \quad \text{.......(4.2)}$$

The normal flexural stress f_x equals

$$f_x = \frac{M}{I}Z \quad \text{.......(4.3)}$$

But, the compression force C equals

$$C = \int f_x\,dA = \int \frac{M}{I} z\,dA = \frac{M}{I} S_y \quad \text{.......(4.4.a)}$$

Hence,

$$\frac{dC}{dx} = \frac{d}{dx}\int f_x\,dA = \frac{S_y}{I}\frac{dM}{dx} \quad \text{.......(4.4.b)}$$

Substituting from Eq. (4.2) and noting that $dM/dx = Q$, one gets

$$q = \frac{Q\,S_y}{I\,b} \quad \text{.......(4.5)}$$

where

Q = shear force acting on the cross section.

I = moment of inertia of the cross section.

S_y = the first moment of hatched area about the y-axis.

b = width of the member where the shear stress are being calculated.

For an uncracked rectangular beam, Eq. (4.5) gives the distribution of shear stresses shown in Fig. (4.1b).

Considering the equilibrium of a small element in the beam, it follows that the horizontal shear stresses should be accompanied by vertical shear stresses of the same magnitude as the horizontal shear stresses. The elements in Fig. (4.2-a) are subjected to combined normal stresses due to flexure, f, and shearing stresses, q. The largest and smallest normal stresses acting on such an element are referred to as principal stresses. The principle tension stress, f_{tmax}, and the principal compression stress, f_{cmax}, are given by:

$$f_{t\max} = \frac{f}{2} + \sqrt{\left(\frac{f}{2}\right)^2 + q^2} \quad \text{...(4.6)}$$

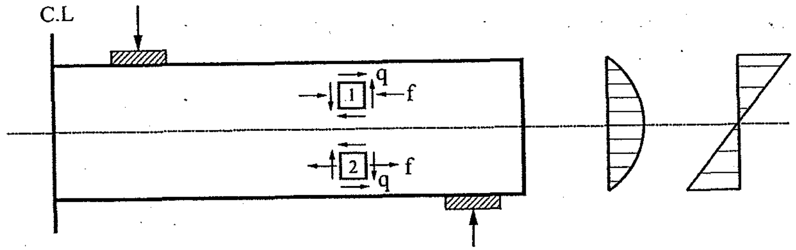
$$f_{c\max} = \frac{f}{2} - \sqrt{\left(\frac{f}{2}\right)^2 + q^2} \quad \text{...(4.7)}$$

The inclination of the principal stresses to the beam axis, θ, is determined by:

$$2\theta = \tan^{-1}\left(\frac{q}{f/2}\right) \quad \text{...(4.8)}$$

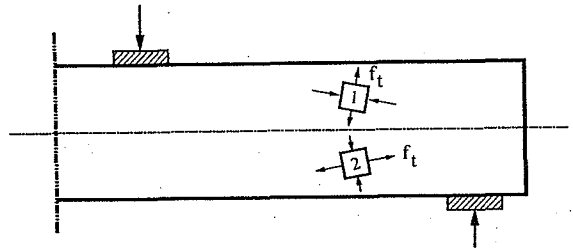
If the principle tensile stresses exceed the tensile strength of concrete, cracking occurs. The direction of cracking at any point is perpendicular to the direction of the principle tensile stress at that point.

Obviously, at different positions along the beam the relative magnitudes of q and f change, and thus the directions of the principal stresses change as shown in Fig. 4.2c. At the neutral axis, the principal stresses will be equal to the shear stresses and will be located at a 45° angle with the horizontal. Diagonal principal tensile stresses, called diagonal tension, occur at different places and angles in concrete beams.



(a) Flexural and shear stresses acting on beam elements

(b) Distribution of shear and normal stresses



(c) Principal stresses on beam elements

Fig.4.2 Normal, shear and principal stresses in homogenous uncracked beam

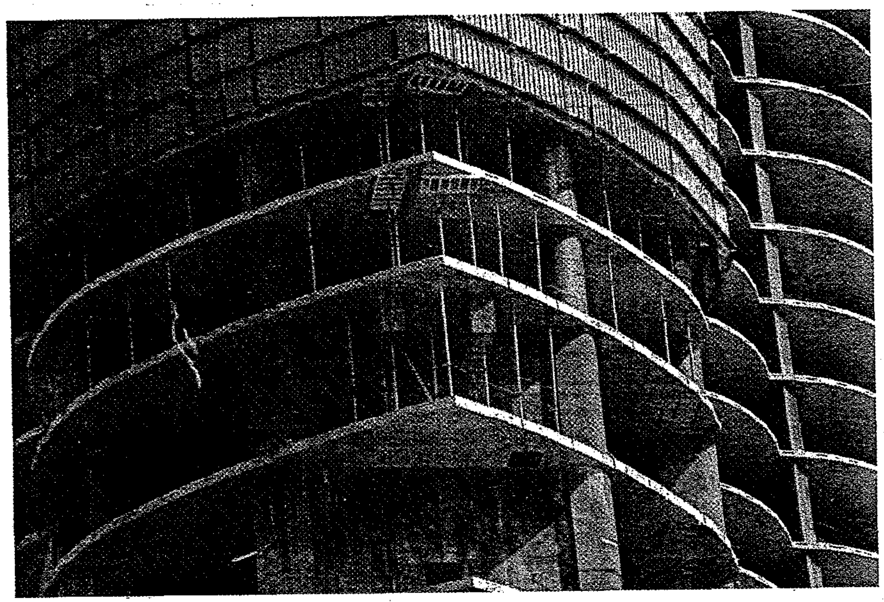


Photo 4.2 Burj-Dubai during construction

4.3 Shear Stresses in Cracked R/C Beams

The general formula that gives the distribution of shear stresses in homogeneous sections subjected to simple bending may be applied to reinforced concrete sections. If one considers the virtual area of the section which consists of the area of concrete in compression plus n-times the area of the steel reinforcement.

$$q = \frac{Q\, S_{nv}}{I_{nv}\, b} \quad \text{.........(4.9)}$$

where S_{nv} is the first moment of area , I_{nv} is the moment of inertia of the full virtual section about the center of gravity and b is the width of the cross section.

It may be observed that, on the tension side of the section, S_{nv} is calculated using the equivalent area of the tension steel reinforcement only.
Consider an infinitesimal portion of length *dx* of a reinforced concrete beam with rectangular cross section subjected to simple bending, where the bending moment is M on one side and M+dM on the other side, as shown in Fig. (4.3). The corresponding compressive forces, which are the resultants of the induced normal stresses, are C and C+dC, respectively.

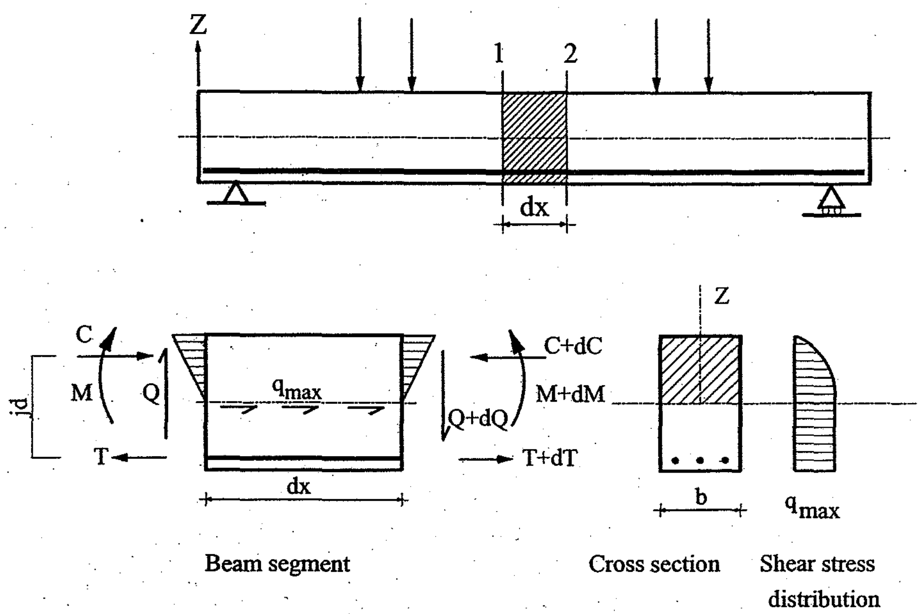


Fig. 4.3 Shear stresses in cracked reinforced concrete beams

Then one should have

$$dC = q_{max}\, b\, dx \quad \text{.........(4.10)}$$

Noting

$$C = \frac{M}{jd} \quad \text{.........(4.11)}$$

where jd is the arm of the internal moment. Then one gets

$$dC = \frac{dM}{jd} \quad \text{.........(4.12)}$$

Substituting Eq. (4.10) into (4.12) and noting that $dM/dx = Q$, one obtains

$$q_{max} = \frac{Q}{jdb} \quad \text{.........(4.13)}$$

The distance *jd* may be taken to be approximately 0.87d.

For routine design, shear strength in reinforced concrete beams is commonly quantified in terms of a nominal shear stress, q, defined as

$$q = \frac{Q}{b \times d} \quad \text{.........(4.14)}$$

4.4 Behavior of Slender Beams Failing in Shear

4.4.1 Inclined Cracking

Two types of inclined cracking occur in concrete beams; web shear cracking and flexure-shear cracking. These two types of inclined cracking are illustrated in Fig. (4.4). Web-shear cracking begins from an interior point in a member when the principal tensile stresses due to shear exceed the tensile strength of concrete. Flexure-shear cracking is essentially an extension of a vertical flexural cracking. The flexure-shear crack develops when the principal tensile stress due to combined shear and flexural tensile stress exceed the tensile strength of concrete. It should be mentioned that web-shear cracks usually occur in thin-walled I beams where the shear stresses in the web are high while the flexural stresses are low.

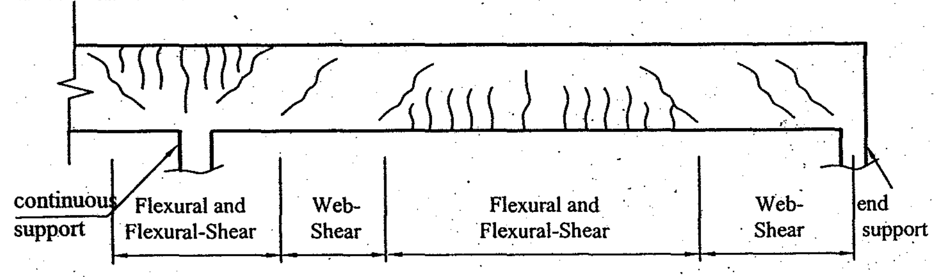


Fig. 4.4 Types of cracking in reinforced concrete beams

4.4.2 Internal Forces in Beams without stirrups

The forces transferring shear forces across an inclined crack in a beam without stirrups are illustrated in Fig. (4.5). In this figure, Q_a is the shear transferred across the crack by interlock of the aggregate particles on the two faces of the crack. Q_{ax} and Q_{ay} are the horizontal and vertical components of this force, respectively. The shear force is resisted by:

Q_{cz}, the shear in the compression zone

Q_{ay}, the vertical component of the shear transferred across the crack by interlock of the aggregate particles on the two faces of the crack.

Q_d, the dowel action of the longitudinal reinforcement

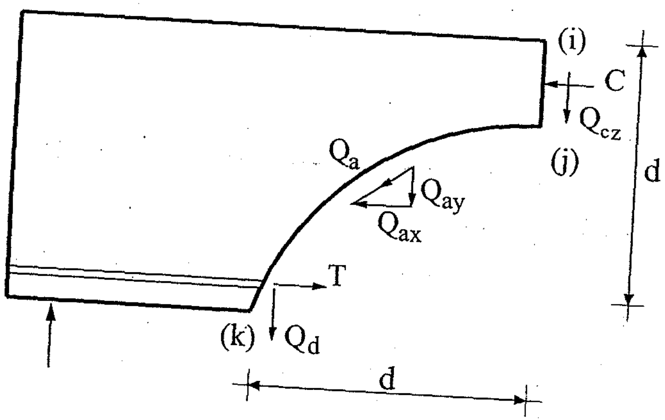


Fig. 4.5 Internal forces in a cracked beam without stirrups

It is difficult to quantify the contributions of Q_{cz}, Q_{ay}, and Q_d. In design, these are lumped together as Q_c, referred to as shear carried by concrete.

$$Q_c = Q_{cz} + Q_{ay} + Q_d \quad \text{.................................(4.15)}$$

Traditionally, Q_c is taken equal to the failure capacity of a beam without stirrups. Since beams without stirrups will fail when inclined cracking occurs, Q_c is equal to the inclined cracking load of the beam without stirrups.

In general, the inclined cracking load of a beam without stirrups, and consequently Q_c, is affected by:

- The tensile strength of concrete: the inclined cracking load is a function of the tensile strength of concrete. As mentioned before, the state of stress in the web of the beam involves biaxial principal tension and compression stresses as shown in Fig. 4.2b (see Section 4.3). A similar biaxial state of stress exists in a split cylinder tension test (Fig. 1.7). This indicates that the inclined cracking load (or the shear carried by concrete) is related to the tensile strength of concrete.

- Longitudinal reinforcement ratio: tests indicate that the shear capacity of beams without stirrups increase with the increase of the longitudinal reinforcement ratio. As the amount of the steel increases, the length and the width of the cracks will be reduced. Hence, there will be close contact between the concrete on the opposite sides of the cracks; improving the shear resistance by aggregate interlocking.

- Shear-span-to-depth ratio (a/d): the shear capacity of beams without stirrups is a function of the shear span, a, to the depth, d, of the beams (see Fig. 4.6). In general, concrete beams can be classified into slender beams and deep beams. Deep beams are those having small (a/d) ratio. They are much stronger than slender beams in shear. Detailed discussion related to this subject can be found in Chapter (7) in this volume and in Volume (3) of this text.

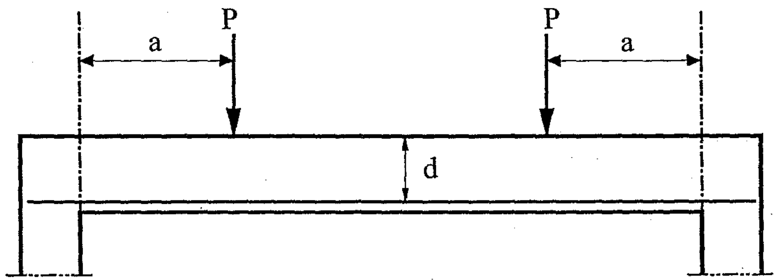


Fig. 4.6 Shear span –to-depth ratio (a/d)

4.4.3 Behavior of Slender Beams with Stirrups

The purpose of web reinforcement is to prevent sudden shear failure and ensure that the full flexural capacity can be developed. Web reinforcement may either be consisting of vertical stirrups, inclined stirrups or bent bars as shown in Fig. (4.7).

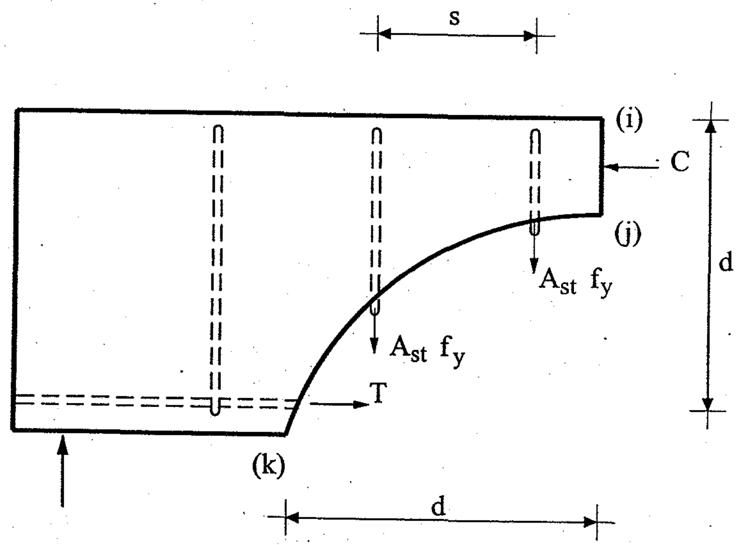


Fig. 4.7 Internal forces in a cracked beam with stirrups

Measurements have shown that web reinforcement is almost free from stress prior to the formation of diagonal cracks. After diagonal cracking, web reinforcement affects the shear resistance of the beam in three separate ways:

1 Part of the shear force is resisted by the web reinforcement traversing the crack.

2 The presence of web reinforcement restricts the growth of diagonal cracks and reduces their penetration into the compression zone; and hence increases the part of the shear force resisted by the compression zone.

3 The presence of stirrups enhances the dowel action.

The forces in a beam with stirrups and an inclined crack are shown in Fig. (4.7).

The shear transferred by tension in the stirrups is defined as Q_s. Assuming that *n* is the number of stirrups crossing a crack, s is the spacing between stirrups, the crack angle is 45 degrees, and that the stirrups yield, then

$$n = \frac{d}{s} \qquad (4.16)$$

$$Q_s = n\,A_{st}\,f_y = \frac{A_{st}\,f_y\,d}{s} \qquad (4.17)$$

where

A_{st} area of stirrups

Shear stress carried by stirrups q_s

$$q_s = \frac{Q_s}{b \times d} \qquad (4.18)$$

Substituting with the value of Q_s in Eq. 4.17 gives

$$q_s = \frac{A_{st} \times f_y \times d / s}{b \times d} = \frac{A_{st} \times f_y}{b \times s} \qquad (4.19a)$$

$$q_s = \frac{A_{st} \times f_y}{b \times s} \qquad (4.19b)$$

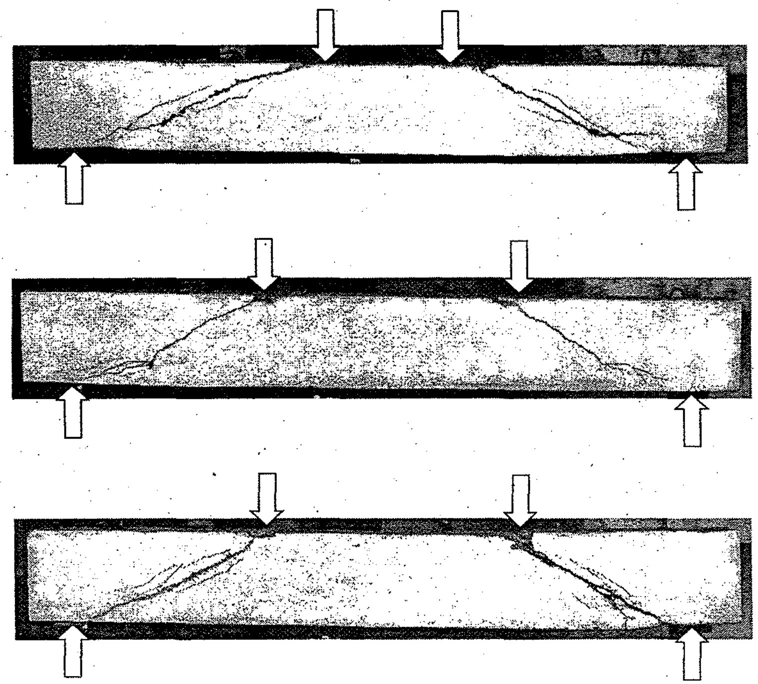


Photo 4.3 Diagonal cracking in the shear span

4.5 Egyptian Code's Procedure for Shear Design

4.5.1 Critical Sections for Shear

The critical sections for shear design are as follows:

1. The critical section is taken at a distance (d/2) from the face of the column provided that column reaction introduces vertical compression in the support zone and no concentrated loads act closer to the support than half the beam depth (Fig. 4.8.a). In such a case, the shear reinforcement obtained for the critical section shall be kept constant through the distance from the critical section to the support.

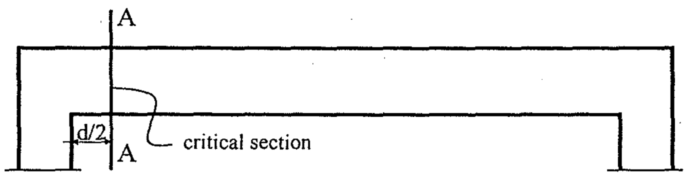


Fig. 4.8a Critical section for shear (general case)

2. If a concentrated load acts within a distance (a) where $(d/2 \le a \le 2d)$, the critical section (A-A) is taken at (d/2) the face of the support. The code allows a reduction of the effect of this force on the shear design by multiplying its effect by (a/2d) as shown in Fig. 4.8b.

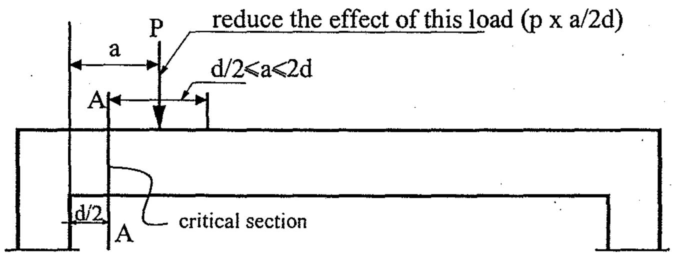


Fig. 4.8b Case of concentrated load $(d/2 \le a \le 2d)$

3. If a concentrated load acts within a distance (a) where a<d/2, the critical section (A-A) is taken at the face of the support. The code allows a reduction of the effect of this force on the shear design by multiplying its effect by (a/2d) as shown in Fig. 4.8c.

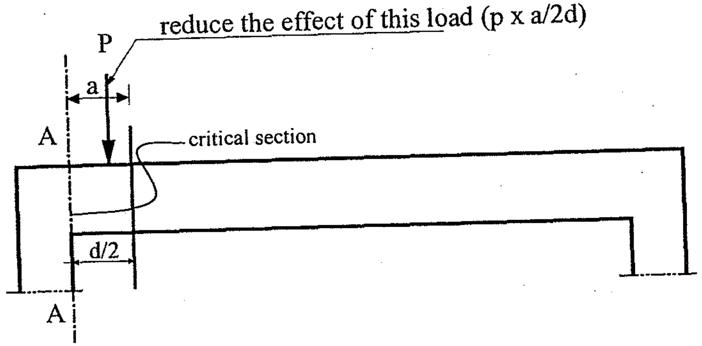


Fig. 4.8c Case of concentrated load (a<d/2)

4. The critical section is taken directly at the face of the column in case the column reaction introduces vertical tension in the support zone as shown in Fig. 4.8d.

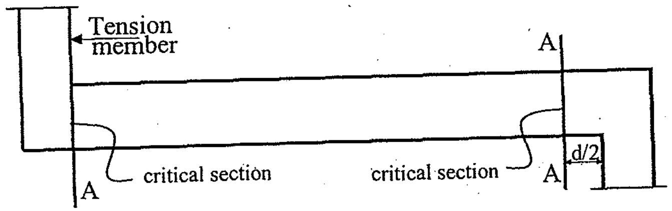


Fig. 4.8d Case of a beam supported by a tension member

4.5.2 Upper limit of Design Shear Stress

In order to avoid shear compression failure and to prevent excessive shear cracking, the ECP 203 limits the design ultimate shear stress to the value given by:

$$q_{u\,max} = 0.7\sqrt{\frac{f_{cu}}{\gamma_c}} \le 4.0\ N/mm^2 \quad \text{..............................(4.20)}$$

The upper limit of q_{umax} in Eq. 4.20 is 4 N/mm^2

If the ultimate shear stress $q_u \rangle q_{u\,max}$, the concrete dimensions of the cross section must be increased.

4.5.3 Shear Strength Provided by Concrete

The code evaluation for the shear strength provided by concrete is as follows:

No axial force

$$q_{cu} = 0.24\sqrt{\frac{f_{cu}}{\gamma_c}} \quad \text{...(4.21)}$$

Combined shear and axial compression

Applying compression force on the cross section will increase the area of concrete in compression and thus enhancing the shear capacity. The ECP 203 gives the following equation

$$\delta_c = \left[1 + 0.07\left(\frac{P_u}{A_c}\right)\right] \le 1.5$$

$$q_{cu} = \delta_c \times 0.24\sqrt{\frac{f_{cu}}{\gamma_c}} \quad \text{......................................(4.22)}$$

Combined shear and axial tension

Applying tension force on the cross section will decrease the area of concrete in compression and speeds up concrete cracking. The ECP 203 gives the following equation

$$\delta_t = \left[1 - 0.30\left(\frac{P_u}{A_c}\right)\right]$$

$$q_{cu} = \delta_t \times 0.24\sqrt{\frac{f_{cu}}{\gamma_c}} \quad \text{......................................(4.23)}$$

Equations 4.22 and 4.23 indicate that the ECP 203 considers the effect of the axial force when calculation the shear strength provided by concrete. An externally applied axial compression force will result in large compression zone leading to enhanced q_{cu}. The opposite would be true for a beam subjected to axial tensile force plus shear and bending.

4.5.4 Shear Strength Provided by Shear Reinforcement

The design ultimate shear stress (q_u) is compared with the nominal shear ultimate shear strength provided by concrete (q_{cu}). Two cases are possible:

a) $q_u \le q_{cu}$, then provide minimum web reinforcement.

b) $q_u \rangle q_{cu}$, then provide web reinforcement to carry q_{su}.

$$q_{su} = q_u - 0.5\,q_{cu} \quad \text{...(4.24)}$$

The code allows the use of three types of shear reinforcement:

1. vertical stirrups
2. inclined stirrups
3. bent up bars

These types are shown in Fig. 4.9

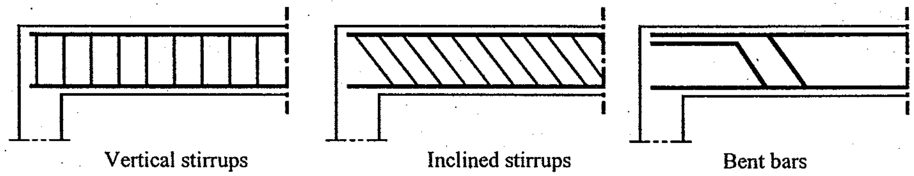


Fig. 4.9 Web reinforcements of reinforced concrete beams

In case of using inclined stirrups or bent up bars, the inclination angle with the beam axis shall not be less than 30^o.

The amount of the shear reinforcement is computed according to the arrangement of the web reinforcement as follows:

➢ ***A: Shear stress provided by vertical stirrups***

$$q_{su} = \frac{A_{st}\,(f_y/\gamma_s)}{b\,.\,s} \quad \text{.......................................(4.25)}$$

where :

A_{st} = area of all vertical legs in one row of stirrups. For two branch stirrup (A_{st}) is twice the area of one bar.

f_y = yield strength of stirrups.

b = beam width.

s = spacing between stirrups (≤ 200mm)

The previous equation contains two unknowns, A_{st} and s, thus either one should be assumed to determine the required shear reinforcement.
If the stirrups spacing are assumed, then Eq. 4.25 becomes

$$A_{st} = \frac{q_{su} \times b \times s}{f_y / \gamma_s} \quad \text{.......................................(4.26)}$$

The area of one branch is determined by

$$A_{st(one\,branch)} = \frac{A_{st}}{n} \quad \text{.......................................(4.27)}$$

where n is the number of branches

If the stirrups area is assumed, then Eq. 4.25 becomes

$$s = \frac{A_{st} \times f_y / \gamma_s}{q_{su} \times b} \leq 200\,mm \quad \text{.......................................(4.28)}$$

➢ ***B: Shear stress provided by inclined stirrups***

In case of using stirrups inclined at angle α with axis of the member

$$q_{su} = \frac{A_{st}\,(f_y/\gamma_s)}{b\,.\,s}(\sin\alpha + \cos\alpha) \quad \text{........................(4.29)}$$

➢ ***C: Shear stress provided by vertical stirrups and two rows or more of bent-up bars***

In case of using two rows of bent-up bars inclined at angle α with axis of the member accompanied by vertical stirrups, then the calculation is as follows:

1. Calculate the total design shear stress q_{su} given by:

$$q_{su} = q_u - 0.5\,q_{cu} \quad \text{.......................................(4.30)}$$

Assume the vertical stirrups area (A_{st}) and spacing (s) then calculate the contribution of the vertical stirrups q_{sus} as follows

$$q_{sus} = \frac{A_{st}\,(f_y/\gamma_s)}{b\,.\,s} \quad \text{.......................................(4.31)}$$

2. Calculate the amount of remaining shear stress that should be carried by the bent-up bars q_{sub} as follows

$$q_{sub} = q_{su} - q_{sus} \quad \text{.......................................(4.32)}$$

3. Calculate the required cross sectional area of the bent-up bars A_{sb}

$$A_{sb} = \frac{q_{sub} \times b \times s}{(f_y/\gamma_s)(\sin\alpha + \cos\alpha)} \quad \text{..........................(4.33)}$$

If the angle (α) is 45^o, Eq. 4.33 becomes

$$A_{sb} = \frac{q_{sub} \times b \times s}{(f_y/\gamma_s)\sqrt{2}} \quad \text{.......................................(4.34)}$$

➢ ***D: Shear stress provided by vertical stirrups and one row of bent-up bars***

In case of using one row of bent-up bars inclined at angle α with axis of the member, then the previous procedure is followed. However, the required cross sectional area A_{sb} is calculated from the following equation

$$A_{sb} = \frac{q_{sub} \times b \times d}{f_y/\gamma_s \times \sin\alpha} \quad \text{.......................................(4.35)}$$

in such a case

$q_{sub} \leq 0.24\sqrt{\frac{f_{cu}}{\gamma_c}}$, If the angle ($\alpha$) is 45^o, Eq. 4.35 becomes

$$A_{sb} = \frac{\sqrt{2} \times q_{sub} \times b \times d}{f_y/\gamma_s} \quad \text{.......................................(4.36)}$$

4.5.5 Code Requirements for Shear Reinforcement

1- A minimum amount of shear reinforcement is required by the code. It is given by

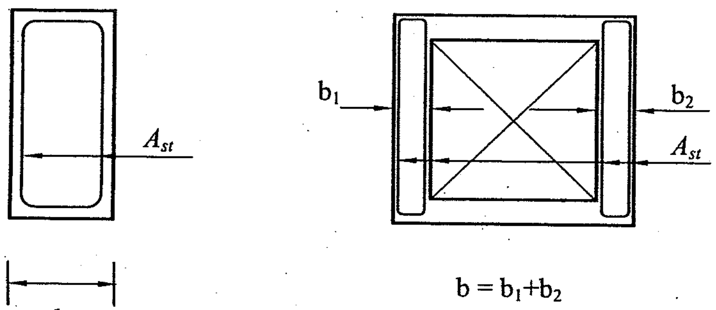
$$A_{st\,(min)} = \frac{0.4}{f_y}\, b \,.\, s \quad \text{..(4.37)}$$

where b is the width of the section as defined in Fig. 4.10.
But not less than

$A_{st(min)} = 0.0015\ b\,.\,s$ for mild steel 24/35

$A_{st(min)} = 0.0010\ b\,.\,s$ for ribbed high-grade steel

but not less than 5 ϕ 6 /m′



a-solid section **b-box section**

Fig. 4.10 Definition of *b* for solid and boxed sections

2- The area of steel $A_{st\,(min)}$ calculated using Eq. (4.37) may be reduced for beams of width exceeding their depth as follows:

$$A_{st\,(min)}\ reduced = A_{st\,(min)} \times \frac{q_u}{q_{cu}} \quad \text{................................(4.38)}$$

where $\frac{q_u}{q_{cu}} < 1$

3- For beams with web width equal to or greater than 400 mm, and in beams of web width exceeding their height, stirrups of at least four branches shall be used. The maximum distance between branches should be less than 250 mm as shown in Fig. 4.11.

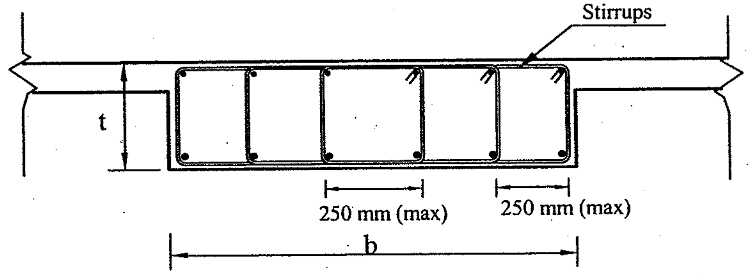


Fig. 4.11 Stirrups arrangement for beams having b>t or b> 400 mm

4- For reinforced beams of depth of not more than 250 mm, the code requires that the design shear stress be resisted by concrete only according to the following relation

$$q_{cu} \le 0.16\sqrt{\frac{f_{cu}}{\gamma_c}} \quad \text{..(4.39)}$$

5- The maximum spacing between vertical stirrups shall not exceed the following value

$$s_{max} \le 200\ mm \quad \text{..(4.40)}$$

6- The maximum spacing between rows of bent up bars is as follows:

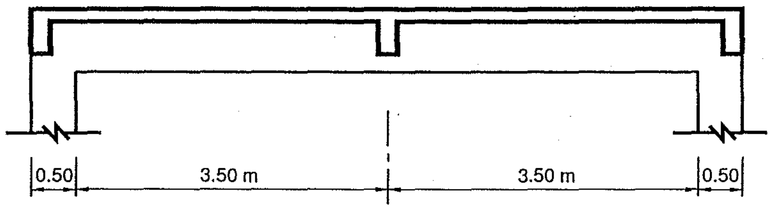
$$\begin{aligned} s_{max} &\le d \\ &\le 1.5d \quad \text{provided } q_u \le 1.5\, q_{cu} \\ or \quad &\le 2d \quad \text{provided } q_u < q_{cu} \end{aligned}$$

7- Construction joints should be generally avoided at location of high shear stresses. Otherwise precautions related to shear friction should be followed.

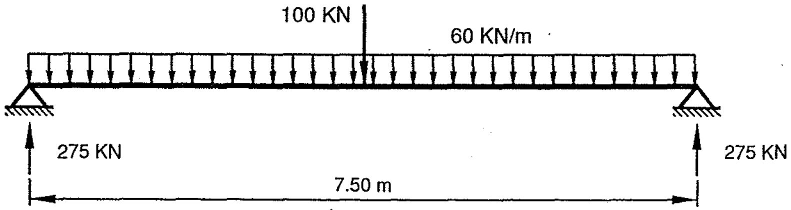
Example 4.1

Figure (EX. 4.1) shows a simply supported reinforced concrete beam that carries a uniformly distributed load having a factored value of 60 kN/m and a central concentrated load of a factored value of 100 kN. It is required to carry out a shear design for the beam according to the following data:

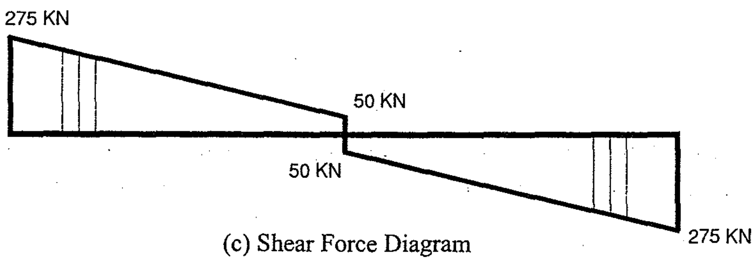
- Beam width = 300 mm
- Beam thickness = 700 mm
- $f_{cu} = 30\,N/mm^2$ and $f_y = 240\,N/mm^2$



(a) Beam Layout



(b) Statical System



(c) Shear Force Diagram

Fig. EX. 4.1

Step No.1: Determine the design shear force

The critical section for shear is located at *d/2* from the face of the support.

Assuming concrete cover of 50 mm

$d = t - cover = 700 - 50 = 650\ mm$

The critical section is located at a distance that is equal to 325mm from the face of the support.

Reaction at the support $= \frac{w \times L}{2} + \frac{P}{2} = \frac{60 \times 7.5}{2} + \frac{100}{2} = 275\,kN$

$$Q_u = \text{Reaction} - w \times \left(\frac{d}{2} + \frac{col.width}{2}\right) = 275 - 60 \times \left(\frac{0.65}{2} + \frac{0.5}{2}\right) = 240.5\,kN$$

Step No.2: Check the adequacy of the concrete dimensions of the section

The concrete dimensions of the section are considered adequate if the shear stress due to the design shear force is less than the ultimate shear strength.

$$q_u = \frac{Q_u}{b \times d} = \frac{240.5 \times 10^3}{300 \times 650} = 1.233\,N/mm^2$$

$$q_{u\max} = 0.7\sqrt{\frac{f_{cu}}{\gamma_c}} \le 4N\ /mm^2$$

$$q_{u\max} = 0.7\sqrt{\frac{30}{1.5}} = 3.13N\ /mm^2 < 4.0\ N\ /mm^2$$

$$q_{u\max} = 3.13N\ /mm^2$$

Since $q_u \langle q_{u\max}$ the concrete dimensions of the section are adequate.

Step No.3: Determine the shear stress carried by concrete

$$q_{cu} = 0.24\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.24\sqrt{\frac{30}{1.5}} = 1.073\,N/mm^2$$

Since the shear stress is greater than the shear stress carried by concrete, web reinforcement is needed.

Step No. 4 Design the web reinforcement

$q_{su} = q_u - 0.5\,q_{cu}$

$q_{su} = 1.233 - 0.5 \times 1.073 = 0.697\ N/mm^2$

$$A_{st} = \frac{q_{su} \times b \times s}{f_y / \gamma_s}$$

Assume that the stirrups spacing is 150 mm

$$A_{st} = \frac{0.697 \times 300 \times 150}{240/1.15} = 150.2\ mm^2$$

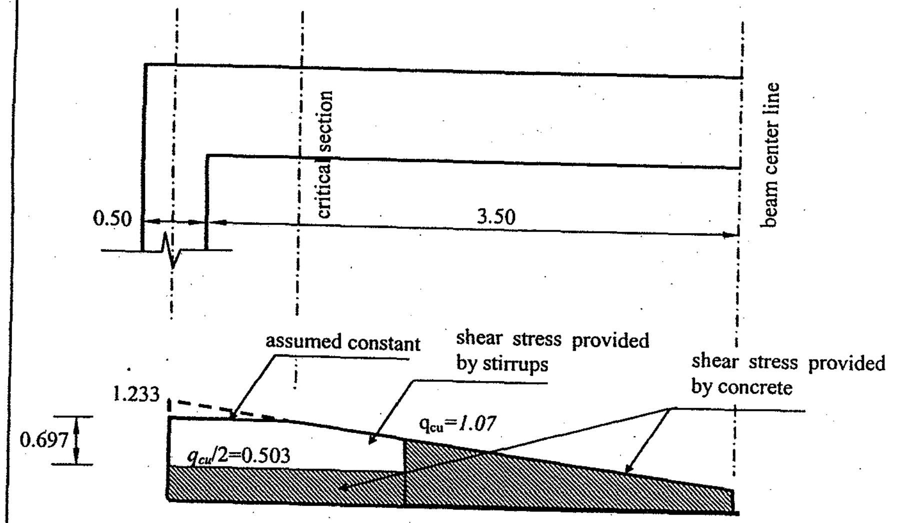
$\therefore$ Required area of one branch $= \frac{150.2}{2} = 75.1 mm^2$, choose ϕ 10 =78.5 mm^2

Use $\phi 10@150mm$ (7 ϕ 10 m′)

Check min shear reinforcement

$$\mu_{min} = \frac{0.4}{f_y} = \frac{0.4}{240} = 0.0017 > 0.0015....ok$$

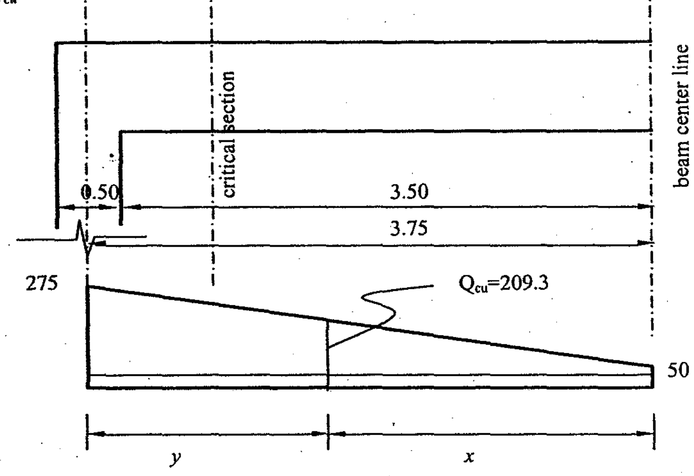
$A_{st(min)} = \mu_{min} \times b \times s = 0.0017 \times 300 \times 150 = 76.5\ mm^2$ < ($A_{st,\ provided}$= 2 x 78.5).....ok



It should be mentioned that using the amount of stirrups obtained from the design of the critical section along the whole span is not economic. A practical approach to get an economic design is to use the minimum required amount of stirrups starting from the section at which the shear stress equals q_{cu}. To compute the location of this section, the following calculations are carried out

The shear force carried by concrete equals

$Q_{cu} = q_{cu} \times b \times d = 1.073 \times 300 \times 650 = 209.3 kN$



Shear force diagram

Referring to the above shear force diagram and similarity of triangular

$$\frac{x}{3.75} = \frac{209.3 - 50}{275 - 50}$$

x=2.65 m

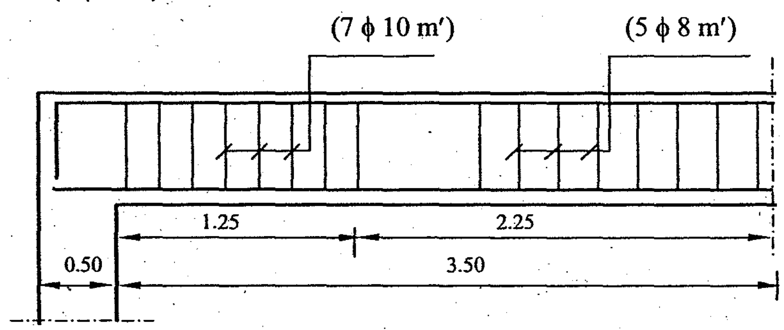
y = 3.75 - 2.65 = 1.09 m ≈ 1.25 m

The calculated stirrups (7 ϕ 10 m′) is provided in the distance y, while a minimum stirrups (s=200 mm) is provided in the distance x

$A_{st(min)} = \mu_{min} \times b \times s = 0.0017 \times 250 \times 200 = 85.0\ mm^2$ (for two branches)

Area of one branch =42 mm^2 (use ϕ 8 =50 mm^2)

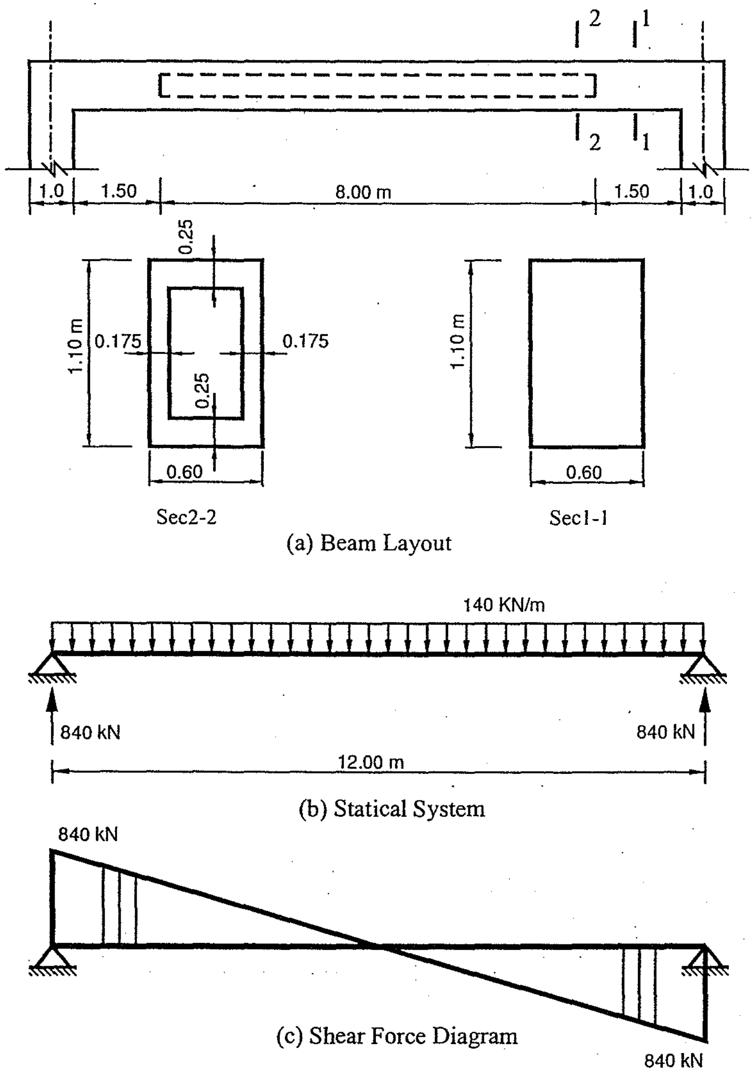
Use (5 ϕ 8 m′)



Final shear design

Example 4.2

Figure (EX. 4.2) shows a simply supported reinforced concrete beam that carries a uniformly distributed load having a factored value of 140 kN/m. It is required to carry out a shear design for the beam according knowing that $f_{cu}=30\,N/mm^2$ and $f_y=360\,N/mm^2$ for the stirrups.



(a) Beam Layout

(b) Statical System

(c) Shear Force Diagram

Fig. EX. 4.2

Step 1: Determine the design shear force

Due to the fact that the girder has a variable cross section, the designer has to check its shear capacity at more than one location. In this example two sections shall be examined as follows:

The critical section in the solid part is located at d/2 from the face of the support.

Assuming a concrete cover of 100 mm

$d = t - \text{cover} = 1100 - 100 = 1000\ mm$

$$\text{Reaction at the support} = \frac{w \times L}{2} = \frac{140 \times 12}{2} = 840\,kN$$

$$Q_{u1} = \text{Reaction} - w \times \left(\frac{d}{2} + \frac{col.width}{2}\right) = 840 - 140 \times \left(\frac{1.0}{2} + \frac{1.0}{2}\right) = 700\,kN$$

The critical section at the hollow part (the box section) is located at the section where the hollow part starts (x=1.5m).

$$Q_{u2} = \text{Reaction} - w \times \left(x + \frac{col.width}{2}\right) = 840 - 140 \times \left(1.5 + \frac{1.0}{2}\right) = 560\,kN$$

Step 2: Check the adequacy of the concrete dimensions

$$q_{u\max} = 0.7\sqrt{\frac{f_{cu}}{\gamma_c}} \le 4.0\ N/mm^2$$

$$q_{u\max} = 0.7\sqrt{\frac{30}{1.5}} = 3.13\,N/mm^2 < 4.0\ N/mm^2$$

$$q_{u\max} = 3.13\,N/mm^2$$

Critical section 1-1

For the critical section at d/2 from the face of the column

$$q_{u1} = \frac{Q_{u1}}{b \times d} = \frac{700 \times 1000}{600 \times 1000} = 1.17\,N/mm^2$$

$\because q_{u1} \le q_{u\max}$ $\therefore$ The concrete dimensions of the section are adequate.

Critical section 2-2

Section 2-2 has a boxed shape and resistance to shear comes from the two webs each having a width of 175 mm

$$q_{u2} = \frac{Q_{u2}}{b \times d} = \frac{560 \times 1000}{(2 \times 175) \times 1000} = 1.60\,N/mm^2$$

$\because q_{u2} \le q_{u\max}$ $\therefore$ The concrete dimensions of the section are adequate.

Step 3: Determine the shear stress carried by concrete

$$q_{cu} = 0.24\sqrt{\frac{f_{cu}}{\gamma_c}}$$

$$= 0.24\sqrt{\frac{30}{1.5}} = 1.073\ N/mm^2$$

Step 4: Design the web reinforcement for each section

For Sec. 1-1

$$q_{su1} = q_{u1} - 0.5\,q_{cu}$$

$$q_{su1} = 1.17 - 0.5\times1.073 = 0.635\ N/mm^2$$

$$A_{st} = \frac{q_{su1}\times b\times s}{f_y/\gamma_s}$$

Assume that the stirrups spacing is 200 mm

$$A_{st} = \frac{0.635\times600\times200}{360/1.15} = 243.4\ mm^2$$

The width of the section is more than 400mm, thus requires more than two branches.

Assuming 4 branches, the area of one branch $= \frac{A_{st}}{n} = \frac{243.4}{4} = 61\ mm^2$

Use Φ 10=78.5 mm^2

Use $\Phi10@200mm$ 4-branches

For Sec. 2-2

$$q_{su2} = q_{u2} - 0.5\,q_{cu}$$

$$q_{su2} = 1.60 - 0.5\times1.07 = 1.065\ N/mm^2$$

$$A_{st} = \frac{q_{su2}\times b\times s}{f_y/\gamma_s}$$

Assume that the stirrups spacing is 200 mm

$$A_{st} = \frac{1.065\times(2\times175)\times200}{360/1.15} = 238.14\ mm^2$$

Each web is provided with one stirrup that has two branches.

Thus, the area of one branch $= \frac{A_{st}}{n} = \frac{238.14}{2\times2} = 59\ mm^2$

Use Φ 10=78.5 mm^2

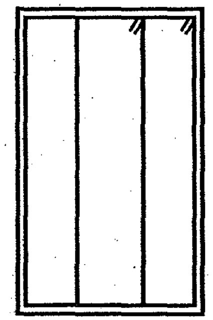
Use $\phi10@200mm$ 4-branches

Check min shear reinforcement

$$\mu_{min} = \frac{0.4}{f_y} = \frac{0.4}{360} = 0.00111 > 0.0010....ok$$

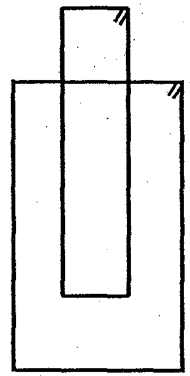
$$A_{st1(min)} = \mu_{min}\times b\times s = 0.00111\times600\times200 = 133\ mm^2 \ < (A_{st,\ provided} = 4 \times 78.5).....ok$$

$$A_{st2(min)} = \mu_{min}\times b\times s = 0.00111\times(2\times175)\times200 = 77.7\ mm^2 \ < (A_{st,\ provided} = 4 \times 78.5).....ok$$



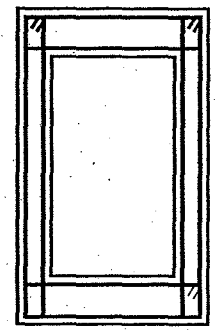
Sec 1-1

(Longitudinal Rft. is not shown)



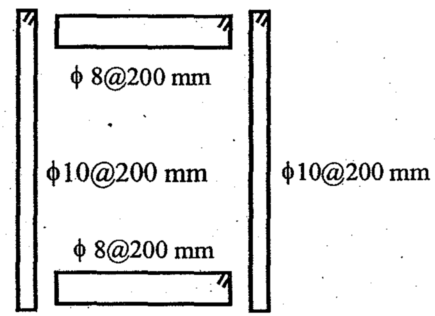
Stirrups Details

2 ϕ10@200 mm



Sec 2-2

(Longitudinal Rft. is not shown)

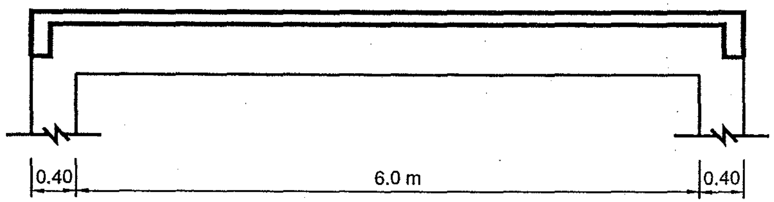


Stirrups Details

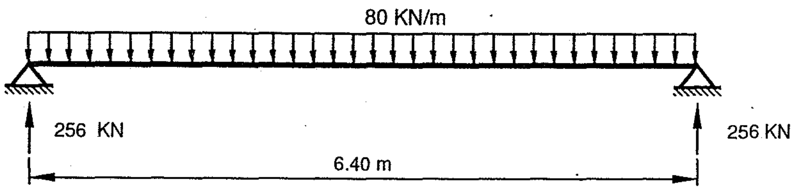
Example 4.3

Figure (EX. 4.3) shows a simply supported reinforced concrete beam that carries a uniformly distributed load having a factored value of 80 kN/m′.
It is required to carry out a shear design for the beam using bent-up bars and vertical stirrups according to the following data:

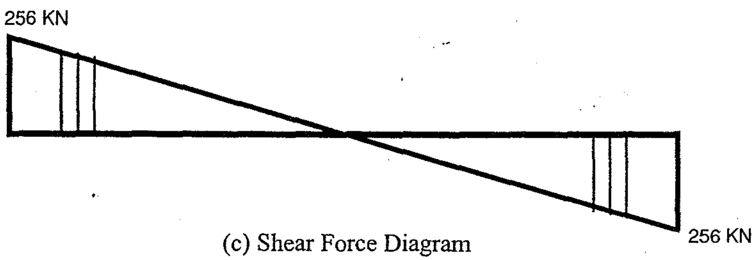
- Beam width = 250 mm
- Beam thickness = 800 mm
- $f_{cu} = 25\,N/mm^2$, $f_y = 240\,N/mm^2$ for the stirrups and f_y=400 N/mm^2 for the bent-up bars



(a) Beam Layout



(b) Statical System



(c) Shear Force Diagram

Fig. EX. 4.3

Step 1: Determine the design shear force

The critical section for shear is located at d/2 from the face of the support.

Assuming concrete cover of 50 mm

$d = t - \text{cover} = 800 - 50 = 750\ mm$

Reaction at the support $= \dfrac{w \times L}{2} = \dfrac{80 \times 6.4}{2} = 256\,kN$

$$Q_u = \text{Reaction} - w \times \left(\frac{d}{2} + \frac{col.width}{2}\right) = 256 - 80 \times \left(\frac{0.75}{2} + \frac{0.4}{2}\right) = 210\,kN$$

Step 2: Check the adequacy of the concrete dimensions

The concrete dimensions of the section are considered adequate if the shear stress due to the design shear force is less than the ultimate shear strength.

$$q_u = \frac{Q_u}{b \times d} = \frac{210 \times 10^3}{250 \times 750} = 1.12\,N/mm^2$$

$$q_{u\max} = 0.7\sqrt{\frac{f_{cu}}{\gamma_c}} \le 4N/mm^2$$

$$q_{u\max} = 0.7\sqrt{\frac{25}{1.5}} = 2.86N/mm^2 < 4.0\ N/mm^2$$

$$q_{u\max} = 2.86\,N/mm^2$$

Since $q_u \langle q_{u\max}$ the concrete dimensions of the section are adequate.

Step No.3: Determine the shear stress carried by concrete

$$q_{cu} = 0.24\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.24\sqrt{\frac{25}{1.5}} = 0.98\ N/mm^2$$

Since the shear stress is greater than the shear stress carried by concrete, web reinforcement is needed.

Step 4: Design the web reinforcement

The web reinforcement consists of two parts: a) vertical stirrups; b) bent-up bars

$$q_{su} = q_u - 0.5q_{cu}$$

$$q_{su} = 1.12 - 0.5 \times 0.98 = 0.63\,N/mm^2$$

Step 4.1: Shear stress carried by vertical stirrups

Assume that a minimum area of stirrups shall be provided

$$\mu_{min} = \frac{0.4}{f_y} = \frac{0.4}{240} = 0.00166 > 0.0015....ok$$

Assume s=200 mm

$$A_{st(min)} = \mu_{min} \times b \times s = 0.00166 \times 250 \times 200 = 83.3\ mm^2$$

Area of one branch=41.65 mm^2, choose ϕ8=50 mm^2

$$q_{sus} = \frac{A_{st}\left(f_y/\gamma_s\right)}{b\ .\ s} = \frac{(2\times50)\,240/1.15}{250\times200} = 0.417\ N/mm^2$$

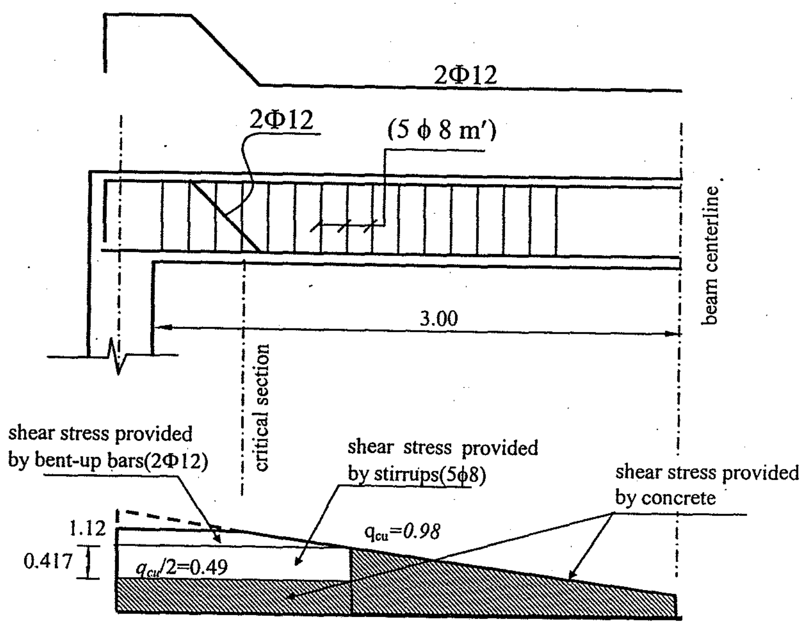
Step 4.2: Design of bent-up bars

$$q_{sub} = q_{su} - q_{sus} = 0.63 - .417 = 0.21\ N/mm^2$$

Using one row of bent-up bars and noting that the yield strength of the flexural steel is 400 N/mm^2, the area of the bars equals

$$A_{sb} = \frac{\sqrt{2}\times q_{sub}\times b\times d}{f_y/\gamma_s} = \frac{\sqrt{2}\times0.21\times250\times750}{400/1.15} = 160\ mm^2$$

Use 2Φ 12 (=226 mm^2)



Example 4.4

Figure (Ex.4.4) shows a simply supported reinforced concrete beam that carries a uniformly distributed load having a factored value of 70 KN/m and a concentrated load of a factored value of 90 KN. It is required to carry out a shear design for the beam according to the following data:

Beam width = 250 mm

Beam thickness = 700 mm

f_{cu} = 25 N /mm^2 and f_y = 240 N /mm^2

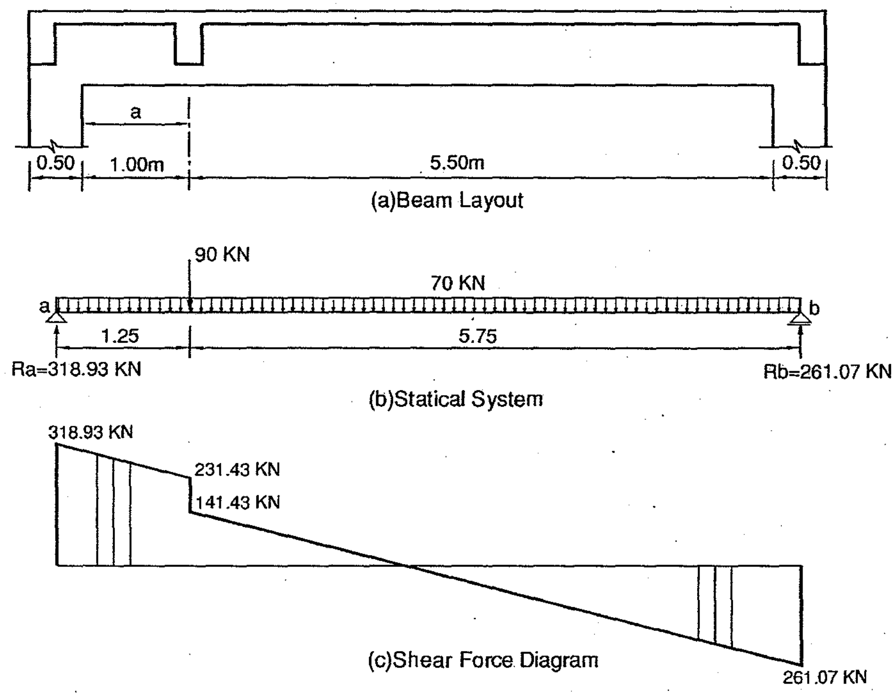


Fig.Ex.4.4

Step 1: Determine the design shear force

The critical section for shear is located at d/2 from the face of the support. Assuming concrete cover of 50 mm

$$d = t - cover = 700 - 50 = 650\ mm$$

The critical section is located at a distance equal to 325mm from the column face. To get the reaction at the support take $\sum M_a = 0$

$$\frac{70\times(7.0)^2}{2} + 90\times1.25 - 7R_2 = 0$$

$$\therefore R_2 = 261.07\ KN$$

$$Q_u = Reaction - w_u \times\left(\frac{d}{2} + \frac{col\,width}{2}\right)$$

$$Q_u = 318.93 - 70\times\left(\frac{0.65}{2} + \frac{0.5}{2}\right) = 278.68KN$$

Step 2: Check the adequacy of the concrete dimensions

The concrete dimensions of the section are considered adequate if the shear stress due to the design shear force is less than the ultimate shear strength.

$$q_u = \frac{Q_u}{b\times d} = \frac{278.68\times10^3}{250\times650} = 1.71 N/mm^2$$

$$q_{u\max} = 0.7\sqrt{\frac{f_{cu}}{\gamma_c}} \le 4 N/mm^2$$

$$q_{u\max} = 0.7\sqrt{\frac{25}{1.5}} = 2.86 N/mm^2 < 4.0\ N/mm^2 \longrightarrow\longrightarrow\longrightarrow q_{u\max} = 2.86 N/mm^2$$

Since $q_u < q_{u\max}$ the concrete dimensions of the section are adequate.
According to code; since the load is at distance "a = 1000mm" between d/2(325mm) & 2d(1300mm), therefore shear stress can be reduced as follows:

$$q_{ur} = q_u \times\frac{a}{2d} = 1.71\times\frac{1000}{2\times650} = 1.32 N/mm^2$$

Step 3: Determine the shear stress carried by concrete:

$$q_{cu} = 0.24\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.24\sqrt{\frac{25}{1.5}} = 0.98 N/mm^2$$

Since the shear stress is greater than the shear stress carried by concrete, web reinforcement is needed.

Step 4: Design the web reinforcement

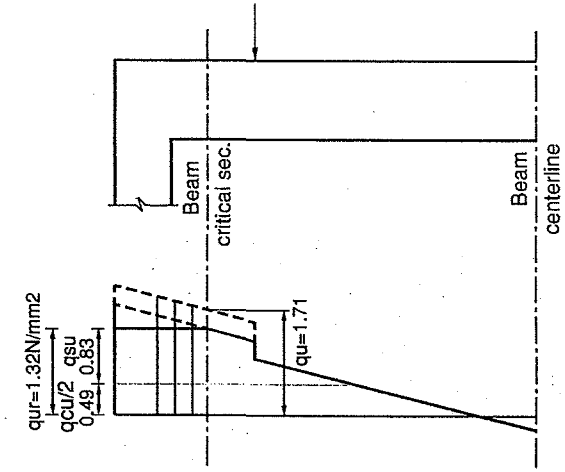
$$q_{su} = q_{ur} - 0.5q_{cu} = 1.32 - 0.5\times0.98 = 0.83 N/mm^2$$

Assume that the stirrups spacing is 150 mm

$$A_{st} = \frac{q_{su}\times b\times s}{f_y/\gamma_s} = \frac{0.83\times250\times150}{240/1.15} = 149.14\ mm^2$$

$\therefore$ Required area of one branch $= \frac{149.14}{2} = 74.57 mm^2$, choose ϕ 10 =78.5 mm^2

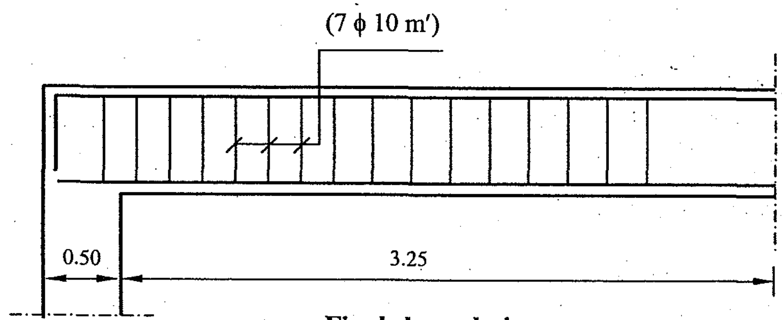
Use $\phi10@150mm$ or (7 ϕ 10 m′)



(c)Shear stress Distribution

Check min shear reinforcement

$$A_{st(\min)} = \frac{0.4}{f_y}\times b\times s = \frac{0.40}{240}\times250\times150 = 62.5\ mm^2 < (A_{st,\,provided} = 2 \times 78.5).....ok$$



Final shear design

5

BOND, DEVELOPMENT LENGTH AND SPLICING OF REINFORCEMENT

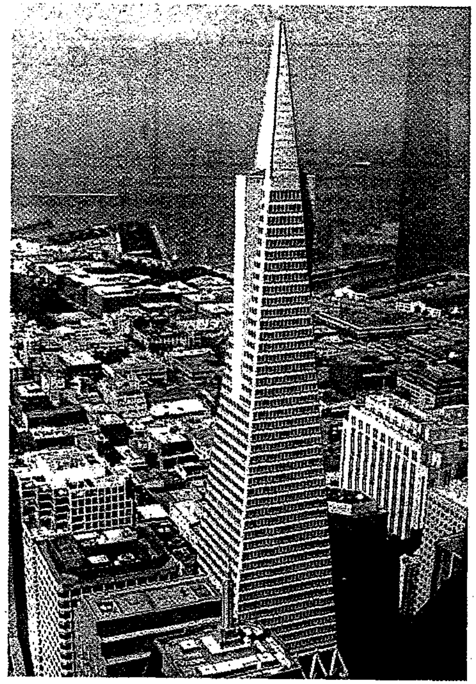


Photo 5.1 Reinforced concrete high rise building (San

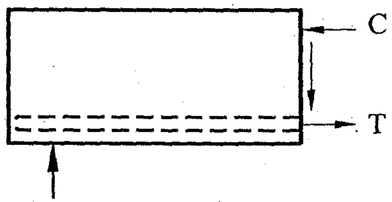
5.1 Introduction

One of the fundamental assumptions of reinforced concrete design is that at the interface of the concrete and the steel bars, perfect bonding exists and no slippage occurs. Based on this assumption, it follows that some form of bond stress exists at the contact surface between the concrete and the steel bars.

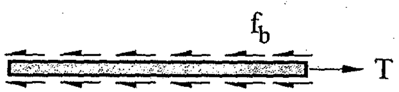
Bond strength results from several factors, such as the adhesion between the concrete and steel interfaces and the pressure of the hardened concrete against the steel bar.

In a reinforced concrete beam, the flexural compressive forces are resisted by concrete, while the flexural tensile forces are provided by reinforcement as shown in Fig. (5.1a). For this process to exist, there must be a force transfer, or bond between the two materials.

The forces acting on the bar are shown in Fig. (5.1b). For the bar to be in equilibrium, bond stresses, f_b, must exist. If these disappear, the bar will pull out of the concrete and the tensile force, T, will drop to zero, causing the beam to fail.



a- Equilibrium of a beam



b- Forces acting on a bar

Fig. 5.1 Bond stress development

5.2 Average Bond Stresses in a Beam

Consider the equilibrium of a segment of a beam of length dx as shown in Fig. (5.2).

$$dM = M_2 - M_1 \quad \text{.......................................} (5.1)$$

Noting that the tension in the reinforcing steel, T, equals

$$T = \frac{M}{Y_{ct}} \quad \text{.......................................} (5.2)$$

Then

$$dT = \frac{dM}{Y_{CT}} \quad \text{.......................................} (5.3)$$

From Eq. 5.3 and 5.1, one can get

$$dT = T_2 - T_1 \quad \text{.......................................} (5.4)$$

In order for the bar to be in equilibrium the change in the force dT should be equal to the average bond stress f_b multiplied by the surface area as shown in Eq. 5.5

$$dT = f_b \cdot \sum O \cdot dx \quad \text{.......................................} (5.5)$$

$$f_b = \frac{dT}{dx \sum O} \quad \text{.......................................} (5.6)$$

where

$\sum O$ = the sum of the perimeters of all tension bars

f_b = average bond stress

Y_{CT} = the lever arm of the internal forces C and T.

Equation (5.6) means that average bond stress is proportional to the rate of change in the tension in the reinforcing steel.

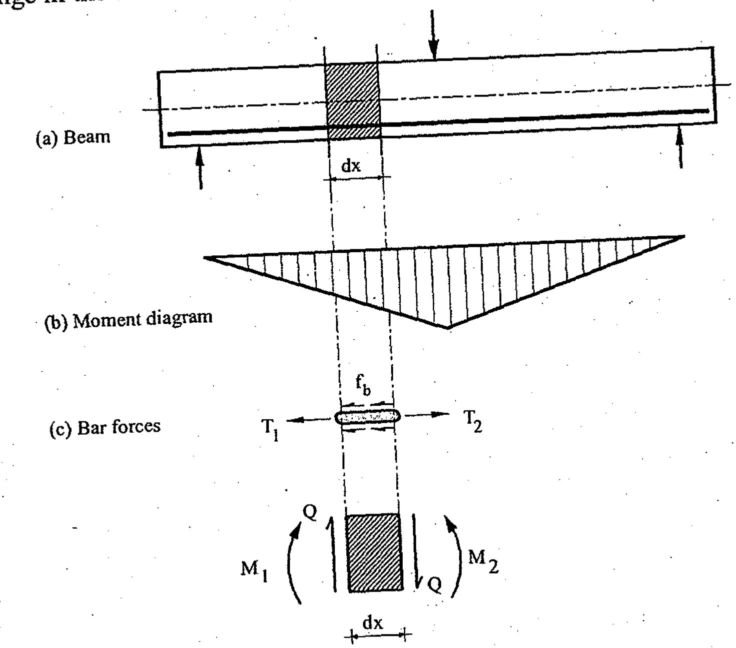


Fig. 5.2 Average flexural bond stress

5.3 True Bond Stresses in a Beam

At the location of the crack, the steel reinforcement carries the tension force. Away from cracks, concrete can pick up part of the tension (T) through bond action. Consequently, tension in steel is reduced between cracks by the amount resisted by the tensile stresses in concrete f_t. It is only at the locations of the cracks that the steel is subjected to tension (T) predicted by Eq. (5.2). Figure (5.3) shows the variations in the tension force in steel (T), the variation in the tensile stresses in concrete (f_t), and the variation of the bond stress (f_b) for a segment of a beam subjected to a constant bending moment (M).

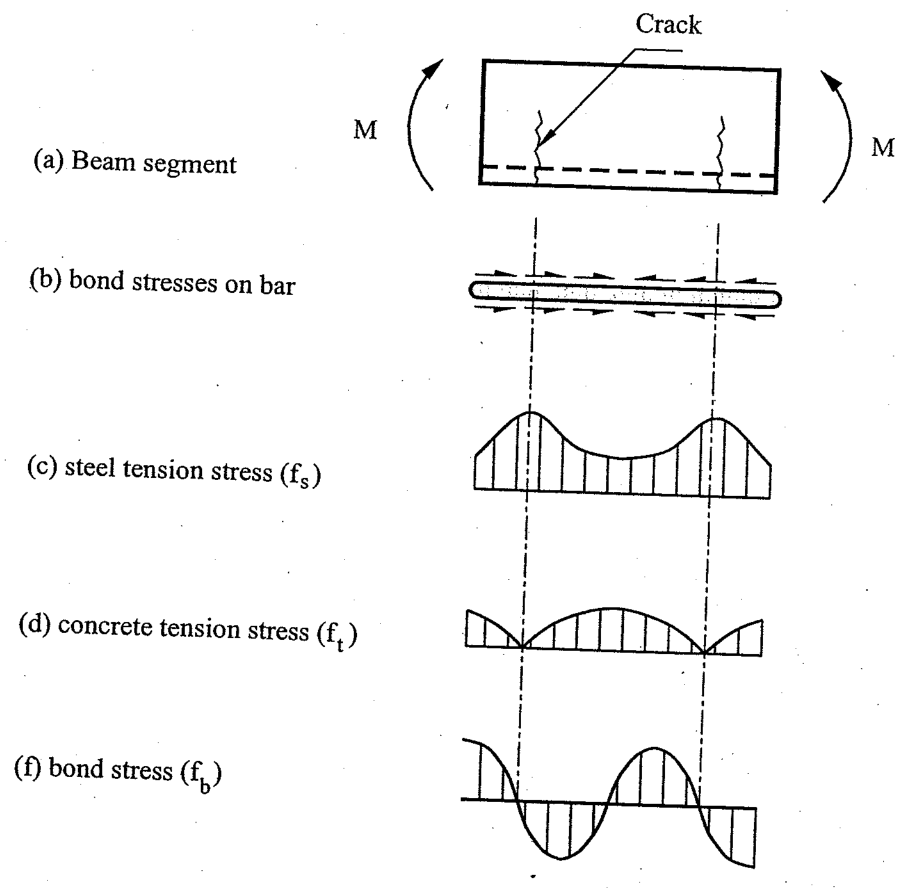


Fig. 5.3 True bond stresses in a beam

5.4 Development Length

5.4.1 Theoretical Considerations

The development length of the bar (L_d) is the necessary embedment length of the bar in concrete in order to ensure that the bar is securely anchored by bond to develop its maximum usable strength. For a bar stressed to its yield strength, the development length is the shortest length of bar in which the stress can increase from zero to yield strength, f_y. If the distance from a point where the bar stress equals f_y to the end of the bar is less than the development length, the bar will pull out of the concrete.

The concept of development length is demonstrated by a bar embedded in a mass of concrete as shown in Fig. (5.4). Equilibrium between internal and external applied forces leads to:

Tension in the bar = Bond force ....(5.7a)

$$\pi \frac{\Phi^2}{4} \frac{f_y}{\gamma_s} = f_b (\pi \Phi) L_d \quad \text{.....(5.7b)}$$

$$L_d = \frac{\Phi (f_y/\gamma_s)}{4 f_b} \quad \text{.....(5.8)}$$

where Φ = the bar diameter
f_b = the average bond stress
A_b = cross sectional area of the bar
f_y =steel yield stress
L_d = Development length

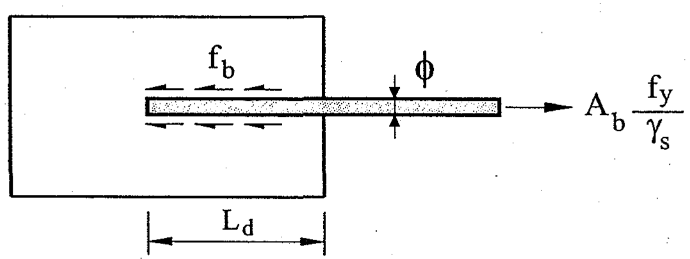


Fig. 5.4 The concept of development length for bars in direct tension

Another approach to understand the concept of development length is presented by considering a beam subjected to two-point loads as shown in Fig. (5.5). Assume that the design tensile force in steel is equal to f_y at point (o). This force is transferred progressively from concrete to steel over the length L. If the length (L) is equal to or larger than the development length L_d calculated from Eq. 5.8, no premature bond failure will occur.

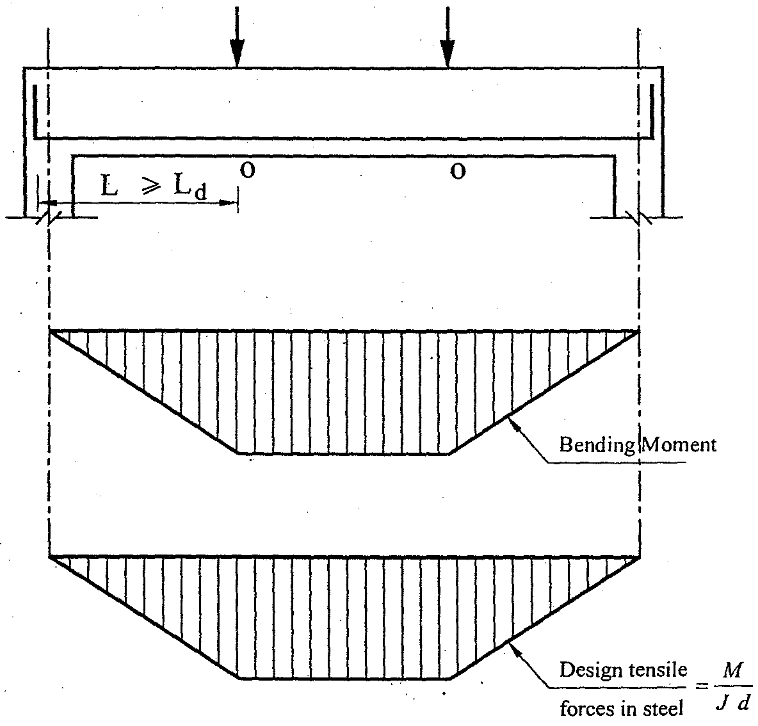


Fig. 5.5 The concept of development length for bars in beams

The necessary development length depends on the following factors:

- The bar diameter.
- The conditions of the bar ends.
- The condition of bar surface.
- The position of bar in the forms during construction.
- The yield strength of steel.
- The characteristic strength of concrete.

5.4.2 Development Length According to ECP 203

In order to develop a tension or a compression force that is equal to the bar yield force at any section, the Egyptian Code present the following equation for calculating the development length L_d

$$L_d = \frac{\alpha \ \beta \ \eta \ (f_y/\gamma_s)}{4 \ f_{bu}} \Phi \ \ldots\ldots (5.9)$$

where

Φ = nominal bar diameter

η = 1.3 for top reinforcement bars below which the concrete depth is more than 300 mm

β = coefficient depending on bar surface condition as defined by Table (5.1)

α = coefficient depending on bar shape as defined by Table (5.2)

f_{bu} = ultimate bond strength calculated from the following equation

$$f_{bu} = 0.30 \sqrt{\frac{f_{cu}}{\gamma_c}} \ \ldots\ldots (5.10)$$

The development length for reinforcing steel bars subjected to tension or compression shall not be less than:

35 ϕ or 400 mm – whichever is bigger- for smooth bars with hooks
40 Φ or 300 mm - whichever is bigger- for deformed bars

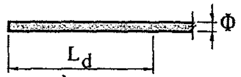
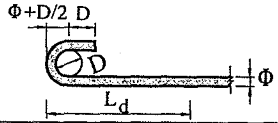
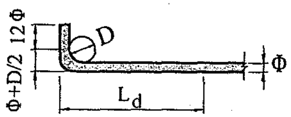
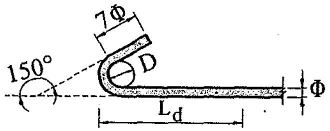
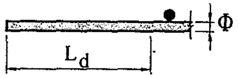
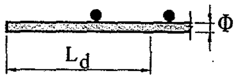
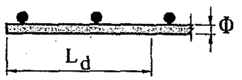
Table 5.1 Values of the correction coefficient (β)

Surface Condition		β	
		Tension	Compression
1	Smooth bar.	1.00	0.70
2	Deformed bar.	0.75	0.45

The development length for bundled bars shall be calculated from equation (5.9) considering the bundle as an individual bar having an equivalent diameter ϕ_e. The equivalent diameter of a bundle consisting of bars of equal diameter shall be calculated as follows:

- In case of using a two-bar bundle $\phi_e = 1.4 \ \phi$
- In case of using a three-bar bundle $\phi_e = 1.7 \ \phi$

Table (5.2) Values of the correction factor α

Type	End Shape	Factor α Tension	Compression
Reinforcing bars	1-Straight 	1.0	1.0
	2- Hook of a 180° bend 	0.75	1.0
	3- Hook of a 90° bend 	0.75	1.0
	4- Hook of a 150° bend 	0.75	1.0
Welded wire mesh	1- Straight bars with no transversal bar in L_d 	1.0	1.0
	2- Straight bars with one transversal bar in L_d 	0.7	0.7
	2- Straight bars with two transversal bars in L_d 	0.5	0.5

D=4 ϕ for steel 240/350.
D= 6ϕ for (ϕ=6-25 mm)
D= 8 ϕ for (ϕ>25 mm)

The increase in the value of the factor (η) from (1.00) to (1.30) for top bars below which more than 300 mm of concrete are placed is justified by test results. These results showed a significant loss of bond strength for bars with 300 mm or more of concrete beneath. This loss is attributed to the tendency of excess water and air in the concrete mix to rise up and accumulate to some extent on the underside of the bar, thus, resulting in weaker bond on the lower part of the bar perimeter (see Fig. 5.6).

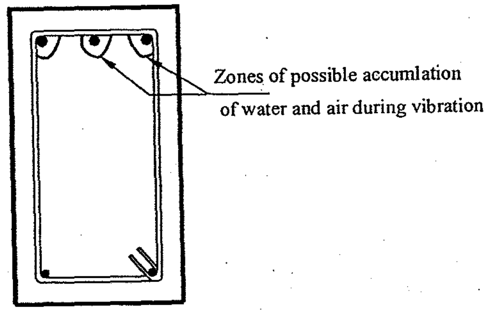


Fig. 5.6 Weak bond strength for top bars

The Egyptian Code does not deal explicitly with cross sections that are provided with reinforcement in excess of that required by calculations. However, the first clause of the code's section regarding development length states that steel bars must be extended on both sides of a section by a length that is proportional to the force in the bar at this section. This statement, effectively, permits reduction of the development length when the provided reinforcement exceed that required by calculations.

For simplicity, the Egyptian Code allows the use of the development length (L_d) as given in Table (5.3) instead of using Eq. 5.9 for values of f_{cu} greater than or equal to 20 N/mm^2. It is to be noted that this length should be increased by 30% for top reinforcing bars.

Table (5-3) Development length (L_d) estimated as multiplier of bar diameter(η=1)*

Concrete Grade N/mm²	Reinforcement Type			
	Deformed bars f_y= 400 N/mm²		Smooth bars f_y= 240 N/mm²	
	Tension	Compression	Tension	Compression
18	65	40	40	35
20	60	40	38	35
25	55	40	36	35
30	50	40	35	35
35	45	40	35	35
40	42	40	35	35
≥45	40	40	35	35

* In case of using deformed bars with hooks, multiply the previous numbers by 0.75
* The use smooth bars without hooks is not allowed.

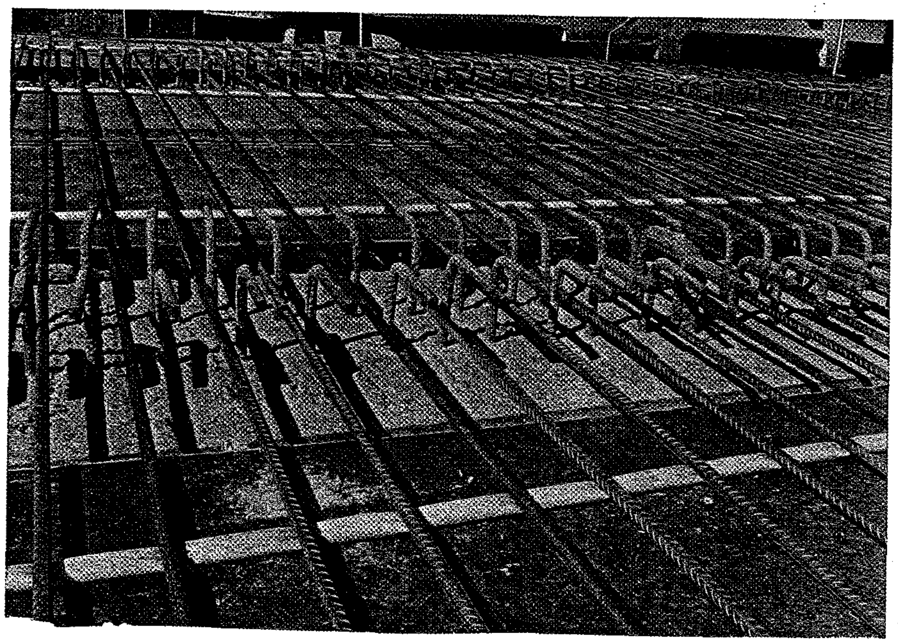


Photo 5.2 Bridge deck during construction (notice the shear connectors in the beam

5.5 Bar Cutoffs in Flexural Members

5.5.1 The Moment of Resistance of a R/C Beam

The moment of resistance of a R/C section is the maximum moment that can be resisted by the section according to its concrete dimensions and steel reinforcement. In other words, it is a property of the cross section. According to the preceding definition, it follows that the moment of resistance of a reinforced concrete beam at any section depends on the concrete dimensions and the amount of steel at that section.

The tension force in steel (T) is related to the design bending moment (M) by the relationship ($T=M/Y_{CT}$). For shallow (slender) reinforced concrete beams, the lever arm (Y_{CT}) could be considered constant. Therefore, it could be assumed that the required areas of steel at various sections in a beam are proportional to the bending moments at these sections.

The simple beam shown in Fig. (5.7a) is subjected to a uniformly distributed load that results in the bending moments shown in Fig. (5.7b).

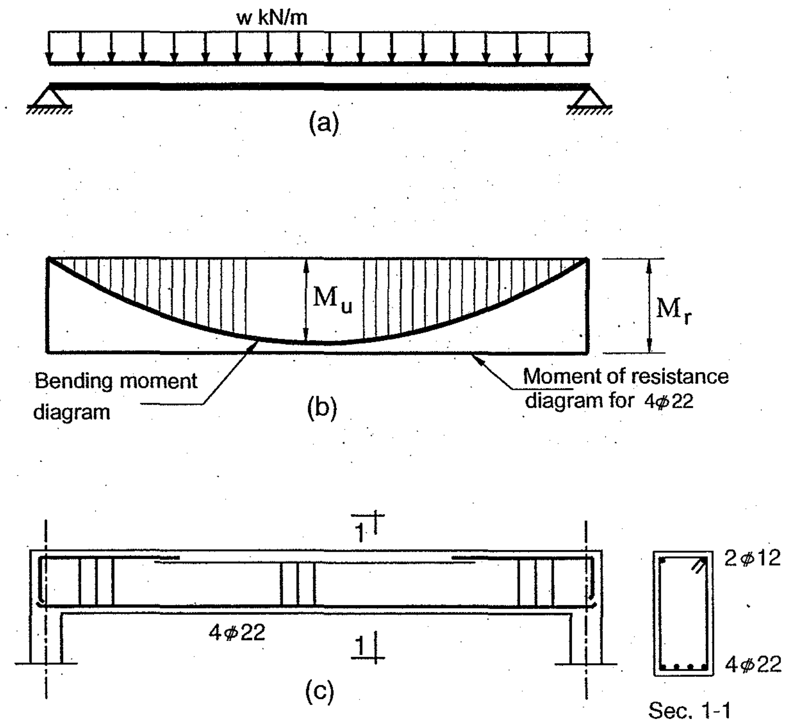


Fig. 5.7 The concept of the moment of resistance

Assuming that the design of the critical section at mid span, which is subjected to a factored moment M_u, results in a required amount of steel $A_{s(req)}$ of a value of 1440 mm^2. Four bars of diameter 22 mm are chosen to reinforce the critical section. The area of the chosen reinforcement is 1520 mm^2. Since the chosen area of steel is larger than the required area of steel, the moment of resistance of the section M_r shall be higher than the applied factored moment M_u. The moment of resistance of a reinforced concrete section can be obtained from

$$M_r = M_u \cdot \frac{A_{s(chosen)}}{A_{s(req)}} \qquad \text{.....(5.11)}$$

If the required area of steel for the section of maximum moment were used along the beam, as shown in Fig. (5.7c), the moment of resistance of the beam sections would be as shown in Fig. (5.7b). At low moment regions, the detailing shown in Fig. (5.7c) results in an uneconomic design, since the moment of resistance of the four bars is much larger than the bending moment diagram. Hence, curtailment of bars should be carried out.

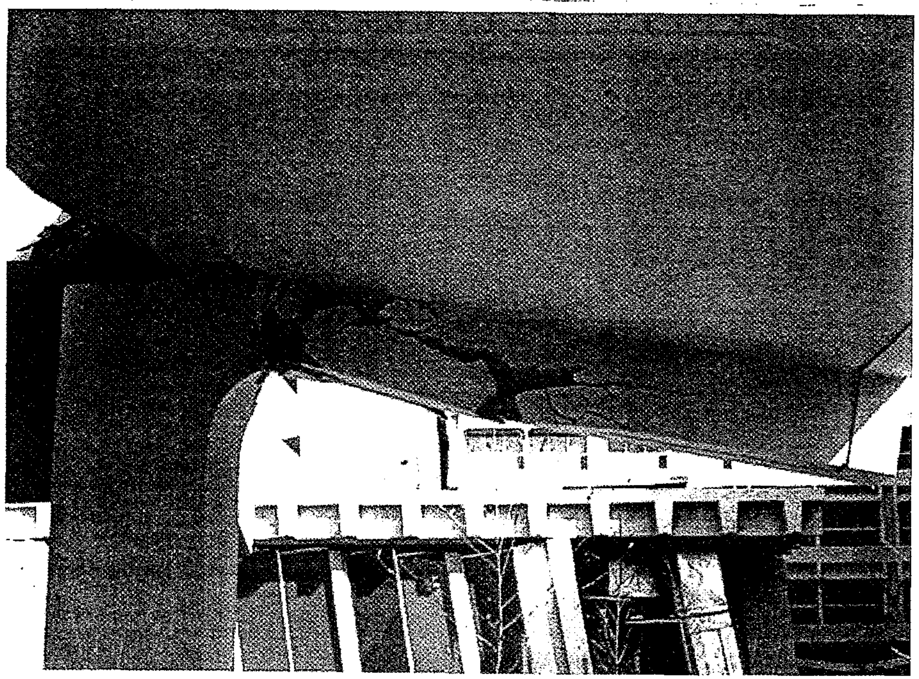


Photo 5.3 Failure of a beam-column connection due to inadequate anchorage of the reinforcement.

5.5.2 Curtailment of Bars in Beams

The maximum required area of steel is needed at the section of maximum bending moment. This amount of reinforcement may be reduced at sections of smaller moments. As a general rule, bars may be cut-off where they are no longer needed for flexural strength. Figure (5.8a) shows a better detailing for the simply supported beam shown in Fig. (5.7a), where two bars of diameter 22mm are extended past the center-line of the support and two bars of diameter 22mm are stopped exactly at the theoretical cut-off points. The moment of resistance diagram for the detailing that involves curtailment of bars is shown in Fig. (5.8b).

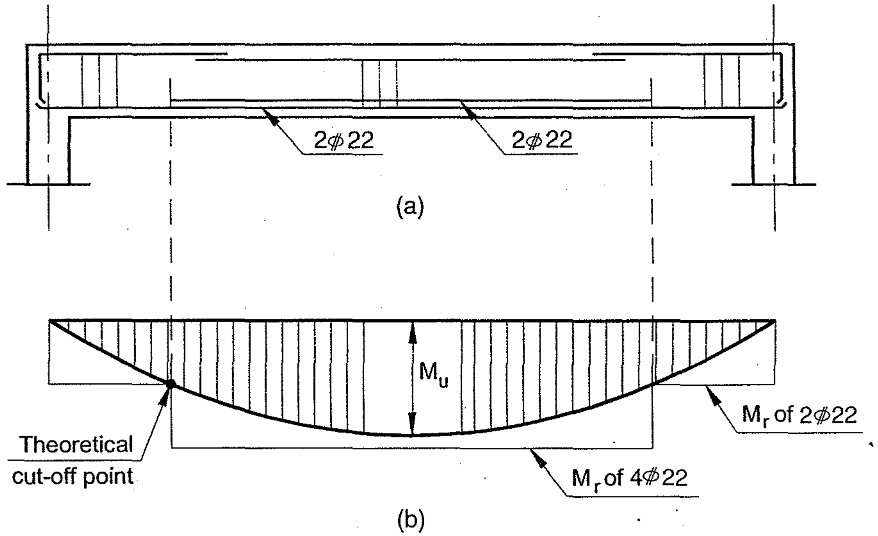


Fig. 5.8 Curtailment of bars in beams

There are practical considerations and sound reasons for not stopping bars exactly at the theoretical cut-off points. These reasons are:

- The bending moment diagrams may differ from those used for design due to approximations in loads and supports.

- After diagonal cracking, the steel tensile force at a crack is governed by the moment at a section not passing vertically above the crack intersection with the steel as indicated in Fig. (5.9), where the tension force (T) at section (1-1) is governed by the moment at section (2-2).
- It is necessary to develop the calculated stress in bars by providing adequate embedment length before the strength of the bar can be achieved.

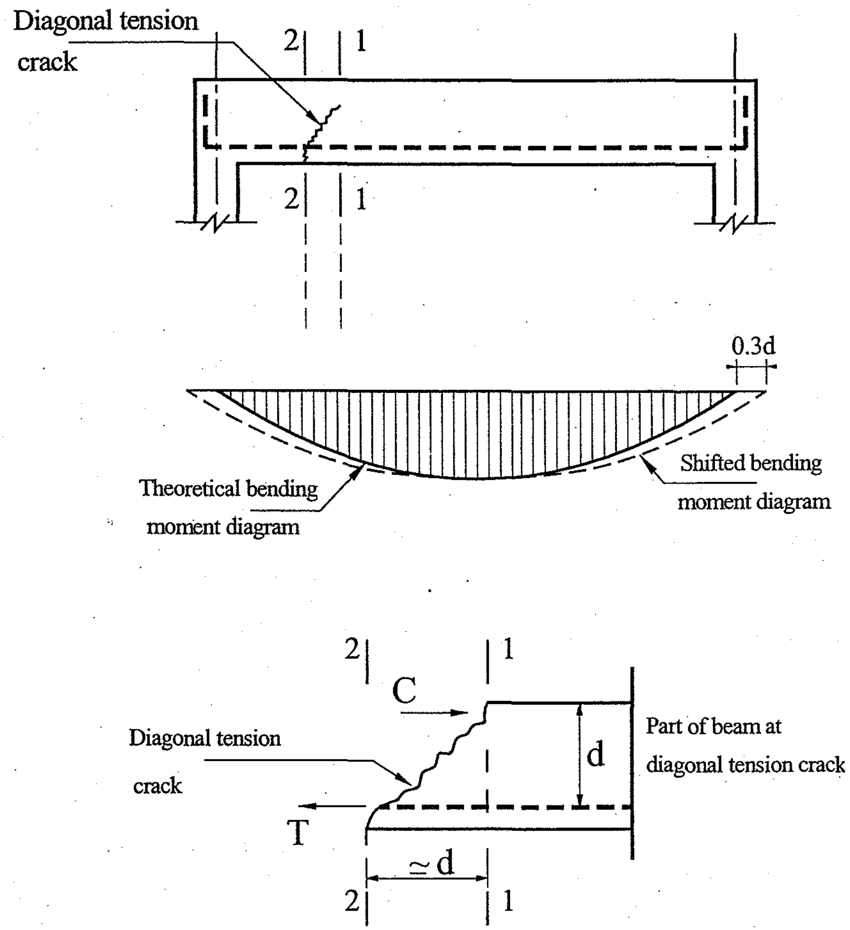


Fig. 5.9 The idea of shifted moment diagram

5.5.3 Egyptian Code's Requirements for Curtailment

To take into account the considerations mentioned in section (5.5.2) of this text, the Egyptian Code sets requirements for bar curtailment as follows:

1- At least one-third of the positive moment steel must be continued uninterrupted along the same face of the beam by a distance of 150 mm past the centerline of the support.

2- At least one-third of the total reinforcement for negative moment must be extended beyond the extreme position of the point of inflection (zero moment) a distance (L_a) not less than the greatest value of:
- d
- $0.3d + 10\Phi$
- $0.3d + (L/20)$

where
d is the effective depth of the beam,
Φ is the bar diameter and
L is the clear span

3- Every bar should be continued at least a distance (L_a) equal to the effective depth (d) or ($0.3d+10\ \Phi$) as shown in Fig. 5.10, whichever is larger, beyond the point at which it is theoretically no longer needed.

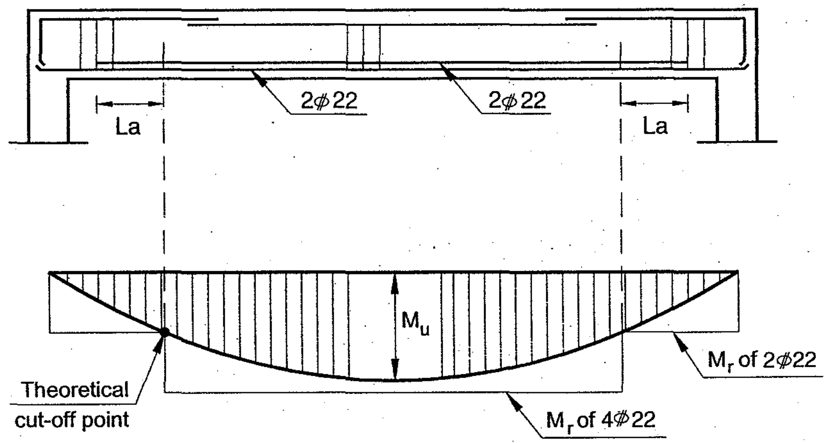


Fig. 5.10 Bar extension beyond the cut-off points (L_a)

4- The full development length (L_d) plus the moment shift i.e., ($L_d + 0.3d$) must be provided beyond critical sections at which the maximum stress exists. These critical sections are located at points where adjacent terminated reinforcement is no longer needed to resist bending moment.

5- No flexural bar shall be terminated in a tension zone unless one of the following conditions is satisfied:

- The shear at the terminated point does not exceed two-thirds that the shear strength of the cross section including shear strength of shear reinforcement q_{su}

$$q_u \leq \frac{2}{3}\left(\frac{q_{cu}}{2} + q_{su}\right) \quad \text{.......................................(5.12)}$$

- Stirrups in excess of those normally required are provided over a distance equal along each terminated bar from the point of cut-off to (0.75 d) as shown in Fig. (5.11). The area of the additional stirrups is not less than;

$$A_{s(add.)} = \frac{0.40\, b.s}{f_y} \quad \text{.......................................(5.13)}$$

The spacing between these stirrups shall not exceed $(d/8\beta)$ where β is the ratio of the area of reinforcement cut-off to the total area of tension reinforcement at the section.

$$s \leq \frac{d}{8\,\beta} \quad \text{.......................................(5.14)}$$

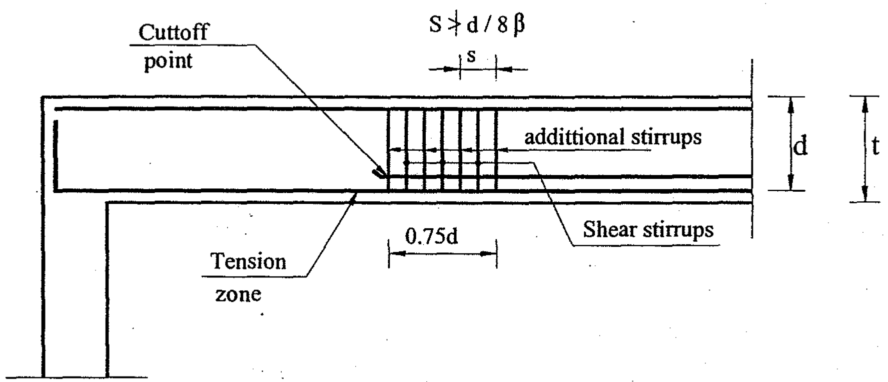
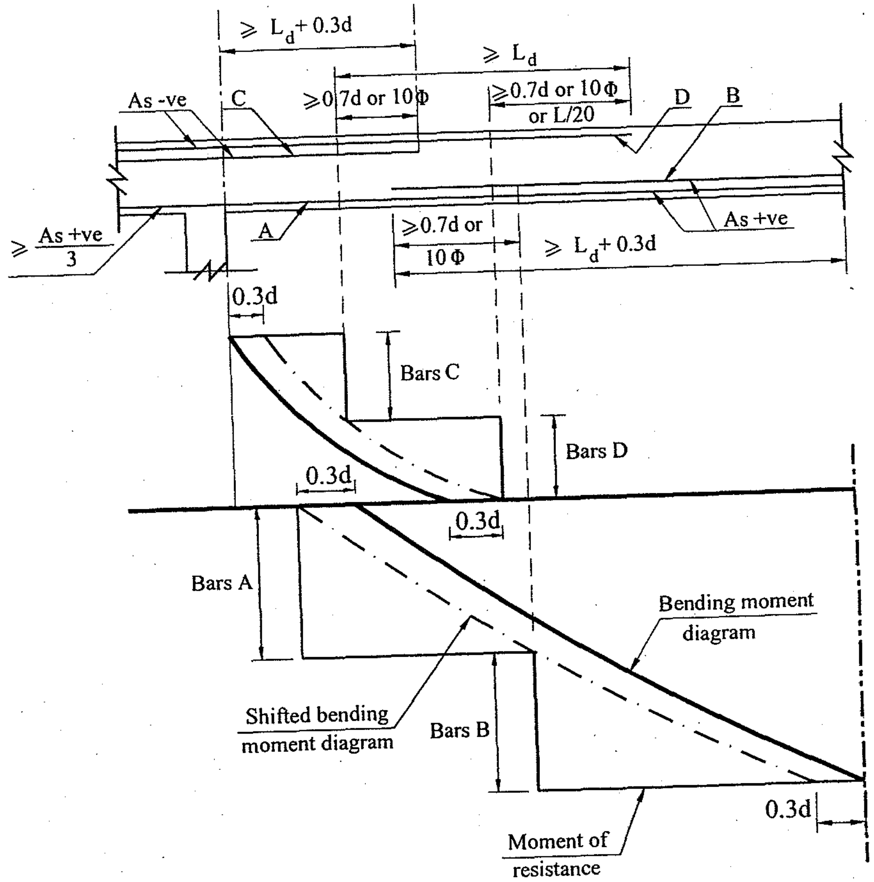


Fig. 5.11 Additional stirrups at cut-off points

ECP 203 requirements for curtailment of bars are summarized in Fig. 5.12



L_d =Development length of bar

d = Beam's effective depth

Φ = Bar diameter

A_s = Area of steel, (-)ve for top steel & (+) ve for bottom steel

Fig. 5.12 Egyptian Code requirements for bars curtailment

5.6 Beams with Bent-up Bars

Bars may be cut-off where they are no longer needed for flexural strength or may be bent to participate in shear resistance and can be extended further to provide tensile top reinforcement as shown in Fig. (5.13). In some cases, particularly for relatively deep beams, it may be impossible to make use of bent-up bars. The cost of labor involved in fabrication and erection of bent-up bars limits their use.

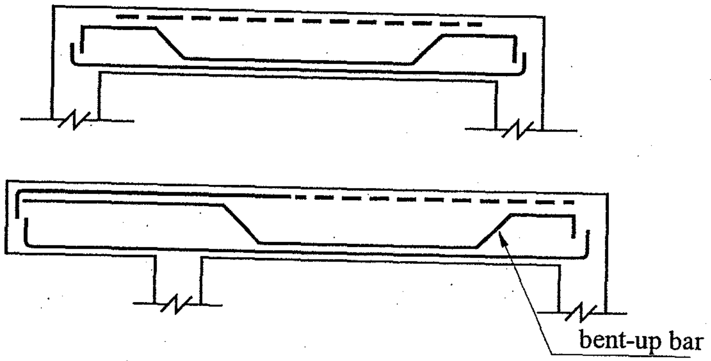


Fig. 5.13 Beams with bent-up bars

5.7 Anchorage of Web Reinforcement

As mentioned in Section 5.6, bars may be bent to participate in shear resistance. In this case, the bar must be extended beyond the point of maximum tensile stresses due to shear. This point may be taken at mid-depth of the beam as shown in Fig. 5.14-a. When stirrups are used as shear reinforcement, they must be properly anchored in the compression zone of the beam. To satisfy this requirement, stirrups are provided with 90° or 150° hooks as shown in Fig. 5.14b.

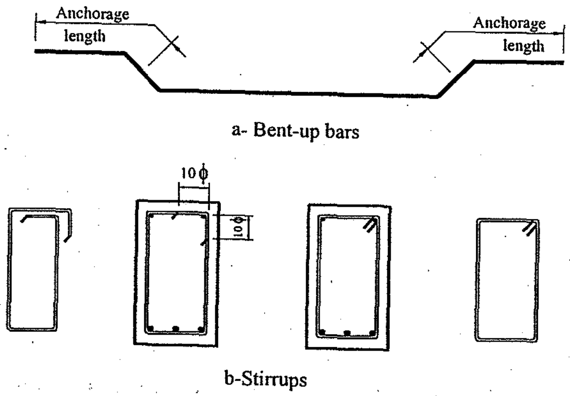


Fig. 5.14 Anchorage length for shear reinforcement

5.8 Splicing of Reinforcement

The need to splice reinforcing steel is a reality due to the limited lengths of steel available. Typical diameter bars are readily available in lengths up 12.0 m. Splicing may be accomplished by welding, utilizing mechanical connections, or, most commonly for bars having diameter 32 mm and smaller, by lapping bars. There are general requirements for proper splicing such as:

(i) Splices in reinforcement at points of maximum stress should be avoided.

(ii) When splices are used, they should be staggered.

5.8.1 Lap splices

In a tensile lapped splice, the force in one bar is transferred to the concrete, which transfers it to the adjacent bar. Bars are lapped a sufficient distance known as the lap length as shown in Fig. (5.15). The lap length shall not be less than the development length. If, however, the ratio of area of steel provided to the required area ($A_{s(provided)}/A_{s(required)}$) is less than 2.0, lap length shall be increased by 30%. If bars of different diameters are lapped, lap should be based on the larger diameter. Lap splices for tension steel should be staggered such that not more than 25% of total area of bars is lapped at one section.

In a compression lap splice, a portion of the force transfer is through bearing of the end of the bar on the concrete. This allows compression lap splices to be much shorter than tension lap splice.

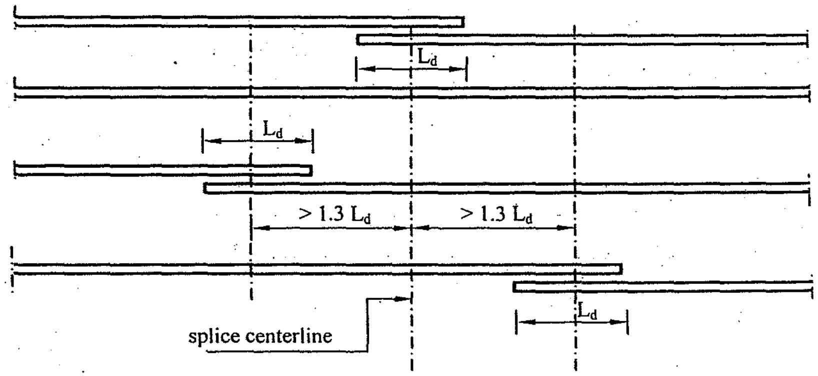


Fig. 5.15 Lap splices

The following conditions are also applied

a- Lap splices are not permitted in tension tie members. Splices in such members shall be made with a full welded splice or a full mechanical connection and splices in adjacent bars shall be staggered by at least 750 mm. The provisions of Section (4-2-5-4-3) of the ECP 203 shall be satisfied.

b- When splicing bars having different diameters, splice length shall be computed based on the larger diameter.

c- Lap splices of bundled bars shall be based on the lap splice length required for individual bars within a bundle calculated in accordance to Section (4-2-5-4-2-c) in the ECP 203, increased by 30%. It is not permitted to splice all the bars in the bundle at a certain section.

d- Lap splices shall not be used for bars having diameter more than 28 mm. For such diameters, welded splices or mechanical connections shall be used.

e- When splicing welded mesh in tension the splice length shall not be less than the following values:

1- For deformed bars, the lap splice length shall be equal to 1.3 L_d but not less than 150 mm (Fig. 5.16a).

2- For smooth bars, the lap splice length shall be equal to 1.5 L_d but not less than 200 mm (Fig. 5.16b).

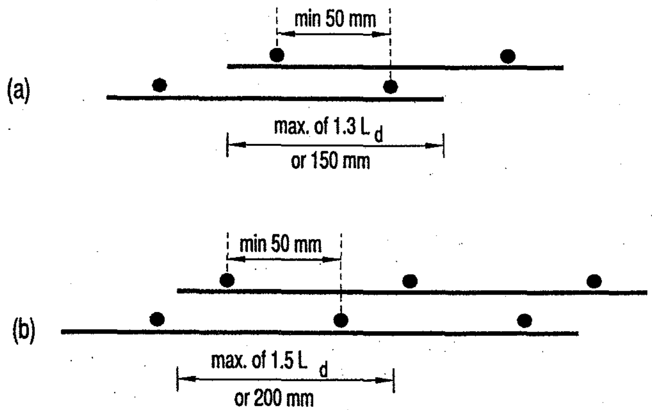


Fig. 5.16 Lap splices for welded mesh

5.8.2 Welded and Mechanical Connections

The code permits the use of welding according to the relevant Egyptian specifications. However, welded connections are not allowed for dynamically loaded structures. Welded bars should maintain their axes collinear. The welded connection shall develop at least 125 percent of the specified yield strength of the bar. This insures that an overloaded spliced bar would fail by ductile yielding away from the splice location.

The following conditions must be also satisfied:

a- Welded splices or mechanical connections not meeting the requirements of Section (4-2-5-4-3-b) of the ECP 203 may be used provided that the distance between splices shall not be less than 600 mm and the splice strength in tension or in compression is not less than the yield strength.

b- Only electrical welding is permitted in applying welding.

c- Welding is not permitted within a distance less than 100 mm from the point at which the bar is hooked provided that internal radius of the hook is not less than 12 times the bar diameter.

d- It is not permitted to splice cold-treated bars except after hot-treating the weld zone.

e- It is not permitted to splice bars by welding in structures subjected to dynamic loads.

6

REINFORCED CONCRETE BEAMS

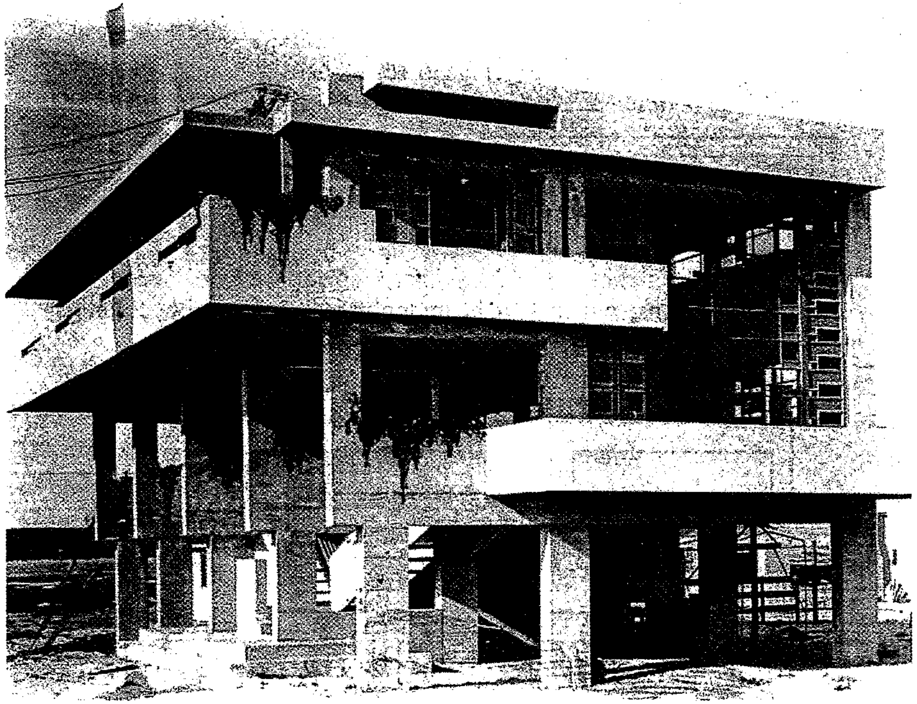


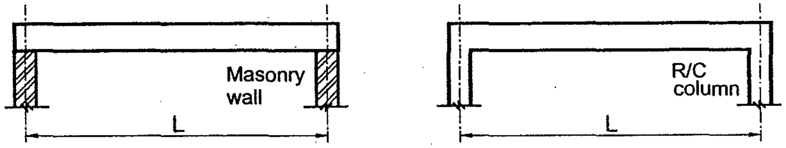
Photo 6.1 Skeleton reinforced concrete structure

6.1 Introduction

This chapter presents the analysis and design of reinforced concrete beams. It starts with introducing the reader to the statical system of R/C beams. Types of loads on beams and method of calculations of such loads are presented. Design of R/C beams to withstand ultimate limit states of failure by flexure, shear, or bond is illustrated. Reinforcement detailing is also presented. The chapter also contains numerous illustrative examples.

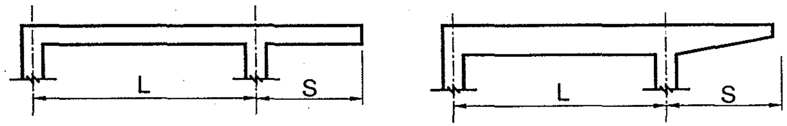
6.1 Statical Systems of R/C Beams

Depending on the conditions at the supports, R/C beams may be classified as:
a- Simple beams, which can be monolithically cast with columns or supported on masonry walls (Figs. 6.1a and 6.1b).



(a) (b)

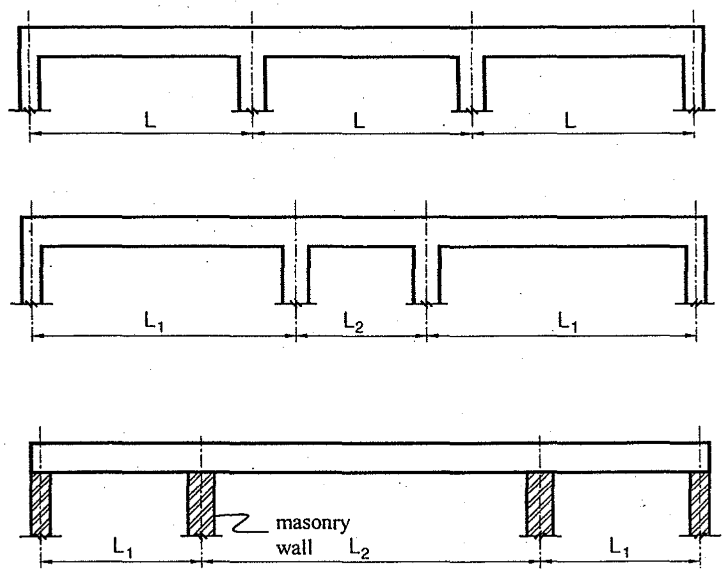
b- Simple beams with cantilevers (Figs. 6.1c and 6.1d).



(c) (d)

c- Continuous beams, which can be monolithically cast with columns (Figs. 6.1e and 6.1f) or supported on masonry walls (Fig. 6.1g).

(e)



(g)

Fig. 6.1 Statical systems of reinforced concrete beams

6.2 The effective span

The span used in the analysis of a reinforced concrete beam is referred to as the effective span, L_{eff}. The value of the effective span may be taken as follows:

For simply supported beams, (refer to Fig. 6.2), the effective span equals to the least value of:

- The distance between the center-lines of the supports (L)
- 1.05 times the clear span(L_c) between the supports
- The clear span between the supports plus the depth of the beam (L_c+d)

For cantilevers, the effective length equals to the lesser value of:

- The length of the cantilever measured from the center of support
- The clear projection of the cantilever plus its largest depth

For continuous beams monolithically cast with supports, the effective span may be taken equal to the lesser value of:

- The distance between the center-lines of the supports
- 1.05 times the clear span between the supports

For continuous beams supported on masonry walls, the effective span may be taken equal to the lesser value of:

- The distance between the center-lines of the supports
- The clear span between the supports plus the depth of the beam

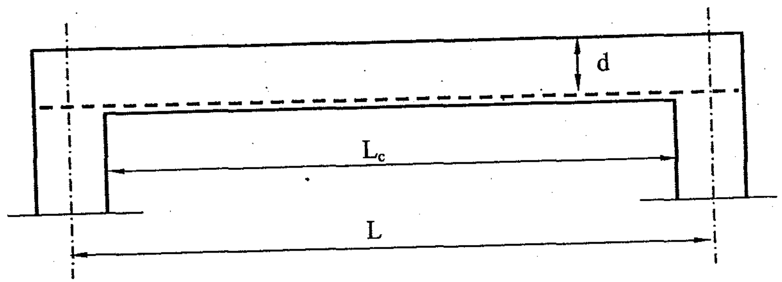


Fig. 6.2 Calculation of the effective depth

6.3 Loads Acting on Beams

A reinforced concrete beam carries the following loads:

- The own weight of the beam.
- The loads transmitted to the beam from the slab.
- The wall loads.
- Any other loads that can be directly transmitted to them.

6.3.1 Own weight of beams

The own weight of a beam is usually calculated per unit meter of its length as shown in Fig. 6.3.

$$\textit{Own weight of the beam (o.w.)} = \gamma_c \; b \; t \quad \text{.............................. (6.1)}$$

where
b = beam width
t = beam thickness
γ_c = density of reinforced concrete (for normal weight concrete =25 kN/m^3)

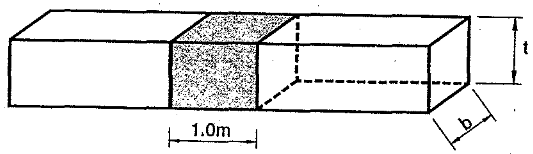


Fig. 6.3a Calculation of the own weight of rectangular beams

For slab-beam systems in which the slabs are cast monolithically with the beams, the own weight of the beam is calculated as follows

$$o.w. = \gamma_c \times b \times (t - t_s) \quad \text{.. (6.2)}$$

where
t_s = slab thickness

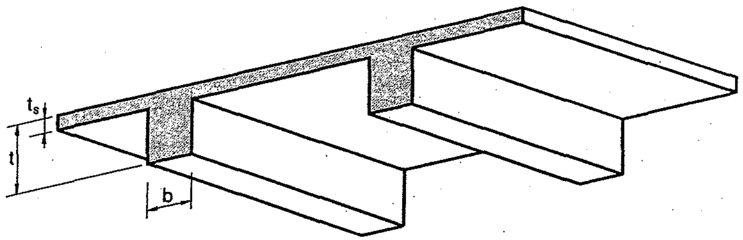


Fig. 6.3b Calculation of the own weight of T- beams

6.3.2 Slab loads

The slab load (kN/m^2) consists of a dead load g_s and a live load p_s. The dead load on the slab consists of its own weight and the weight of the flooring above. Dead load of the slab g_s is calculated from:

$$g_s = t_s \times \gamma_c + \textit{Flooring} \quad \text{.. (6.3)}$$

The floor covering (*flooring*) is usually taken from 1.5 to 2.5 kN/m^2 depending on the used materials.
The live loads p_s on the slab depend on the usage of the structure as given in Chapter 1.

In cases where the slab is supported on all four sides and the ratio of length to width is larger than 2 (Fig. 6.3), the short direction of the slab is stiffer than the long one. In such a case, the slab carries the load in its short direction and acts as a one-way slab. Accordingly, only the beams (AB and CD) of long spans support the loads from the slab.

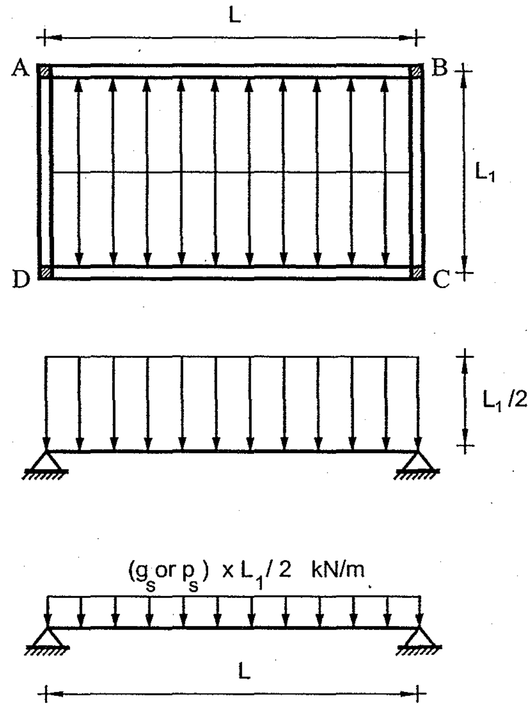


Fig. 6.3 Loads on beams supporting one-way slabs L/L$_1$>2

In cases where the slab is supported on all four sides and the ratio of length to width is smaller than or equal to 2 (Fig. 6.4), the load is carried by all the beams surrounding the slab and the slab is called a two-way slab.

For two-way slabs supported on relatively similar beams on the four sides, the lines defining the slab area associated to each beam bisect the corner between the two edges and are inclined 45° to either edge. Such distribution means that the loads on beams supporting two-way slabs are either triangular or trapezoidal.

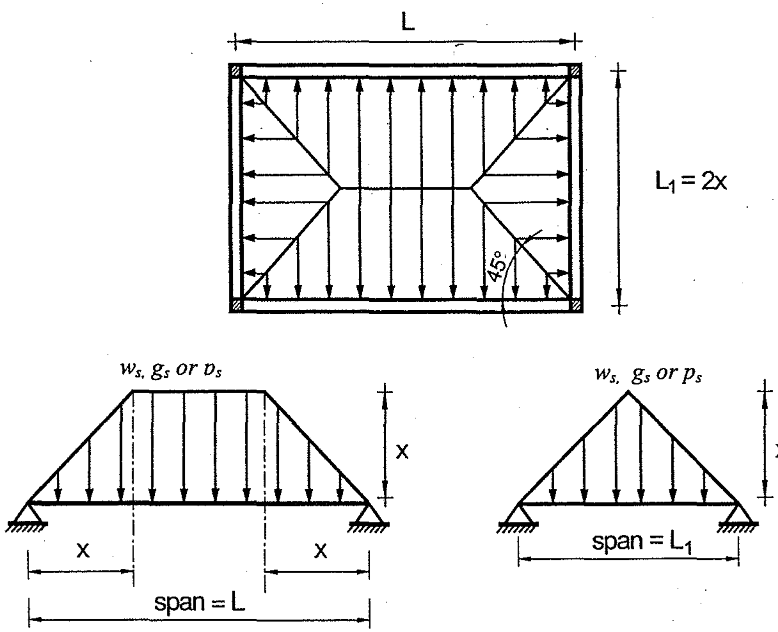


Fig. 6.4 Loads on beams supporting two-way slabs $1 \le L/L_1 \le 2$

In case where the triangular or the trapezoidal load satisfies the following conditions:

- The triangular or the trapezoidal load has a symmetrical distribution with the maximum intensity at the mid-span of the beam.
- The triangular or the trapezoidal load covers the span and vanishes at the supports.

Then, the triangular or the trapezoidal load can be replaced by uniformly distributed loads along the span of the beam, except for cantilever beams. Referring to Fig. (6.5), the following definitions are recalled:

Equivalent uniform load for shear (g_{sh} or p_{sh}): a uniform load that replaces the triangular or trapezoidal loads and gives the same maximum shear.

$$g_{sh} = \beta\ g_s\ x \qquad (6.4)$$

$$p_{sh} = \beta\ p_s\ x \qquad (6.5)$$

where g_s is the slab dead load given by Eq. 6.3 and p_s is the live load.

Equivalent uniform load for bending (g_b or p_b): a uniform load that replaces the triangular or trapezoidal loads and gives the same maximum bending moment at midspan.

$$g_b = \alpha\ g_s\ x \qquad (6.6)$$

$$p_b = \alpha\ p_s\ x \qquad (6.7)$$

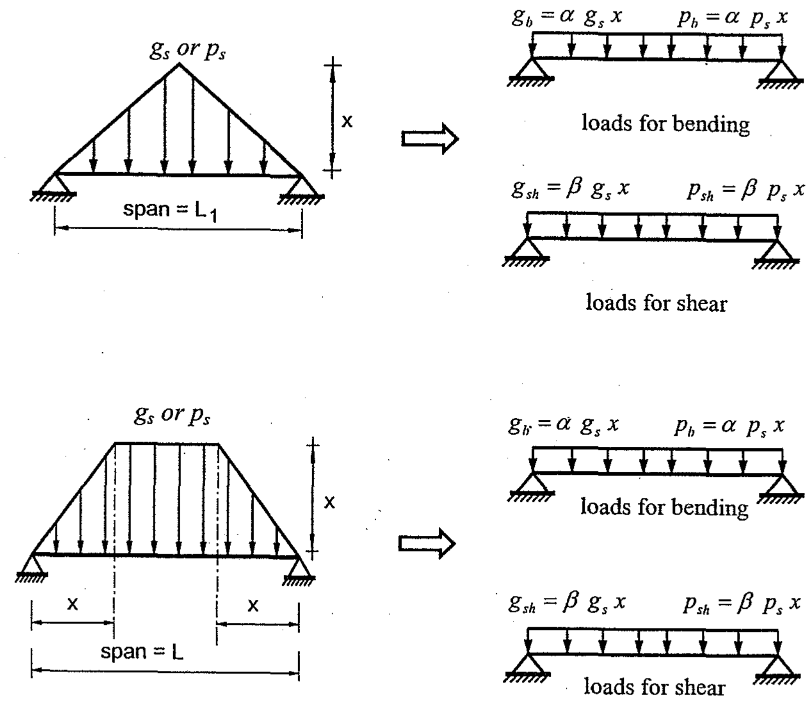


Fig. 6.5 The concept of equivalent uniform loads

In order to show the way in which this is done, consider the case of a beam supporting a triangular load with maximum intensity w_s at the middle as shown in Fig. 6.4. The values of α and β for a triangular load can be easily derived. Consider a slab load of an intensity w_s, then:

The maximum shear force due to the triangular load at support $= w_s L_1/4$

The maximum bending moment due to the triangular load at midspan $= w_s L_1^2/12$

If the triangular load is replaced by a uniform load of intensity w_β that gives the same value of maximum shear, then:

$$\frac{w_s L_1}{4} = \frac{w_\beta L_1}{2} \quad \text{or} \quad w_\beta = 0.5\, w_s \quad \beta = 0.5$$

If the triangular load is replaced by a uniform load of intensity w_α, that gives the same value of maximum bending moment at mid-span, then:

$$\frac{w_s L_1^2}{12} = \frac{w_\alpha L_1^2}{8} \quad \text{or} \quad w_\alpha = 0.667\, w_s \quad \alpha = 0.667$$

In case of trapezoidal loading, it can be easily shown that:

$$\alpha = (1 - \frac{1}{3r^2}) \qquad \beta = (1 - \frac{1}{2r}) \quad \text{.. (6.8)}$$

where $r = \dfrac{Long\ direction}{Short\ direction} = \dfrac{L}{L1} = \dfrac{L}{2\,x} \geq 1$

The coefficients α and β for are given in Table 6.1 for different values of L/2x.

Table 6.1 Coefficients of equivalent uniform loads on beams

$L/2x$	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
α	0.667	0.725	0.769	0.803	0.830	0.853	0.870	0.885	0.897	0.908	0.917
β	0.500	0.554	0.582	0.615	0.642	0.667	0.688	0.706	0.722	0.737	0.750

In case one of the conditions mentioned before is not met, as shown in Fig. 6.6, one should not use the coefficients α and β. In such a case, the non-uniform load could be approximated to an equivalent uniform load for calculating both the shear and the bending moments. The intensity of this load w is given as:

$$w = w_s\ (loaded\ area/loaded\ length) \quad \text{....................... (6.9)}$$

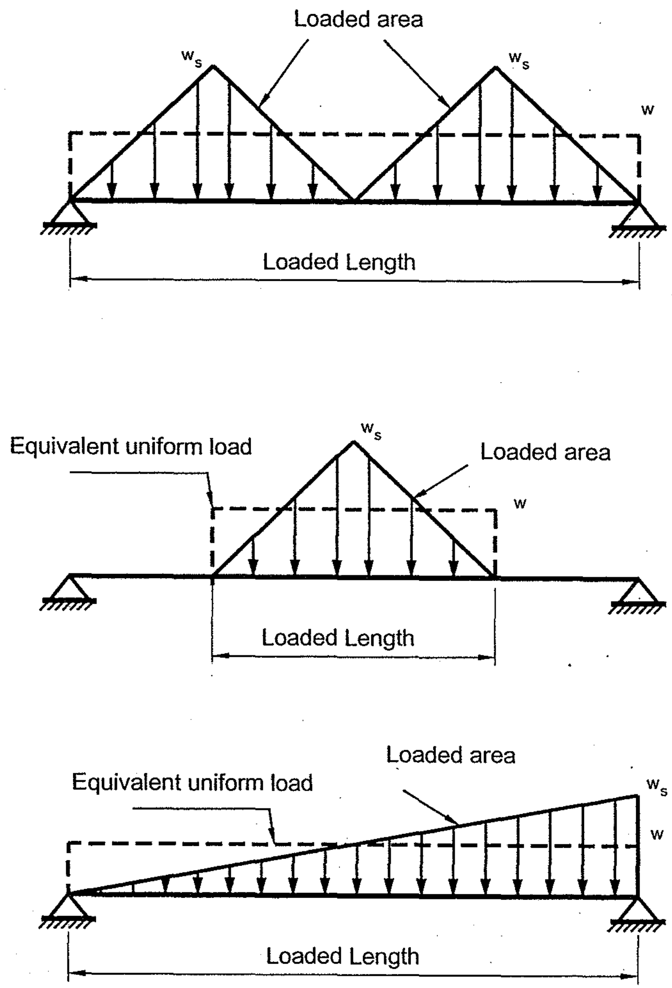


Fig. 6.6 Examples of cases where coefficients α and β can not be used.

6.3.3 Wall loads

Figure 6.7 shows an elevation of a skeletal concrete structure in which the simple beam with cantilever (beam B) supports a masonry wall having a clear height h_w. For the wall panel bounded between two columns, only wall loads bounded by 60° lines from columns cause bending moments and shearing forces in the beam (Fig. 6.7.b). This is mainly due to arching action of the wall.

The trapezoidal wall load can be replaced by an equivalent uniform load giving the same maximum internal forces. The coefficients α and β depend on the ratio $L_o/2x$, where $x = h_w/\sqrt{3}$ and can be determined from Table 6.1. It should be mentioned that for walls supported on cantilever beams no arching action occurs and the total wall load is transmitted to the beam.

$$g_w = \gamma_w \times t_w + \text{plastering weight} \qquad (6.10)$$

where

γ_w = specific weight of wall material ranges between (12-19) kN/m²

t_w = thickness of the wall

plastering weight from two sides can be assumed (0.8-1.0) kN/m²

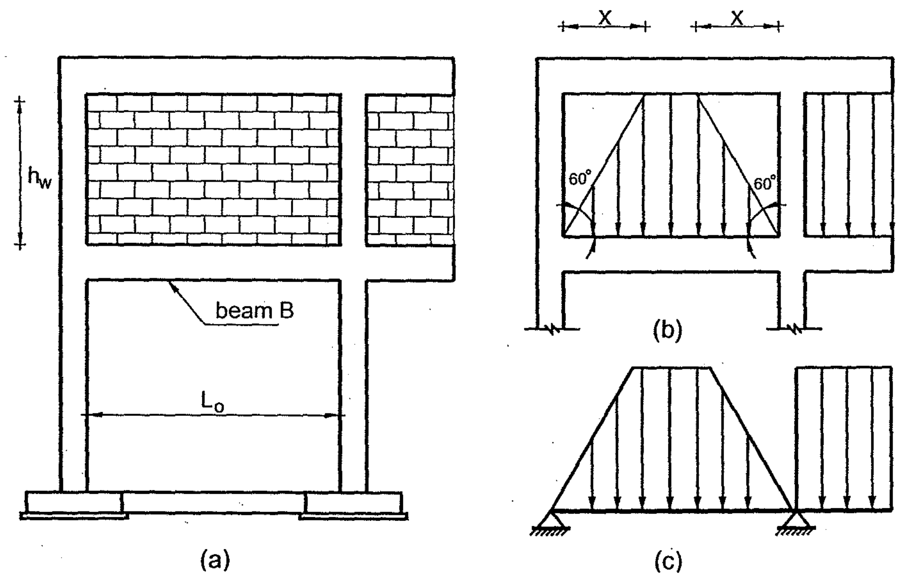


Fig. 6.7 Wall loads Calculations

For calculating wall load on beams we consider the arch action of the wall by considering a triangular part of wall load to be carried by the column while the remaining part of the wall load (trapezoidal part) is carried by the beam (refer to Fig. 6.7b). Hence, when calculating loads on columns the triangular part of the wall load must be added to columns loads.

$$\text{wall load for bending}(g_{wb}) = \alpha\, h_w\, g_w \qquad (6.11)$$

$$\text{wall load for shear}(g_{ws}) = \beta\, h_w\, g_w \qquad (6.12)$$

Referring to Fig. 6.7, assume that the h_w=2.4 m and the clear distance between the supports is 5.0 m and the own weight of the wall, g_w, equals 4.75 kN/m².

For $x = h_w/\sqrt{3} = 2.4/\sqrt{3} = 1.39\ m$ $\qquad L_o/2x = 1.80 \qquad \alpha = 0.897, \beta = 0.722$

Wall load for calculating bending moments for the part of the beam between the supports $= \alpha \times g_w \times h_w = 0.897 \times 4.75 \times 2.4 = 10.23$ kN/m

Wall load for calculating shearing forces for the part of the beam between the supports $= \beta \times g_w \times h_w = 0.722 \times 4.75 \times 2.4 = 8.23$ kN/m

The weight of the part of the wall supported on the cantilever beam is totally transmitted to the beam and is used for calculating the shear and the moments

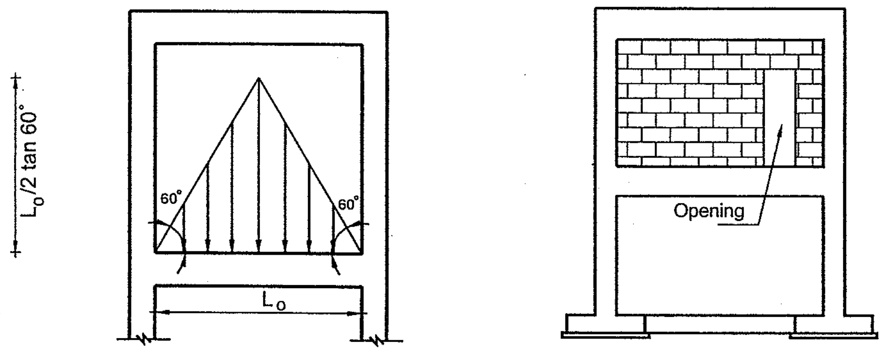
$= g_w \times h_w = 4.75 \times 2.4 = 11.4$ kN/m

<u>**Note 1**</u>: Figure 6.8a shows a case in which the wall dimensions result in triangular load on the beam. In such a case, the equivalent wall load is calculated as follows:

$$\text{wall load for bending} = 2/3\ (L_o/2\ \tan 60^\circ)\, g_w$$

$$\text{wall load for shear} = 1/2\ (L_o/2\ \tan 60^\circ)\, g_w$$

Note 2: In case of walls containing openings, the arch action is not fully developed and the total value of the wall load should be transferred to the beam (see Fig. 6.8b).



a. Short span walls

b. Walls with openings

Fig. 6.8 Calculation of wall loads for some special cases

Photo 6.2 Typical Slab-beam structural system

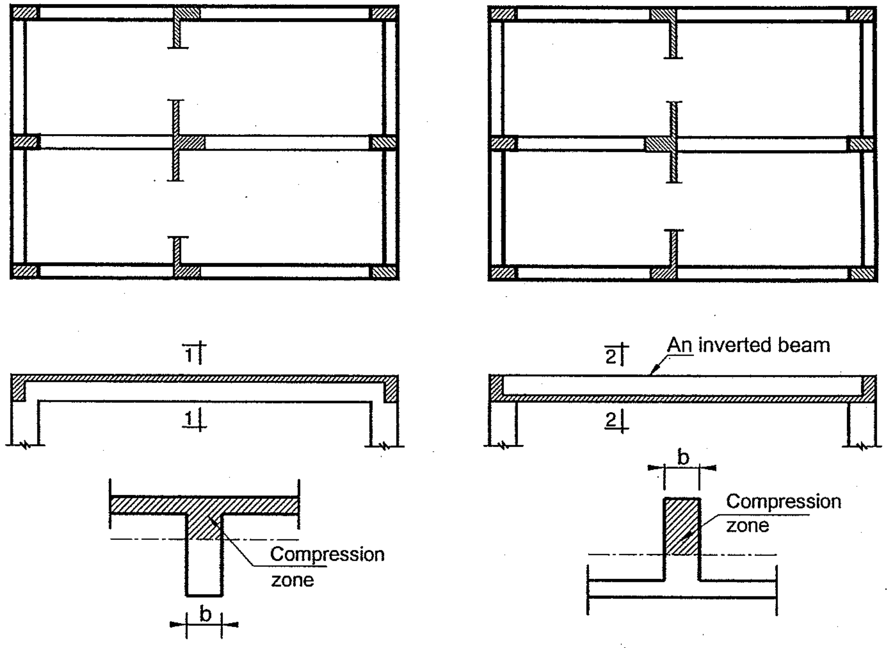
6.4 Slenderness limits for beams

The compression zone of a reinforced concrete beam is normally laterally restraint against sideway buckling. This lateral restraint is maintained by floor slabs attached to the compression zone (see Fig. 6.9a).

Figure (6.9b) shows a case in which the compression zone of the beams is not laterally supported against sideway buckling by the floor slabs. In such a case, and in other cases where the floor slabs does not exist, the code sets the following limits on the clear distance between points of inflections in the lateral direction:

(a) For simply supported or continuous beams, the lesser of 40 b or 200 b^2/d

(b) For cantilever beams with lateral restraint only at support: 20 b or $80b^2/d$ whichever is less:



(a): Laterally supported beam

(b): Laterally unsupported beam

Fig. 6.9 Lateral restraint against sideway buckling

6.5 Linear Elastic Analysis of Continuous Beams

The ECP 203 adopts linear elastic analysis for the determination of bending moments and shear forces in continuous beams. Critical load arrangement normally requires that alternate spans are loaded and adjacent spans are unloaded.

The ECP 203 permits the calculation of bending moments and shear forces based on the assumption of rigid knife-edge supports. This assumption may lead to very conservative values of the bending moments especially when the columns are stiff and beam to column joints are monolithic. It should be mentioned that for exterior columns moments induced by column restraint must be taken into account.

A further simplification is offered by the Code for continuous beams of nearly equal spans and depths under uniformly distributed loads, provided that variations in spans do not exceed 20% of the longest span, bending moments and shear forces may be estimated as shown in Fig. 6.10. The bending moments obtained using the coefficients of Fig. 6.10a should not be redistributed. For cases of particularly heavy live loads, the coefficients of the figure may not be applicable; an analysis of the continuous beams as being supported on knife-edge supports would be required.

I- Moment = w x L^2/k_m

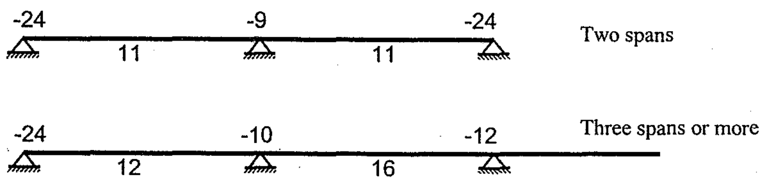


Fig. 6.10a Moment coefficients (k_m) for continuous beams

II- Shear = k_s x w x L

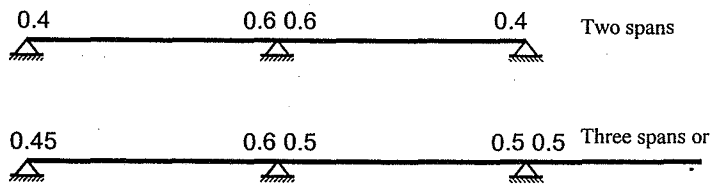


Fig. 6.10b Shear coefficients (k_s) for continuous beams

6.6 Reinforcement Detailing in R/C Beams

The effective depth, d, of a beam is defined as the distance from the extreme compression fiber to the centroid of the longitudinal tensile reinforcement (see Fig. 6.11).

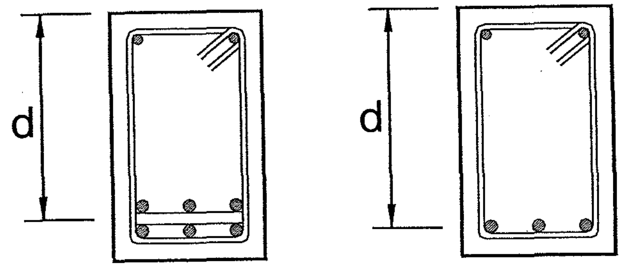


Fig. 6.11 Effective depth for a reinforced concrete beam

6.6.1 Concrete Cover

It is necessary to have a concrete cover (a) between the surface of the beam and the reinforcing bars (refer to Fig. 6.12) for three reasons:

1. To bond the reinforcement to the concrete so that the two elements act together. The efficiency of bond increases as the cover increases. A cover of at least one bar diameter is required for this purpose in beams.
2. To protect the reinforcement against corrosion. In highly corrosive environments such as beams constructed near ocean spray the cover should be increased.
3. To protect the reinforcement from strength loss due to overheating in the case of fire.

Table 6.2 summarizes the requirement ECP 203 regarding the value of the clear cover

Table (6.2) Minimum concrete cover** (mm)

Category of structure - Table (4.1)	All element except walls and slabs		Walls and Solid slabs	
	$f_{cu} \le 25$ N/mm^2	$f_{cu} > 25$ N/mm^2	$f_{cu} \le 25$ N/mm^2	$f_{cu} > 25$ N/mm^2
One	25	20	20	20
Two	30	25	25	20
Three	35	30	30	25
Four	45	40	40	35

** The concrete cover should not be less than the largest bar diameter

6.6.2 Bar Spacing

The arrangement of bars within a beam must allow:

1. Sufficient concrete on all sides of each bar to transfer forces into or out of the bars.
2. Sufficient space so that the fresh concrete can be placed and compacted around the bars.
3. Sufficient space to allow a vibrator to reach through the bottom of the beam.

Referring to Fig. 6.12, the Egyptian code requires that the distance b should be

$$b = \text{Larger of} \begin{cases} \text{Largest used bar diameter } \varphi_{max} \\ 1.5 \text{ max aggregate size} \end{cases}$$

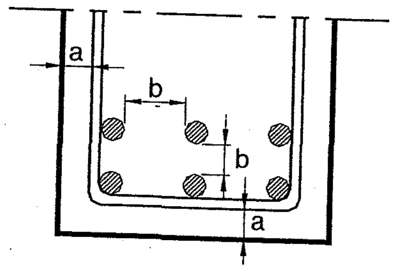


Fig. 6.12 Spacing between individual bars

6.6.3 Egyptian Code Recommendations

The ECP 203 requires when bars are placed in two more layers the bars in the top layer must be directly over those in the other layers, to allow the concrete and vibrators to pass through the layers.

If the drop of the beam below the slab exceeds 600 mm, longitudinal skin reinforcements (shrinkage side bars) are to be provided along both side faces of the beam. The area of this reinforcement should not be less than 8% of the area of the main reinforcement; the spacing should not exceed 300 mm (see Fig. 6.13).

Beam flange should be built integrally with or effectively connected to the web to justify the design of the beam as having a flanged section. Also, top reinforcement (normal to the beam axis) should be provided in the flange as a condition of utilizing the monolithic action between flange and web. The area of such reinforcement should not be less than 0.3% of flange's cross section. It should be extended to cover the total width of the effective flange. This transverse reinforcement shall be space not farther apart than 200 mm. Stirrups in the web should extend to the top of the flange to ensure monolithic action between the flange and web.

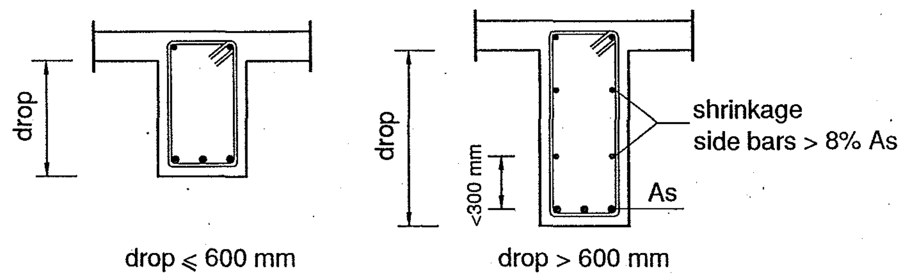
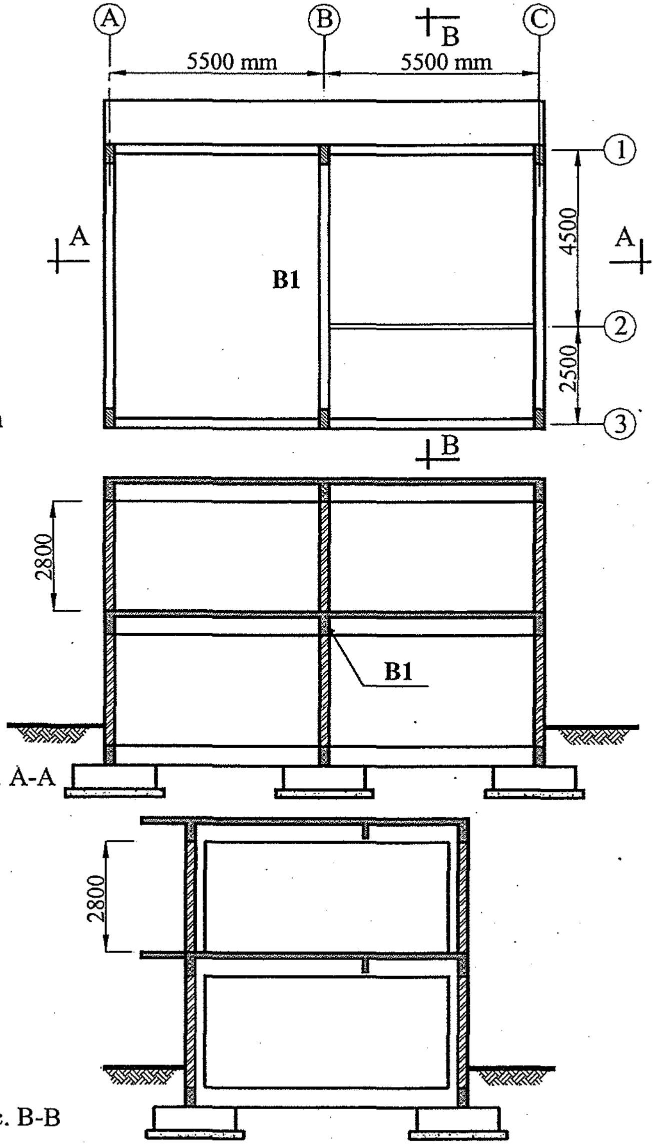


Fig. 6.13 Provisions for shrinkage reinforcement

Example 6.1

Figure (EX. 6.1) shows a plan and two sectional elevations of a reinforced concrete structure. It is required to calculate the loads acting on the simple beam B1. Live Load = 2.0 kN/m^2, Flooring =1.5 kN/m^2 and own weight of brick wall = 4.5 kN/m^2. Assume the thickness of the slabs = 150 mm.



(a) Plan

(b) Sec. A-A

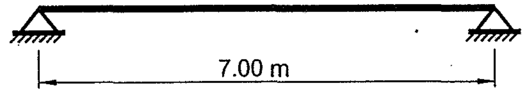
(c) Sec. B-B

Fig. EX. 6.1 Skeletal structure

Solution

Step 1: Statical System of the Beam

Simply supported beam having a span of 7.0 m



Statical system of the beam

Step 2: Calculation of Loads

Step 2.1: Own weight of the beam

The width of the wall on axis (B) is 250 mm, accordingly the width of the beam is taken as 250 mm. For simply supported beams, it is reasonable to assume the thickness of the beam as (span/10).

Thus the cross sectional dimensions of the beam = 250 mm × 700 mm

Own weight of the beam = width x (beam thickness – slab thickness) ×γ_c

= 0.25 x (0.70-0.15) x 25 = 3.44 kN/m

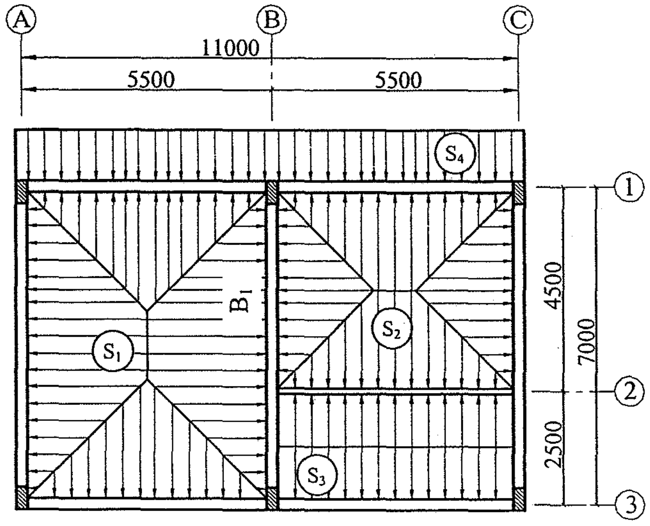
Step 2.2: Loads transmitted to the beam through the slab

Dead load of slab, g_s = Own weight of the slab + Flooring
= 0.15 x 25 + 1.50 = 5.25 kN/m^2

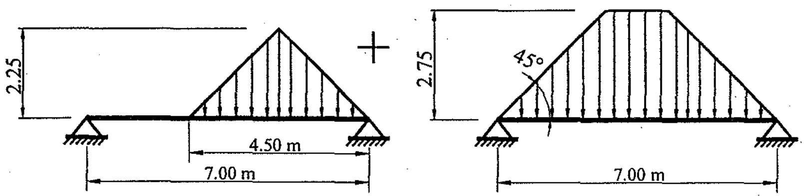
Live load of slab, p_s = 2.0 kN/m^2

The slab load distribution is shown schematically on the plan. Slabs S_1 and S_2 are classified as two-way slabs. Slab S_3 is a one way slab that transmits its load to the long span beams located on axes 2 and 3. Slab S_4 is a cantilever slab that transmits its load directly to the beams on axis (1).

The slab load is transmitted to beam B1 in two parts; the first part is transmitted directly and is composed of a trapezoidal load and a triangular load while the second part is transmitted indirectly as a concentrated load through the beam on axis (2).



Load distribution of the slabs



Direct slab load on beam B1

The trapezoidal load has its maximum value at the mid-span and vanishes at the supports. Thus, it can be replaced by uniform loads covering the whole span and giving the same maximum value of the internal force under consideration. The coefficients α and β are obtained as follows:

$\frac{L}{2x}=\frac{7.0}{5.5}=1.27$ $\qquad \alpha=0.793$ and $\beta=0.605$

Equivalent uniform dead load for bending = 5.25 x 0.793 x 2.75 = 11.45 kN/m′

Equivalent uniform dead load for shear = 5.25 x 0.605 x 2.75 = 8.73 kN/m′

Equivalent uniform live load for bending = 2.00 x 0.793 x 2.75 = 4.36 kN/m′

Equivalent uniform live load for shear = 2.00 x 0.605 x 2.75 = 3.33 kN/m′

The slab triangular load does not have its maximum value at the middle of the span and it does not vanish at the support. Hence, it will be considered with its average value both for calculating the bending moments and shearing forces.

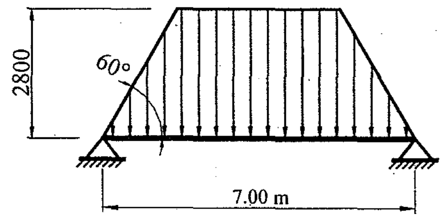
Equivalent uniform dead load for the loaded part of the beam = $\frac{g_s \text{ x loaded area}}{\text{loaded length}}$

$$= \frac{5.25 \text{ x } (0.5 \text{x} 4.5 \text{x} 2.25)}{4.5} = 5.9 \quad \text{kN/m}$$

Equivalent uniform live load for the loaded part of the beam = $\frac{p_s \text{ x loaded area}}{\text{loaded length}}$

$$= \frac{2.00 \text{ x } (0.5 \text{x} 4.5 \text{x} 2.25)}{4.5} = 2.25 \quad \text{kN/m}$$

Step 2.3: Wall Load



Direct wall load on the beam

From the figure the height of wall = 2.80 m

In order to simplify the analysis, the trapezoidal wall load is transformed into equivalent uniform loads for calculating the maximum bending moment and the maximum shear force.

$$\frac{L}{2x} = \frac{L}{2(h_w \tan 30)} = \frac{7.00}{2 \times 2.8 \times \tan 30^\circ} = 2.16 \rangle 2.0$$

Since L/2x > 2, the equivalent uniform wall Load for calculating the bending moment or the shear force of the beam

$$g_{wb} = g_{ws} = g_w \times h_w = 4.5 \times 2.80 = 12.6 \, \text{kN/m}$$

Step 2.4: Calculation of the concentrated load

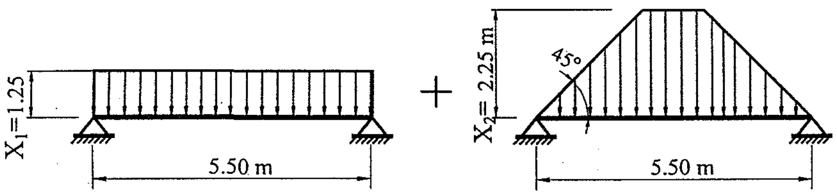
In addition to the previously calculated equivalent uniform loads, beam B1 supports the reaction of the secondary beam on axis (2).

The beam on axis (2) supports loads from the slab and its own weight. Assume that the dimensions of the beam are 120 mm x 700 mm.

Own weight of the beam = width x (beam thickness – slab thickness) x γ_c
= 0.12 x (0.7-0.15) x 25 = 1.65 kN/m

The slab load is transmitted to the beam on axis (2) directly and is composed of a rectangular uniform load and a trapezoidal load. Since we are interested in the maximum reaction of the beam on axis (2), only loads for shear are calculated.

$\frac{L}{2x}=\frac{5.5}{4.5}=1.22$ $\beta=0.589$



Slab load on the secondary beam on axis 2

It is clear from sec. B-B that no wall is present on the beam. Thus, wall load=0

Slab dead load = $g_s \times X_1 + g_s \times X_2 \times \beta = 5.25 \times 1.25 + 5.25 \times 2.25 \times 0.589 =$ 13.52 kN/m

Slab live load = $p_s \times X_1 + p_s \times X_2 \times \beta = 2.0 \times 1.25 + 2.0 \times 2.25 \times 0.589 =$ 5.15 kN/m

Total equivalent uniform dead load g_{sh} = ow.+ Slab load =1.65 + 13.52 = 15.17 kN/m

Total equivalent uniform live load p_{sh} = slab load = 5.15 kN/m

Reaction due to dead load (G)= $\frac{g_{sh} \times L}{2} = \frac{15.17 \times 5.5}{2} = 41.7$ kN

Reaction due to live load (P)= $\frac{p_{sh} \times L}{2} = \frac{5.15 \times 5.5}{2} = 14.2$ kN

Summary

Equivalent dead load for bending

Part ab

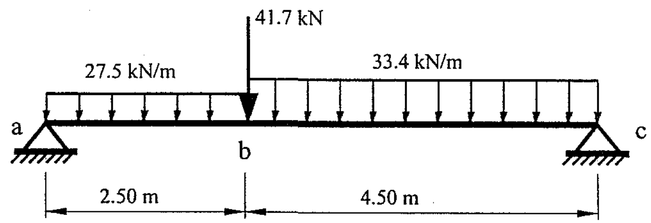
$g_b = o.w + slab\ load\ for\ bending + wall\ load\ for\ bending$

$g_b = 3.44 + 11.45 + 12.6 = 27.5\ kN/m$

Part bc

$g_b = 3.44 + (11.45 + 5.9) + 12.6 = 33.4\ kN/m$

The concentrated load is the reaction due to dead load as calculated from the analysis of the beam located on axis (2)



Equivalent dead load for shear

Part ab

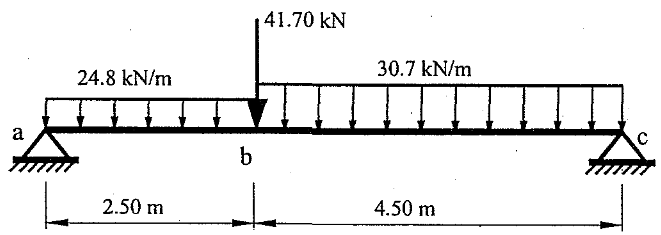
$g_{sh} = o.w + slab\ load\ for\ shear + wall\ load\ for\ shear$

$g_{sh} = 3.44 + 8.73 + 12.6 = 24.8\ kN/m$

Part bc

$g_{sh} = 3.44 + (8.73 + 5.9) + 12.6 = 30.7\ kN/m$

The concentrated load is the reaction due to dead load as calculated from the analysis of the beam located on axis (2)



Equivalent live load for bending

Part ab

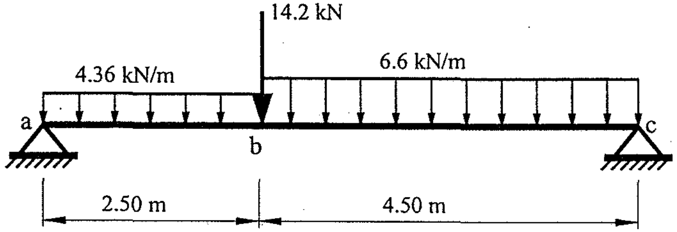
p_b = *slab live load for bending*

$p_b = 4.36 kN/m$

Part bc

$p_b = 4.36 + 2.25 = 6.6 kN/m$

The concentrated load is the reaction due to live load as calculated from the analysis of the beam located on axis (2)



Equivalent live load for shear

Part ab

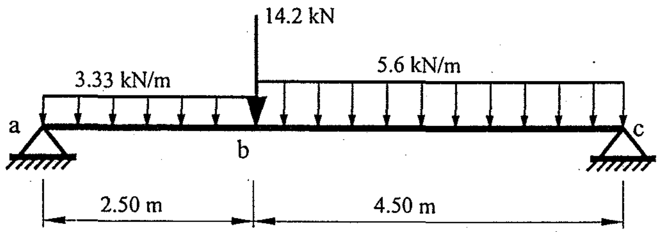
p_{sh} = *slab live load for shear*

$p_{sh} = 3.33 kN/m$

Part bc

$p_{sh} = 3.33 + 2.25 = 5.6 kN/m$

The concentrated load is the reaction due to live load as calculated from the analysis of the beam located on axis (2)



Example 6.2

Figure EX. 6.2a shows an architectural plan of a typical story of an office building. It is required to:

a- Propose the structural system of the floor as a slab-beam type system.

b- Draw the load distribution from the slabs to the surrounding beams.

Solution

The structural engineer works very closely with the architect when proposing the structural system of the floor in order to meet the architectural requirements.
Generally, one provides beams at the locations of the masonry walls. The width of the beam is usually equal to that of the wall. Beams are also provided in order to get reasonable slab dimensions.

According to the previously mentioned points, the structural plan of the floor is shown in Fig. EX. 6.2b. The following points can be observed:

1- There are two terraces in the floor plan. They are both cantilever slabs since the span of the cantilever is relatively small (1.2m). For spans of about 2.0m, cantilever slabs can be economically utilized. For longer spans, deflection considerations limited the use of cantilever slabs.

2- In the corridor area, no walls exist on axes 3, 4 and 5. Hence, we can either provide beams between the columns on axes C and D or leave the corridor area beam free. If it is architecturally accepted, then providing beams improves the framing action of the building.

3- The slabs of the structural plan of Fig. EX. 6.2b have relatively reasonable dimensions. No need to provide beams to reduce the area of such slabs. For example, the floor slab of the meeting room has dimensions of 4.0m x 7.6m. The area of such a slab could be divided into two parts if one provides a beam on axis E, between axes 5 and 6. However, such a beam is not architecturally acceptable. Accordingly, the structural engineer should provide the required thickness and steel reinforcement of such slab of dimensions 4.0m x 7.6m in order to preserve the architectural requirements. The same observation applies to the floor slab of room (5), where a beam could have been provided on axis 7 between the columns located on axes A and C.

4- Most of the floor beams are supported directly on columns. In some cases, however, beams can be supported on other beams. For example, the beam located on axis 8 is supported on cantilevers extended from the beams located on axes A and C.

The load distribution from the slabs to the surrounding beams is shown in Fig. EX. 6.2c.

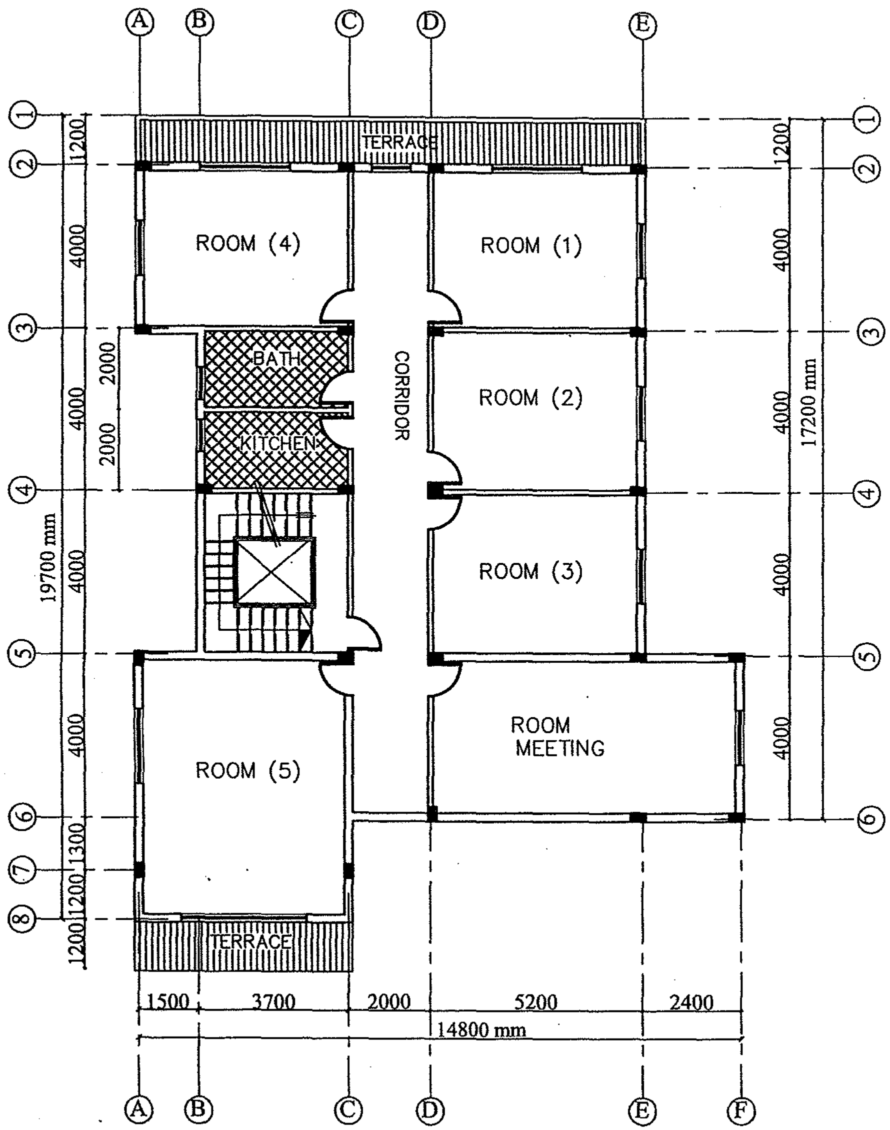


Fig. EX. 6.2a Architectural plan for a typical floor of an office building

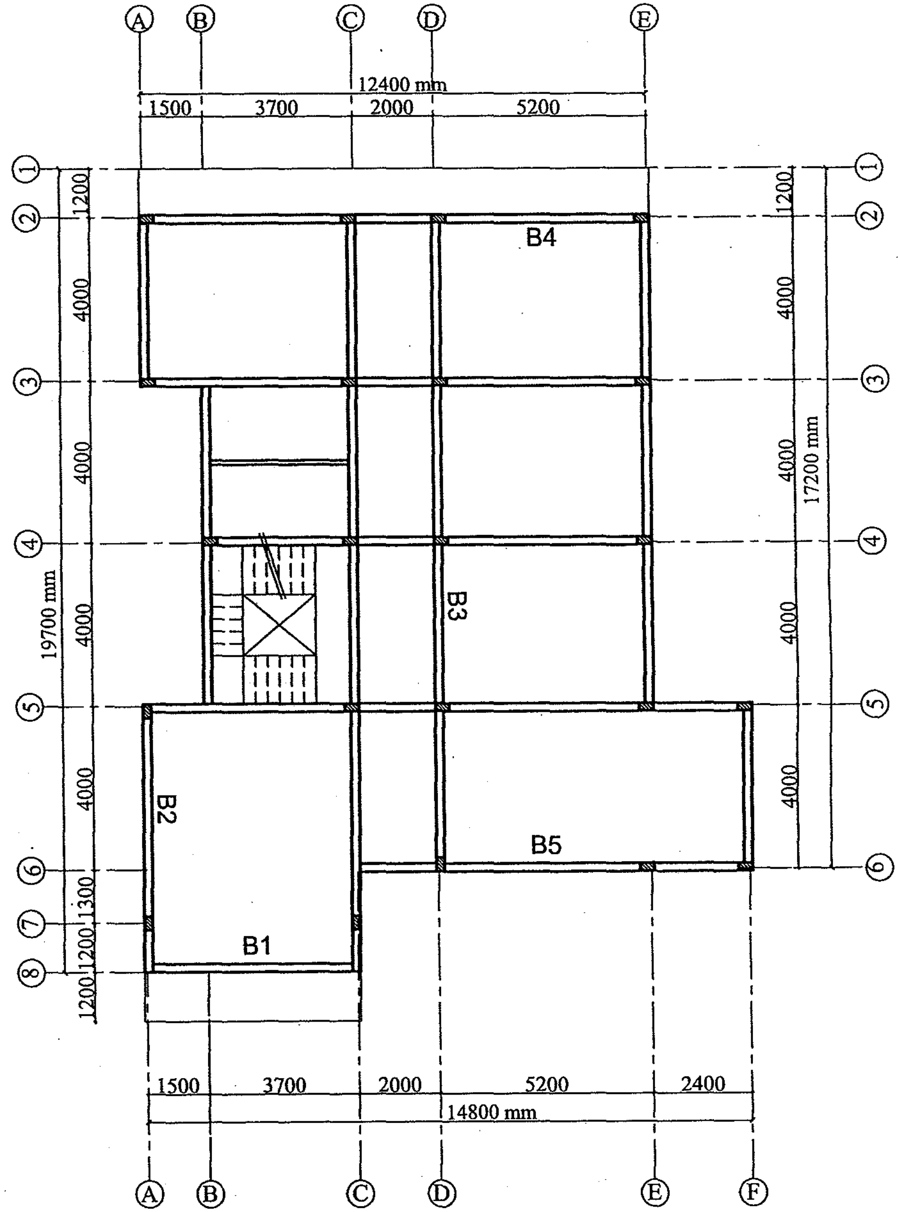


Fig. EX. 6.2b Structural plan (slab-beam system)

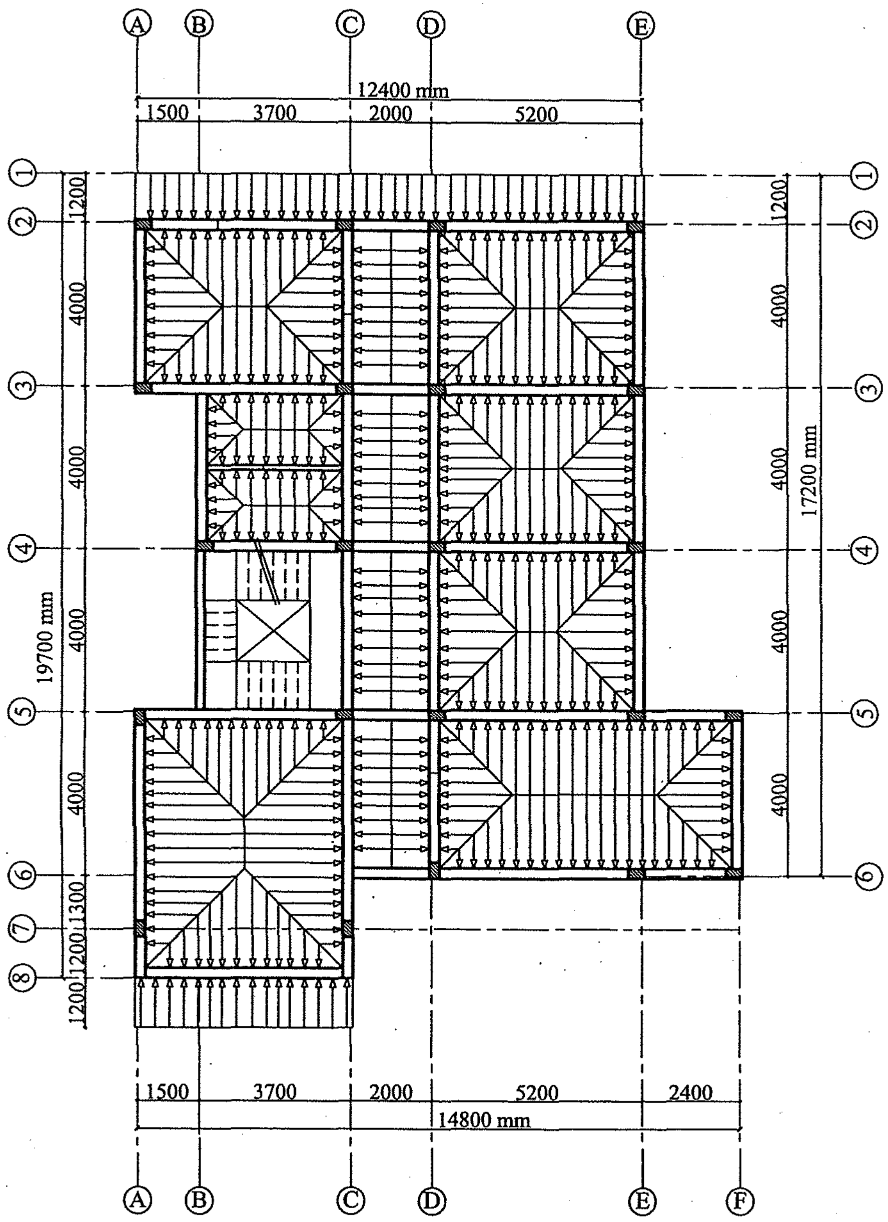


Fig. EX. 6.2c Load distribution from the slab to the surrounding beams

Example 6.3

It is required to calculate the loads acting on the simple beam with cantilever B2 shown in Fig. (EX. 6.2b).

Data

Live Load = 2.0 kN/m^2

Flooring = 1.5 kN/m^2

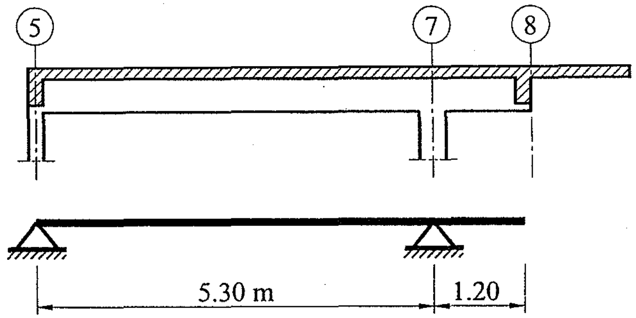
Floor Height = 3.50 m.

Specific weight of the brick wall is 14.8 kN/m^3

Solution

Step 1: Statical System of the Beam:

Simple beam with a cantilever



Statical System

Step 2: Calculation of Loads

Step 2.1: Own weight of the beam

Assume that slab thickness is equal to 150 mm.

Assume that the thickness of the beam in the range of (span/10) ($t \cong 600\ mm$)

Thus the cross sectional dimensions of the beam = 250 mm x 600 mm

Own weight of beam = width x (thickness of beam–thickness of slab) x γ_c

$$= 0.25 \times (0.60-0.15) \times 25 = 2.81\ \text{kN/m}$$

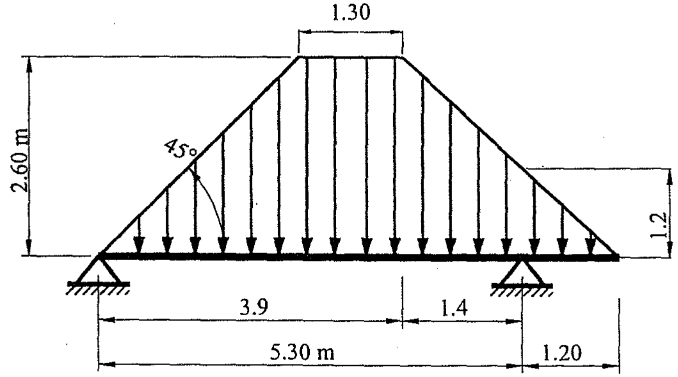
Step 2.2 Loads transmitted to the beam through the slab

Own weight of slab = thickness of slab x γ_c = 0.15 x 25 = 3.75 kN/m^2

Dead Load, g_s = Own weight of slab + Flooring = 3.75 + 1.50 = 5.25 kN/m^2

Live Load p_s = 2.0 kN/m^2

The slab load is transmitted to beam B2 in two parts; the first part is transmitted directly and is composed of a trapezoidal load and the second part is transmitted indirectly as a concentrated load through the secondary beam B1.



Direct slab load transmitted to B2

The slab trapezoidal load does not have its maximum value at the middle of the span and it does not vanish at the support. Hence, the loads will be calculated using the area-method for calculating the bending moments and shearing forces.

$$\text{Average uniform load} = \frac{(g_s \text{ or } p_s) \text{ x loaded area}}{\text{loaded span}}$$

1. Between supports

$$loaded\ area = \frac{2.6\times 2.6}{2} + 2.6\times 1.3 + \left(\frac{2.6+1.2}{2}\right)\times 1.4 = 9.42\,m^2$$

$$\text{Thus, the uniform dead load} = \frac{g_s \times loaded\ area}{loaded\ span} = \frac{5.25 \text{ x } (9.42)}{5.30} = 9.33 \text{ kN/m}'$$

$$\text{and the uniform live load} = \frac{p_s \times loaded\ area}{loaded\ span} = \frac{2.0 \text{ x } (9.42)}{5.30} = 3.55 \text{ kN/m}'$$

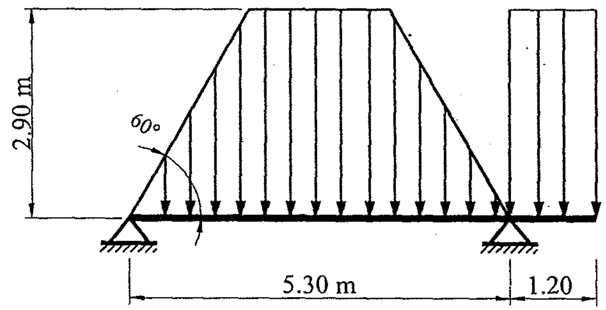
2. Cantilever part

$$loaded\ area = \frac{1.2^2}{2} = 0.72\,m^2$$

$$\text{Thus, the uniform dead load} = \frac{g_s \times loaded\ area}{loaded\ span} = \frac{5.25 \text{ x } (0.72)}{1.2} = 3.15 \text{ kN/m}'$$

$$\text{and the uniform live load} = \frac{p_s \times loaded\ area}{loaded\ span} = \frac{2.0 \text{ x } (0.72)}{1.2} = 1.2 \text{ kN/m}'$$

Step 2.3: Wall Load



Wall load transmitted to B2

$$g_w = \gamma_w \times t_w + plastering\ weight = 14.8\times 0.25 + 0.8 = 4.5\,kN/m^2$$

Height of the wall = floor height – beam thickness = 3.5 – 0.60 = 2.90 m

$$x = \frac{h}{\tan 60} = \frac{h}{\sqrt{3}} = \frac{2.9}{\sqrt{3}} = 1.67\ m$$

$$\frac{L}{2\,x} = \frac{5.3}{2\times 1.67} = 1.58$$

α=0.867 and β=0.684

1. Between supports

- Equivalent uniform wall load for bending (g_{wb})

  $$g_{wb} = \alpha \times g_w \times h_w = 0.867\times 4.5\times 2.9 = 11.31\,kN/m'$$

- Equivalent uniform wall load for shear (g_{ws})

  $$g_{ws} = \beta \times g_w \times h_w = 0.684\times 4.5\times 2.9 = 8.93\,kN/m'$$

2. Cantilever part

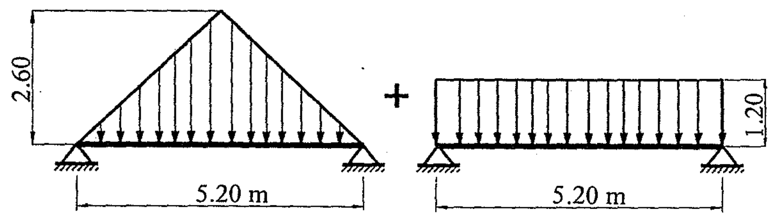
- Since the wall load is rectangular, the equivalent uniform wall-load for bending equals the load for shear.

  $$g_{wb} = g_{ws} = g_w \times h_w = 4.5\times 2.9 = 13.05\,kN/m'$$

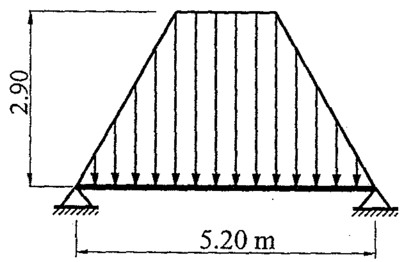
Step 2.4: Calculation of the Concentrated Load (Beam B1)

In addition to the previously calculated average uniform loads, beam B2 supports the reaction of the secondary beam on axis (8) (B1).

Beam B1 supports loads from the slabs, the walls, and its own weight. When calculating its reaction, it should be analyzed for the **load for shear**. The slab load is composed of a uniform rectangular load and a triangular load.



Direct slab loads on B1



Direct wall load on B1

Self weight

Assume the concrete dimension of B1 as 250 mm x 600 mm

Own weight of the beam = width x (beam thickness – slab thickness) x γ_c

= 0.25 x (0.6-0.15) x 25 = 2.81 kN/m

Slab load

Slab dead load for shear = load from triangular part + load from one way slab

$= \beta \cdot g_s \cdot x_1 + g_s \cdot x_2$

= 0.5 x 5.25 x 2.6 +5.25 x 1.2 = 13.125 kN/m′

Slab live load for shear $= \beta \cdot p_s \cdot x_1 + p_s \cdot x_2$

= 0.5 x 2.0 x 2.6 + 2.0 x 1.2 = 5.0 kN/m′

Wall load

Height of the wall = Floor height – Beam thickness= 3.5 – 0.60 = 2.90 m

$x = \frac{h_w}{\sqrt{3}} = \frac{2.9}{\sqrt{3}} = 1.67\ m$ $\qquad \frac{L}{2\,x} = \frac{5.2}{2 \times 1.67} = 1.55$

α=0.862 and β=0.678

$g_w = \gamma_w \times t_w + plastering\ weight = 14.8 \times 0.25 + 0.8 = 4.5\,kN/m^2$

Equivalent uniform wall load for calculating the shear (g_{ws})

$g_{ws} = \beta \times g_w \times h_w = 0.678 \times 4.5 \times 2.9 = 8.85\,kN/m'$

Total load

Total equivalent uniform dead load for shear (g_{sh}) =

$g_{sh} = o.w + slab\,load\ for\ shear + wall\ load\ for\ shear$

$g_{sh} = 2.81 + 13.125 + 8.85 = 24.79\,kN/m$

Total equivalent uniform live load for shear (p_{sh})= 5.0 kN/m′

Reactions

Reaction due to dead load (G)= $\frac{g_{sh} \times L}{2} = \frac{24.79 \times 5.2}{2} = 64.4\,kN$

Reaction due to live load (P)= $\frac{p_{sh} \times L}{2} = \frac{5.0 \times 5.2}{2} = 13.0\,kN$

Step 3: Total loads acting on the beam B2

Step 3.1: Equivalent dead load for bending

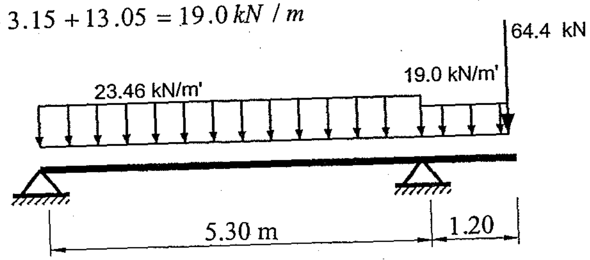
between supports

$g_b = o.w + slab\,load + wall\ load\ \ for\,bending$

$g_b = 2.81 + 9.33 + 11.31 = 23.46\,kN/m'$

cantilever part

$g_b = 2.81 + 3.15 + 13.05 = 19.0\,kN/m$



Step 3.2: Equivalent dead load for shear

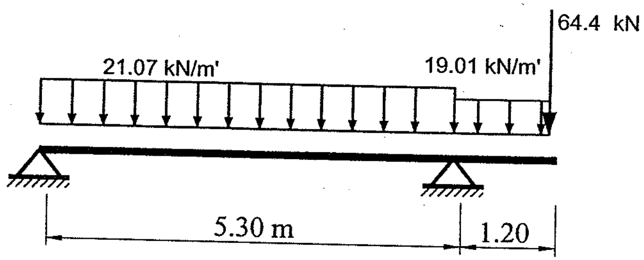
between supports

$g_{sh} = o.w + slab\ load\ + wall\ load\ \ for\ shear$

$g_{sh} = 2.81 + 9.33 + 8.93 = 21.07\,kN/m'$

cantilever part

$g_{sh} = 2.81 + 3.15 + 13.05 = 19.01\,kN/m'$



Step 3.3: Equivalent live load for shear and bending

Since the slab load is calculated using the area method, equivalent live load for shear equals equivalent live load for bending

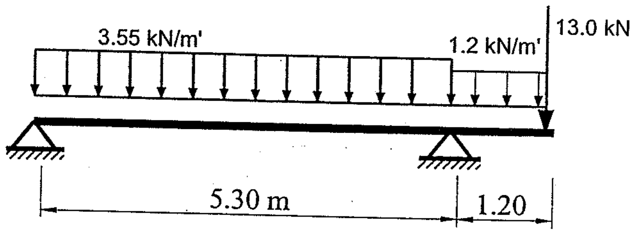
between supports

$p_b = slab\ live\ load$

$p_b = 3.55\ kN/m'$

cantilever part

$p_b = 1.2\,kN/m'$



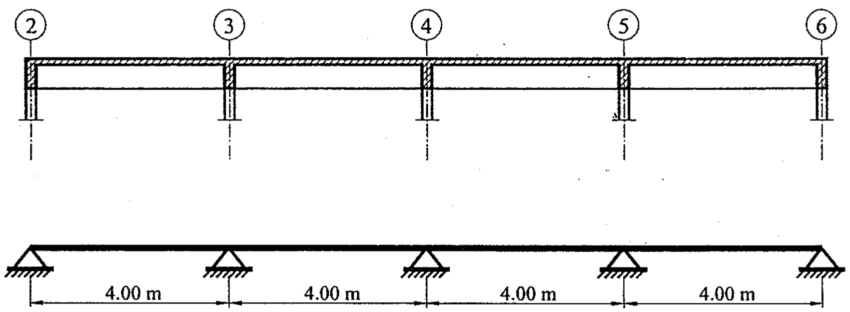
Example 6.4

It is required to find the loads acting on the beam B3 shown in Fig. (EX. 6.2b). Live Load = 2.0 kN/m^2, Flooring = 1.5 kN/m^2, Floor Height = 3.50 m. Specific weight of the brick wall is 14.8 kN/m^3.

Solution

Step 1: Statical System of the Beam:

Continuous beam with four equal spans



Statical system

Step 2: Calculation of Loads

Step 2.1: Own weight of the beam

Assume the cross sectional dimensions of the beam = 0.12 m x 0.60 m
Assume that average slab thickness is120 mm.
Own weight of beam = width x (thickness of beam–thickness of slab) x γ_c
= 0.12 x (0.60-0.12) x 25 = 1.44 kN/m

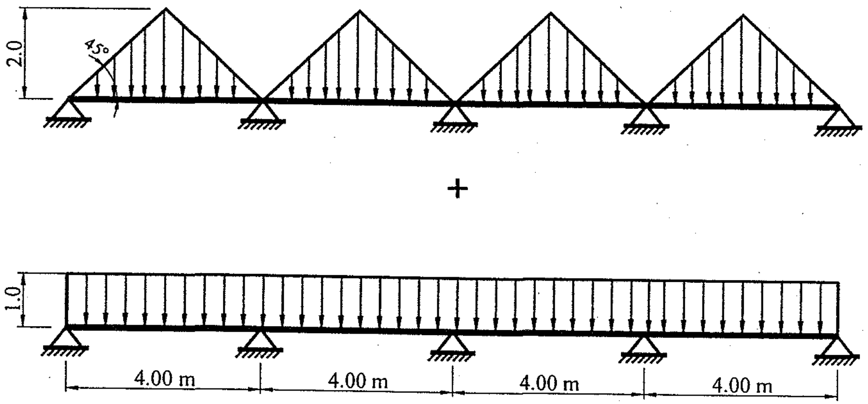
Step 2.2: Load transmitted to the beam through the slab

Own weight of slab = $t_s \times \gamma_c$
= 0.12 x 25 = 3.00 kN/m^2

Dead Load, g_s = Own weight of slab + Flooring
= 3.00 + 1.50 = 4.50 kN/m^2

The continuous beam supports two-way slabs from one side and one-way slabs from the other side. The two-way slabs transmit triangular loads to the beam,

while the one-way slabs transmit uniform loads. In order to simplify the beam analysis, the triangular loads are transformed into equivalent uniform loads.



Slab Loads

Equivalent uniform slab dead load for bending

g_b = triangular load from two way slab + load from one way slab

$= \alpha \cdot g_s \cdot x_1 + g_s \cdot x_2$

= 0.67 x 4.50 x (4.0/2) + 4.50 x 2/2 = 10.53 kN/m′

Equivalent uniform slab dead load for the shear

$g_{sh} = \beta \cdot g_s \cdot x_1 + g_s \cdot x_2$

= 0.5 x 4.50 x (4.0/2) + 4.50 x 2/2 = 9.0 kN/m′

Equivalent uniform slab live load for bending

$p_b = \alpha \cdot p_s \cdot x_1 + p_s \cdot x_2$

= 0.67 x 2.0 x (4.0/2) + 2.0 x 2/2 = 4.68 kN/m′

Equivalent uniform slab live load for the shear

$p_{sh} = \beta \cdot p_s \cdot x_1 + p_s \cdot x_2$

= 0.5 x 2.0 x (4.0/2) + 2.0 x 2/2 = 4.0 kN/m′

Step 2.3: Wall Load

$g_w = \gamma_w \times t_w + plastering\ weight = 14.8 \times 0.12 + 0.8 = 2.58\,kN/m^2$

Height of the wall = Floor height – Beam thickness= 3.5 – 0.60 = 2.90 m

$x = \frac{h}{\sqrt{3}} = \frac{2.9}{\sqrt{3}} = 1.67\ m$

$\frac{L}{2\,x} = \frac{4.0}{2 \times 1.67} = 1.19$

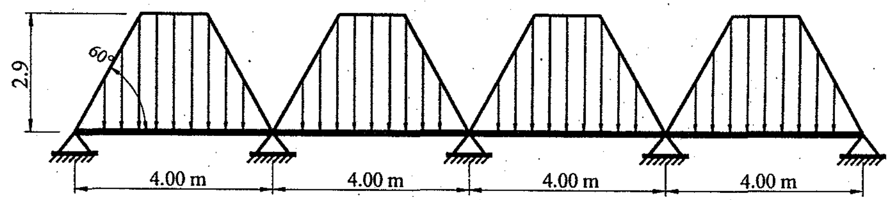
α=0.766 and β=0.581

Equivalent uniform wall load for bending (g_{wb})

$g_{wb} = \alpha \times g_w \times h_w = 0.766 \times 2.58 \times 2.9 = 5.73\,kN/m'$

Equivalent uniform wall load for shear (g_{ws})

$g_{ws} = \beta \times g_w \times h_w = 0.581 \times 2.58 \times 2.9 = 4.34\,kN/m'$

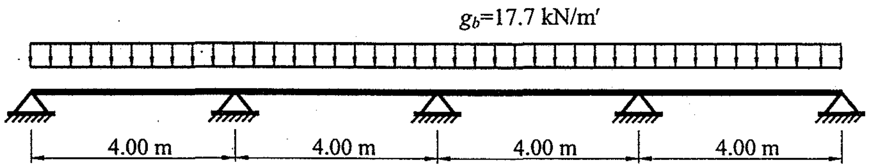


Wall load

Step 3: Total loads acting on Beam
Step 3.1: Equivalent Dead Load for bending

$g_b = o.w + slab\,load + wall\,load\ for\,bending$

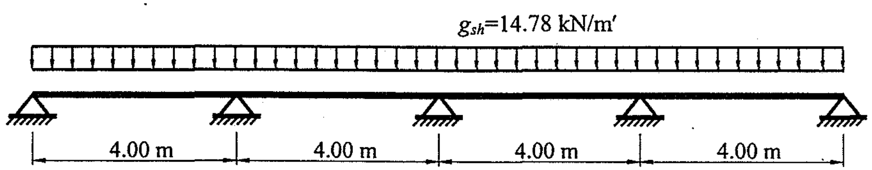
$g_b = 1.44 + 10.53 + 5.73 = 17.7\,kN/m'$



Step 3.2: Equivalent Dead Load for Shear

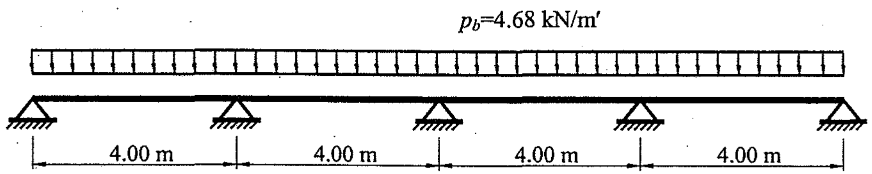
$g_{sh} = o.w + slab\ load\ + wall\ load\ \ for\ shear$

$g_{sh} = 1.44 + 9.0 + 4.34 = 14.78\,kN\ /m'$



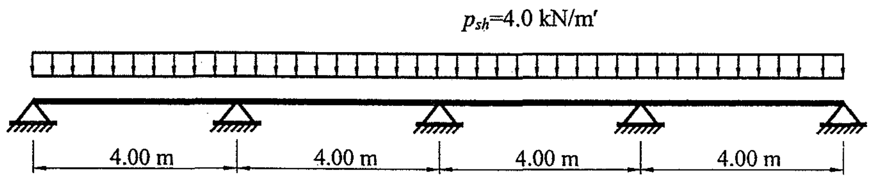
Step 3.3: Equivalent Live Load for bending

p_b = slab load =4.68 kN/m′



Step 3.4: Equivalent Live Load for shear

p_{sh} = slab load=4.0 kN/m′



Example 6.5

It is required to carry out an integrated design for the simple beam shown in Fig. EX. 6.5a. The beam is arranged every 5.0 m. The beam carries a uniformly distributed unfactored dead load (including its own weight) of a value of 20 kN/m and unfactored live load of a value of 15 kN/m. This uniform unfactored load could be considered for the bending and shear designs. The characteristics compressive strength of concrete f_{cu} = 30 N/mm^2. The yield strength of the longitudinal steel f_y = 360 N/mm^2 and for the stirrups = 240 N/mm^2.

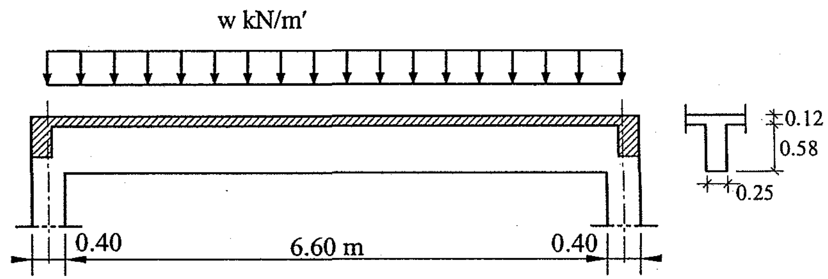


Fig. Ex-6.5a Simple beam

Solution
Step 1: Flexural design
Step 1.1: Calculation of maximum moments

Factored design load, w_u = 1.4 D.L. + 1.6 L.L.

= 1.4 x 20 + 1.6 x 15 = 52 kN/m

For obtaining the maximum moments, one needs to calculate the effective span.

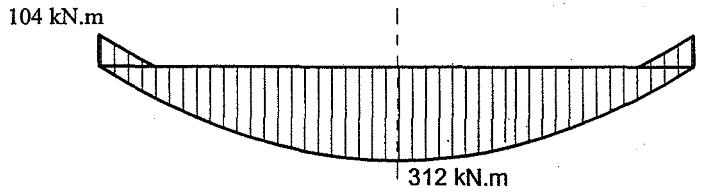
Assume a concrete cover to the C.L. of the steel of 50 mm

d = 700 – 50 = 650 mm

L_{eff} = The smallest of:

- distance C.L. to C.L. between the support = 7000 mm
- clear span + d = 6600 +650 =7250 mm
- 1.05 x clear span = 1.05x6600 =6930 mm

The bending moment diagram is shown in the figure below



Bending moment diagram

Maximum positive bending moment at mid-span

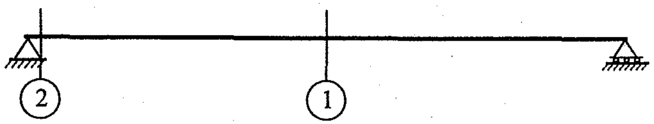
$$M_{u\max(+ve)} = 52.0 \times \frac{6.93^2}{8} = 312 \quad kN.m$$

Maximum negative moment at the support (due to partial prevention of beam rotation)

$$M_{u\max(-ve)} = 52.0 \times \frac{6.93^2}{24} = 104 \quad kN.m$$

Step 1.2: Design of Critical Sections

The critical sections are shown below

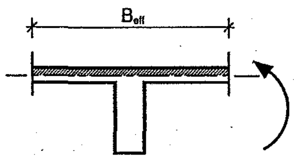


Section at midspan (Sec 1):- T-section

B_{eff} = The smallest of:

- $16t_s + b = 16 \times 120 + 250 = 2170\,mm$
- $\frac{L}{5} + b = \frac{7000}{5} + 250 = 1650\,mm$
- C.L. to C.L. between the beams on plan = 5000 mm

B_{eff} = 1650 mm



Assume that the neutral axis is inside the flange ($a < t_s$)

$$d = C_1 \sqrt{\frac{M_u}{f_{cu} B}}$$

$$650 = C_1 \sqrt{\frac{312 \times 10^6}{30 \times 1650}} \qquad C_1 = 8.19$$

The point is outside the C_1-J curve $\therefore \frac{c}{d} < (\frac{c}{d})_{min}$

Take $\frac{c}{d} = (\frac{c}{d})_{min} = 0.125 \quad C = 0.125 \times 650 = 81\,mm$ and $J = 0.826$

$a = 0.80 \times 81 = 65\,mm < t_s$ (as assumed)

$$A_s = \frac{M_u}{f_y \cdot j.d}$$

$$A_s = \frac{312 \times 10^6}{360 \times 0.826 \times 650} = 1614\ mm^2$$

$(A_s)_{min}$ = the smaller of:

$$\frac{0.225\sqrt{f_{cu}}}{f_y} \times b \times d = \frac{0.225\sqrt{30}}{360} \times 250 \times 650 = 556\,mm^2$$

1.3 A_s(required) =1.3x1614=2098mm^2
but not less than

$$\frac{0.15}{100} \times b \times d = \frac{0.15}{100} \times 250 \times 650 = 244\,mm^2$$

$(A_s)_{min.}$ = 556 mm^2 < $(A_s)_{required}$

Choose A_s=5Φ 22 $\qquad A_{s(chosen)}$ =1900 mm^2

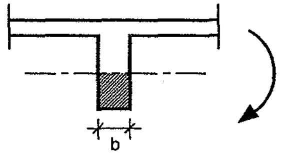
$$M_r = M_u \times \frac{A_{s(chosen)}}{A_{s(required)}} = 312 \times \frac{1900}{1614} = 367\,kN.m$$

Stirrup hangers: The minimum required area of steel used as stirrup hanger is 10 % of the main steel (Use 2ϕ12).

Section at support(Sec 2):- Rectangular-section

b=250 mm

$$d = C_1\sqrt{\frac{M_u}{f_{cu}\, b}}$$



$$650 = C_1\sqrt{\frac{104\times10^6}{30\times 250}} \qquad C_1 = 5.52$$

The point is outside the C_1-J curve $\therefore \frac{c}{d} < (\frac{c}{d})_{min}$

Take $\frac{c}{d} = (\frac{c}{d})_{min} = 0.125$

$$C = 0.125\times 650 = 81mm \qquad J = 0.826$$

$$A_s = \frac{M_u}{f_y . j . d} = \frac{104\times10^6}{360\times0.826\times650} = 538\ mm^2$$

$(A_s)_{min} = 556\ mm^2 > A_{s(req)}$ .....use A_{smin}

Choose A_s=3Φ16=600.0 mm^2

$$M_r = M_u \times \frac{A_{s(chosen)}}{A_{s(required)}} = 104\times\frac{600}{538} = 116.0\ kN.m$$

Step 1.3: Calculation of the development Length

$$L_d = \{\frac{\alpha.\beta.\eta.(\frac{f_y}{\gamma_s})}{4 f_{bu}}\}.\phi$$

$$f_{bu} = 0.30\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.30\sqrt{\frac{30}{1.5}} = 1.34\ N/mm^2$$

η_{bottom}=1.0 $\qquad$ η_{top}=1.3

For bars in tension:-

α=1.0 (Straight Bars) and β=0.75 (deformed bars)

$$L_{d(bottom)} = \{\frac{1.0\times0.75\times1.0\times(360/1.15)}{4\times1.34}\}.\phi = 44\phi$$ (962 mm) for Φ 22

$$L_{d(top)} = \{\frac{1.0\times0.75\times1.3\times(360/1.15)}{4\times1.34}\}.\phi = 57\phi$$ (912 mm) for Φ 16

For bars in compression:-

α=1.0 (Straight Bars) and β=0.50 (deformed bars)

$$L_{d(bottom)} = \{\frac{1.0\times0.50\times1.0\times(360/1.15)}{4\times1.34}\}.\phi = 29\phi$$ (642 mm) for Φ 22

$$L_{d(top)} = \{\frac{1.0\times0.50\times1.3\times(360/1.15)}{4\times1.34}\}.\phi = 38\phi$$ (456 mm) for Φ 12

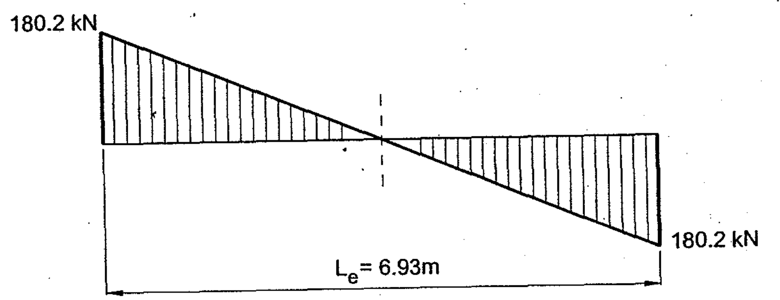
Or we can directly use the coefficients from Table (5.3) with f_{cu} = 30 N/mm²

For Tension: $L_{d(bottom)}$=50Φ, $L_{d(top)}$=65Φ

For Compression: $L_{d(bottom)}$=40Φ, $L_{d(top)}$=52Φ

Step 2: Shear Design

The shear force diagram is shown in the figure below



Shear force diagram

For the case of uniform load, the critical section is at d/2 from the face of support.

$$Q_u = \text{Reaction at the support (R)} - w_u(d/2 + \textit{half column width})$$

$$R = 52\times\frac{6.93}{2} = 180.2\,kN$$

$$Q_u = 180.2 - 52\times(\frac{0.65}{2} + 0.20) = 152.88\ kN$$

$$q_u = \frac{Q_u}{b\times d} = \frac{152.88\times10^3}{250\times650} = 0.94\ N/mm^2$$

$$q_{u(\max)} = 0.7\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.7\sqrt{\frac{30}{1.5}} = 3.13 \;\; N/mm^2 \; < 4.0 N/mm^2$$

$$q_{u(\max)} = 3.13 N/mm^2$$

$q_u \leq q_{u(\max)}$ $\therefore$ Concrete dimensions of the section are adequate for shear.

$$q_{cu} = 0.24\sqrt{\frac{f_{cu}}{\gamma_c}}$$

$$q_{cu} = 0.24\sqrt{\frac{30}{1.5}} = 1.07 \;\; N/mm^2$$

$q_u < q_{cu}$ Use minimum stirrups

According to the ECP 203, $\mu_{\min} = \frac{0.4}{f_y} = \frac{0.4}{240} = 0.00167$ (not less than 0.0015)

$A_{st(\min.)} = \mu_{\min}.b.s$

Take s=200 mm

$A_{st(\min.)} = 0.00167 \times 250 \times 200 = 84.0 \; mm^2$ (for two branches)

Area of one branch = 42 mm^2 →→Use Stirrups 5ϕ8/m

Curtailment check for bottom straight bars

At the cut-off locations, ECP 203 requires that the spacing between the stirrups (s) should be greater than $d/8\beta$ where $\beta = \frac{A_s(cut)}{A_s(total)} = \frac{2\Phi22}{5\Phi22} = 0.4$

$$S \leq \frac{650}{0.8 \times 0.4} = 203 \; mm$$

Additional stirrups is added (ϕ8 @200 mm) in a distance of 0.75 d. Thus the final spacing is decreased to 100 mm for the distance of 0.75 d.

Step 3: Reinforcement detailing

Figure EX. 6.5b shows the curtailment of bars and the moment of resistance diagram as well as the original bending moment diagram for the case of straight longitudinal bars. The rules mentioned in Chapter 5 were followed for bar curtailment.

Complete reinforcement detailing for the beam is shown in Figs EX. 6.5c and EX. 6.5d for the case of straight and bent bars, respectively.

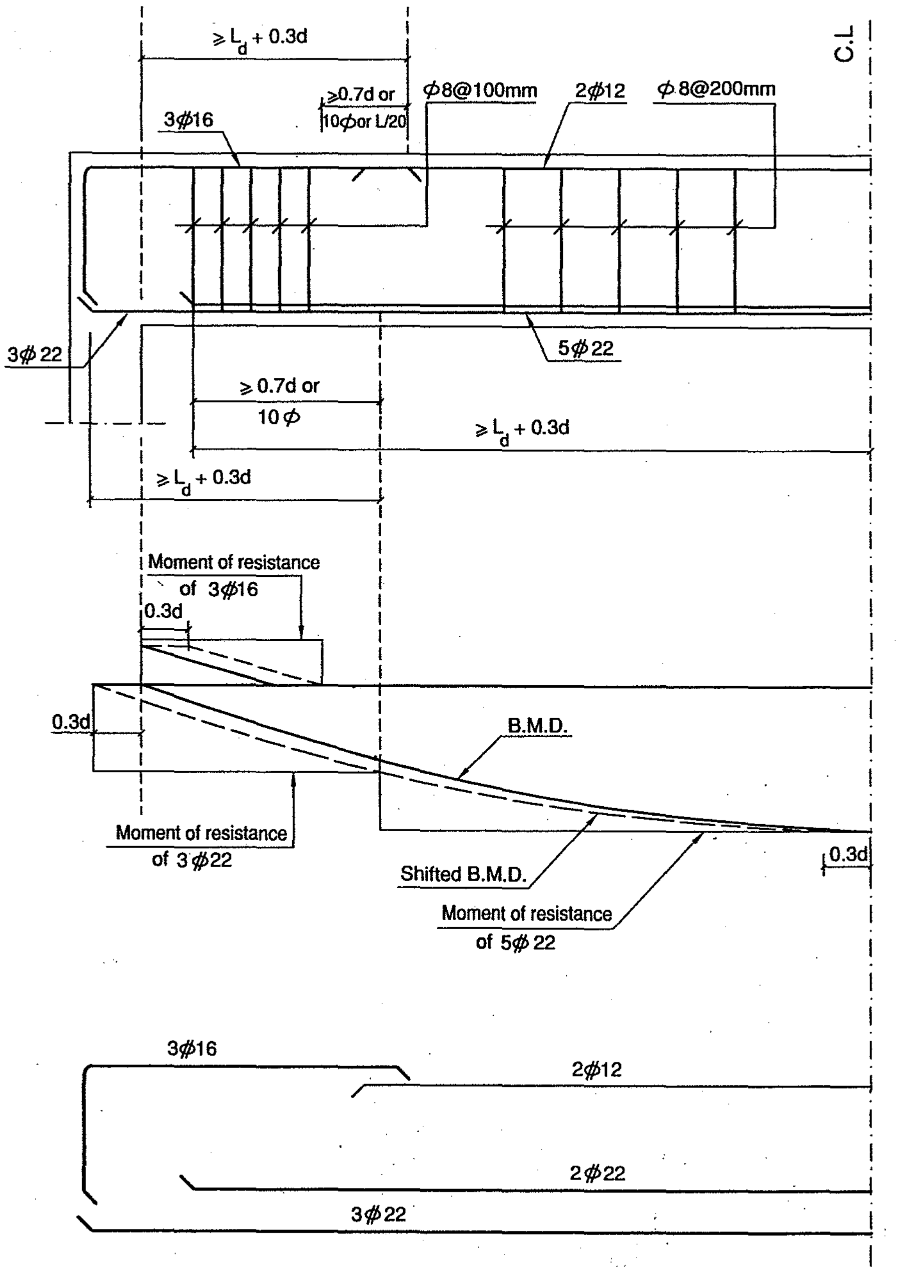


Fig. EX. 6.5b Curtailment of bars for beam B1

(Case of straight bars)

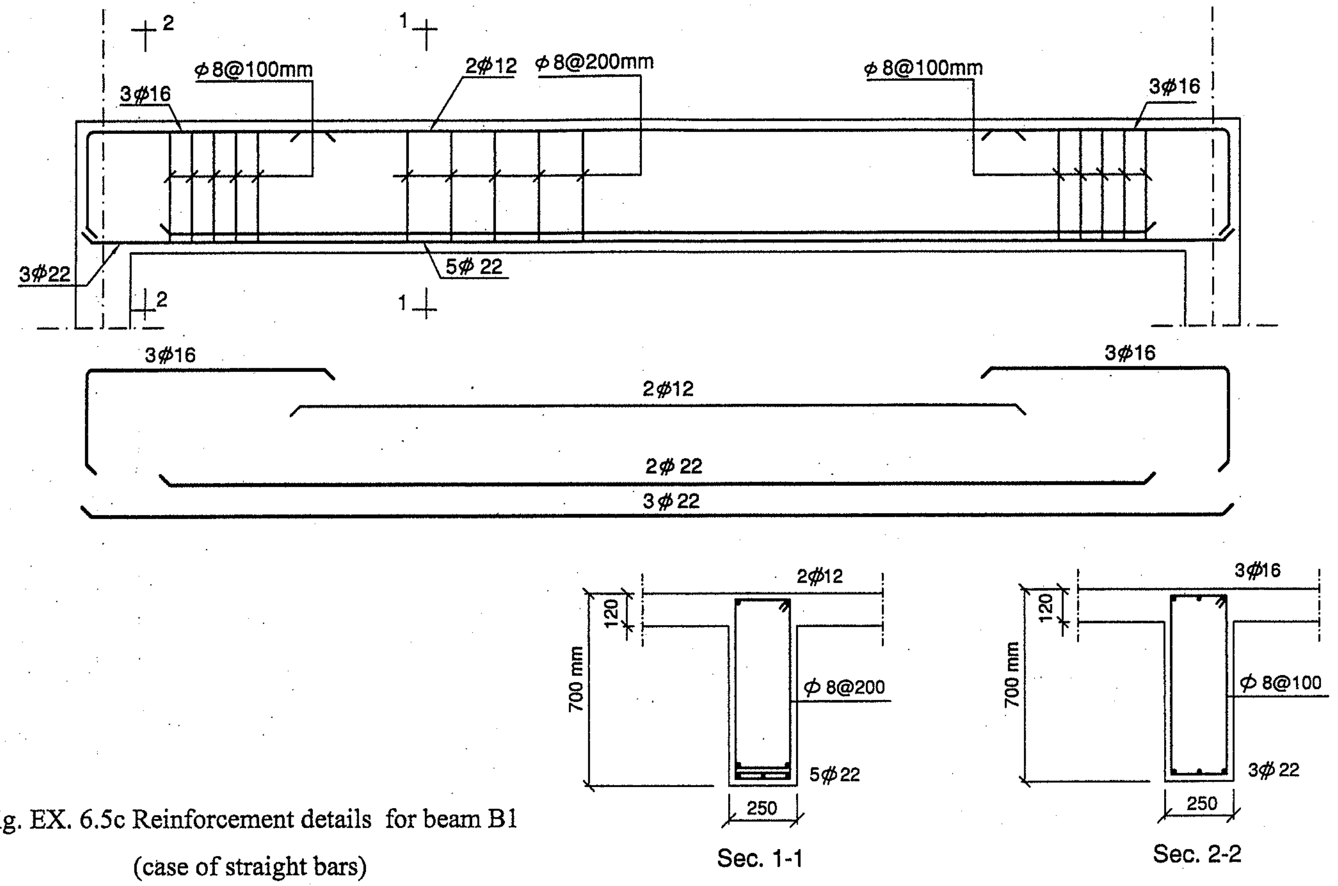


Fig. EX. 6.5c Reinforcement details for beam B1
(case of straight bars)

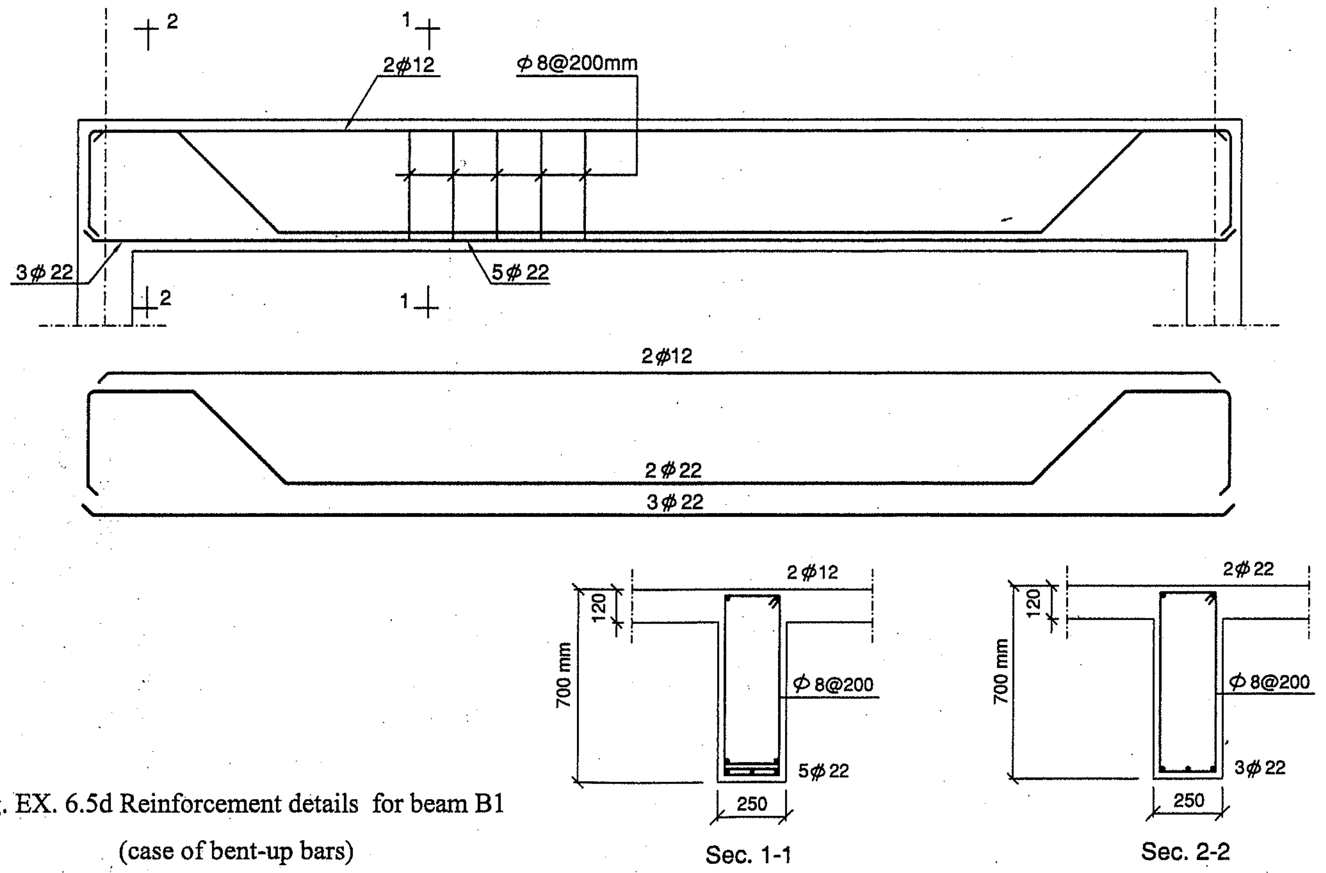
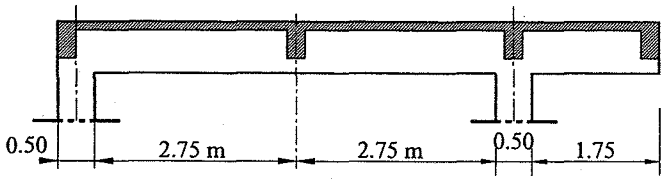


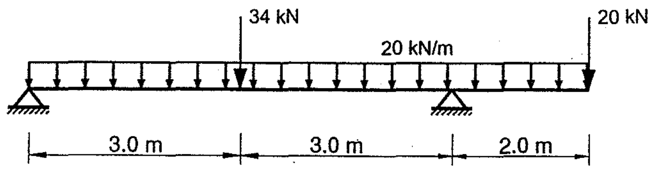
Fig. EX. 6.5d Reinforcement details for beam B1
(case of bent-up bars)

Example 6.6

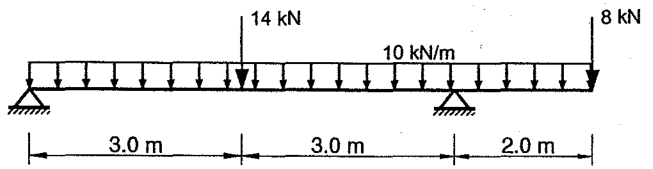
It is required to carry out an integrated design for the simple beam with cantilever shown in Fig. (EX. 6.6a). The beam is arranged every 5.0 m. The unfactored dead and live loads acting on the beam are also given. The cube compressive strength of concrete f_{cu} = 30 N/mm^2 and the yield strength of steel f_y = 360 N/mm^2.



a) Simple beam with cantilever



b) Unfactored dead loads



c) Unfactored live loads

Fig. EX. 6.6a Simple beam with cantilever

Solution

Step 1: Flexural design

Step 1.1: Calculation of maximum moments

Maximum +ve moment at mid-span

Since the live loads is less than 0.75 D.L., the ultimate factor of 1.5 may be used. For obtaining the maximum moment at mid-span, the full live load is

applied between the support, while only 0.9 D.L. (minimum dead load required by the code) without live loads are applied at the cantilever part.

The calculations of the loads are carried as follows:

Between the supports:

$w_u = 1.5 \times (D.L. + L.L)$

$w_u = 1.5 \times (20.0 + 10.0) = 45.0\ kN/m$

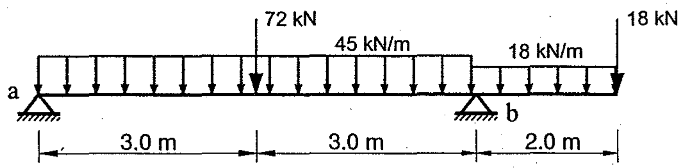
$P_u = 1.5 \times (34.0 + 14.0) = 72.0\ kN$

Cantilever loads:

$w_u = 0.90 \times D.L.$

$w_u = 0.90 \times 20.0 = 18.0\ \ kN/m$

$P_u = 0.90 \times 20.0 = 18.0\ kN$



Load case 1

$$M_{u(+ve)} = \frac{w \times L^2}{8} + \frac{P \times L}{4} - M_{b(-ve)} = \left(\frac{45 \times 6^2}{8} + \frac{72 \times 6}{4}\right) - \frac{72}{2} = 274.5\,\text{kN.m}$$

Maximum -ve moment at the cantilever

For obtaining the maximum negative moment at the cantilever

Between the supports:

$w_u = 0.90 \times D.L$

$w_u = 0.90 \times 20.0 = 18.0\ \ kN\ /m$

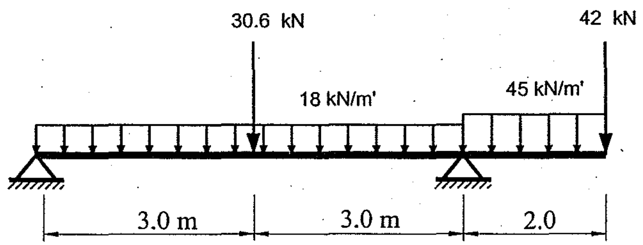
$P_u = 0.90 \times 34.0 = 30.6\ kN$

Cantilever loads:

$w_u = 1.5 \times (D.L. + L.L)$

$w_u = 1.5 \times (20.0 + 10.0) = 45.0\,kN/m$

$P_u = 1.5 \times (20.0 + 8.0) = 42.0\ kN$



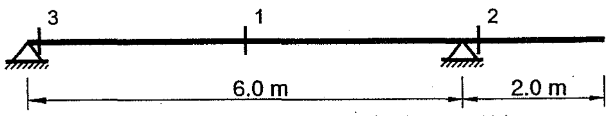
Load case 2

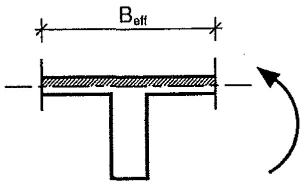
$$Mu(-ve)@cantilever = 42\times2+\frac{45\times2^2}{2}=174.0\,kN.m$$

$$Mu(+ve)@mid\,span=\frac{30.6\times6}{4}+\frac{18\times6^2}{8}-\frac{174}{2}=39.9\,kN.m$$

Step 1.2: Design of critical sections

The critical sections are shown in the figure below





Section No. 1 **T-section**

M_u = 274.50 kN.m

B_{eff} = The smallest of:

- $16t_s+b=16\times120+250=2170\,mm$
- $\frac{L_2}{5}+b=\frac{0.8\times6000}{5}+250=1210\,mm$
- C.L. to C.L. between the beams on plan = 5000 mm

Note: The factor 0.8 is used because the span is continuous from one end.

B_{eff}= 1210 mm

$$Assume\ beam\ thickness\,(t)=\frac{span}{10-12}\cong700\ mm$$

$$d=t-cover=700-50=650\ mm$$

Assume that the N.A. is inside the flange ($a < t_s$)

$$d=C_1\sqrt{\frac{M_u}{f_{cu}B}}$$

$$650=C_1\sqrt{\frac{274.5\times10^6}{30\times1210}} \qquad C_1=7.47\ \&\ J=0.826$$

The point is outside the curve $\therefore \frac{c}{d}<(\frac{c}{d})_{min}$

Take $\frac{c}{d}=(\frac{c}{d})_{min}=0.125$

$$C=0.125\times650=81.25\,mm$$

$a=0.80\times81.25=65\,mm$ < t_s=120 mm (as assumed)

$$A_s=\frac{M_u}{f_y.j\,d}=\frac{274.50\times10^6}{360\times0.826\times650}=1420\,mm^2$$

$(A_s)_{min}$ = the smaller of:

$$\frac{0.225\sqrt{f_{cu}}}{f_y}\times b\times d=\frac{0.225\sqrt{30}}{360}\times250\times650=556\,mm^2$$

1.3 A_s(required) =1.3x1420=1846 mm^2

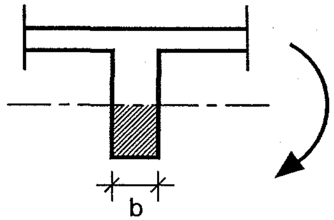
$(A_s)_{min.}$ = 556 mm^2 < $(A_s)_{required}$

Choose A_s=6Φ18 $\qquad A_{s(chosen)}$ = 1527 mm^2

$$M_r=M_u\times\frac{A_{s(chosen)}}{A_{s(required)}}=274.5\times\frac{1524}{1420}=294.6\ kN.m$$

Section 2: **Rectangular section**

b= 250 mm



M_u=174.0 kN.m

$$d=C_1\sqrt{\frac{M_u}{f_{cu}\ b}}$$

$$650=C_1\sqrt{\frac{174.0\times10^6}{30\times250}}$$

$C_1=4.26$ & $J=0.81$

$$A_s=\frac{M_u}{f_y.j\,d}=\frac{174.00\times10^6}{360\times0.81\times650}=918\,mm^2$$

Choose 4Φ18 $\qquad A_{s(chosen)}$ = 1016 mm^2

$$M_r=M_u\times\frac{A_{s(chosen)}}{A_{s(required)}}=174.0\times\frac{1016}{918}=192.6\ kN.m$$

Section No. 3: **Rectangular section**

b= 250 mm

At the simple support, the Code requires a design for the moment that develops due to partial prevention of the beam rotation. For a beam that carries a uniformly distributed load kN/m, the Code gives the moment at the simple support as $w \times L^2/24$. This value equals to half the fixed end moment. Extending this concept to our case, one could assume that the moment that shall be developed is equal to half the value of the fixed end moment developed for a beam that carries a uniformly distributed load plus a concentrated load at mid-span.

Fixed end moment at simply supported span

$$= \frac{w \times L^2}{12} + \frac{P \times L}{8} = \frac{45 \times 6^2}{12} + \frac{72 \times 6}{8} = 189\, kN.m$$

$$M \text{ at section } 3 = 0.5 \times 189 = 94.50\ kN.m$$

$$d = C_1 \sqrt{\frac{M_u}{f_{cu}\, b}}$$

$$650 = C_1 \sqrt{\frac{94.5 \times 10^6}{30 \times 250}} \qquad C_1 = 5.79 \quad \& \text{ J=0.826}$$

$$A_s = \frac{M_u}{f_y . j\, d} = \frac{94.5 \times 10^6}{360 \times 0.826 \times 650} = 489\, mm^2$$

$(A_s)_{\min}$ = the smaller of:

$$\frac{0.225\sqrt{f_{cu}}}{f_y} \times b \times d = \frac{0.225\sqrt{30}}{360} \times 250 \times 650 = 556\, mm^2$$

1.3 A_s(required) =1.3x489=635.7 mm^2

A_{smin}=556 mm^2

Choose A_s=3Φ16 (or 2Φ18+2Φ12 in case of bent bars)

Step 1.3: Calculation of the development length

$$L_d = \{\frac{\alpha.\beta.\eta.(\frac{f_y}{\gamma_s})}{4 f_{bu}}\}.\phi$$

$$f_{bu} = 0.30\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.30\sqrt{\frac{30}{1.5}} = 1.34\ N/mm^2$$

η_{bottom}=1.0 $\qquad$ η_{top}=1.3

For bars in tension:-

α=1.0 (Straight Bars) and β=0.75 (deformed bars)

$$L_{d(bottom)} = \{\frac{1.0 \times 0.75 \times 1.0 \times (360/1.15)}{4 \times 1.34}\}.\Phi = 44\Phi$$

$$L_{d(top)} = \{\frac{1.0 \times 0.75 \times 1.3 \times (360/1.15)}{4 \times 1.34}\}.\Phi = 57\Phi$$

For bars in compression:-

α=1.0 (Straight Bars) and β=0.50 (deformed bars)

$$L_{d(bottom)} = \{\frac{1.0 \times 0.50 \times 1.0 \times (360/1.15)}{4 \times 1.34}\}.\Phi = 29\Phi$$

$$L_{d(top)} = \{\frac{1.0 \times 0.50 \times 1.3 \times (360/1.15)}{4 \times 1.34}\}.\Phi = 38\Phi$$

Or we can directly use the coefficients from Table (5.3) with f_{cu}=30 N/mm^2

For Tension: $L_{d(bottom)}$=50Φ, $L_{d(top)}$=65Φ

For Compression: $L_{d(bottom)}$=40Φ, $L_{d(top)}$=52Φ

Step 2: Shear design

For calculating the design shear forces, the total dead and live loads have to be placed on the beam as shown in the figure below

$$w_u = 1.5 \times (20.0 + 10.0) = 45.0\ kN/m$$

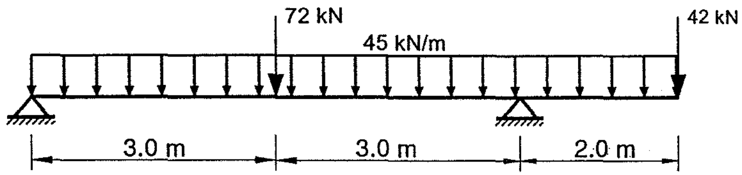
$$P_{u1} = 1.5 \times (34.0 + 14.0) = 72.0\ kN$$

$$P_{u2} = 1.5 \times (20.0 + 8.0) = 42.0\ kN$$

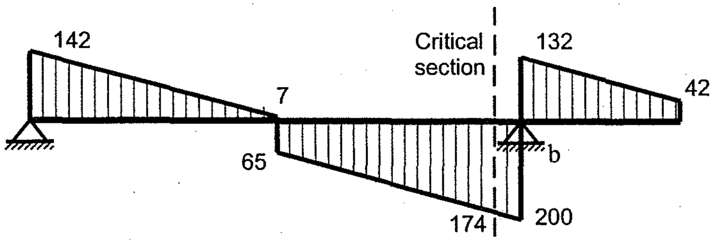
Maximum shear force is at d/2 from the left of the intermediate support.

$$R_b = \frac{72\times3+42\times8+45\times8\times4}{6} = 332\ kN$$

$$Q_b = 332-42-45\times2 = 200\ kN$$



Load case 3



Shear force diagram

$$Q_u = Q_b - w_u\,(d/2 + \text{half the column width})$$

$$Q_u = 200.0-45.0\times(\frac{0.65}{2}+0.25)=174kN$$

$$q_u = \frac{Q_u}{b\times d} = \frac{174.00\times10^3}{250\times650} = 1.07\,N/mm^2$$

$$q_{u(\max)} = 0.7\sqrt{\frac{f_{cu}}{\gamma_c}}$$

$$q_{u(\max)} = 0.7\sqrt{\frac{30}{1.5}} = 3.13\ \ N/mm^2 < 4.0\,N/mm^2$$

$$q_{u(\max)} = 3.13\,N/mm^2$$

$q_u \le q_{u(\max)}$ the concrete dimensions of the section are adequate for shear.

$$q_{cu} = 0.24\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.24\sqrt{\frac{30}{1.5}} = 1.07\,N/mm^2$$

Since $q_u = q_{cu}$, one has to use the minimum stirrups

According to the ECP 203, $\mu_{\min} = \frac{0.4}{f_y} = \frac{0.4}{240} = 0.00167$ (not less than 0.0015)

$A_{st(\min.)} = \mu_{\min}.b.s$

take s=200 mm

$A_{st(\min.)} = 0.00167\times250\times200 = 84.0\ mm^2$ (for two branches)

Area of one branch = 42 mm^2

Use 5 ϕ 8/m′

Curtailment Check for bottom straight bars

At the cut-off locations, ECP 203 requires that the spacing between the stirrups (s) should be greater than $d/8\beta$ where $\beta = \frac{A_s(cut)}{A_s(total)} = \frac{2\Phi18}{6\Phi18} = 0.33$

$$s \le \frac{650}{8\times0.333} = 244$$

Additional stirrups is added (ϕ8 @200 mm) in a distance of 0.75 d. Thus the final spacing is decreased to 100 mm for the distance of 0.75 d.

Step 3: Reinforcement detailing

Fig. EX. 6.6b and EX. 6.6c shows the moment of resistance diagram as well as the original bending moment diagram for the case of straight and bent-up longitudinal bars receptively. The rules mentioned in chapter 5 were followed for bar curtailment.

Complete reinforcement detailing for the beam is shown in Figs EX. 6.6d and EX. 6.6e for the case of straight and bent bars, respectively.

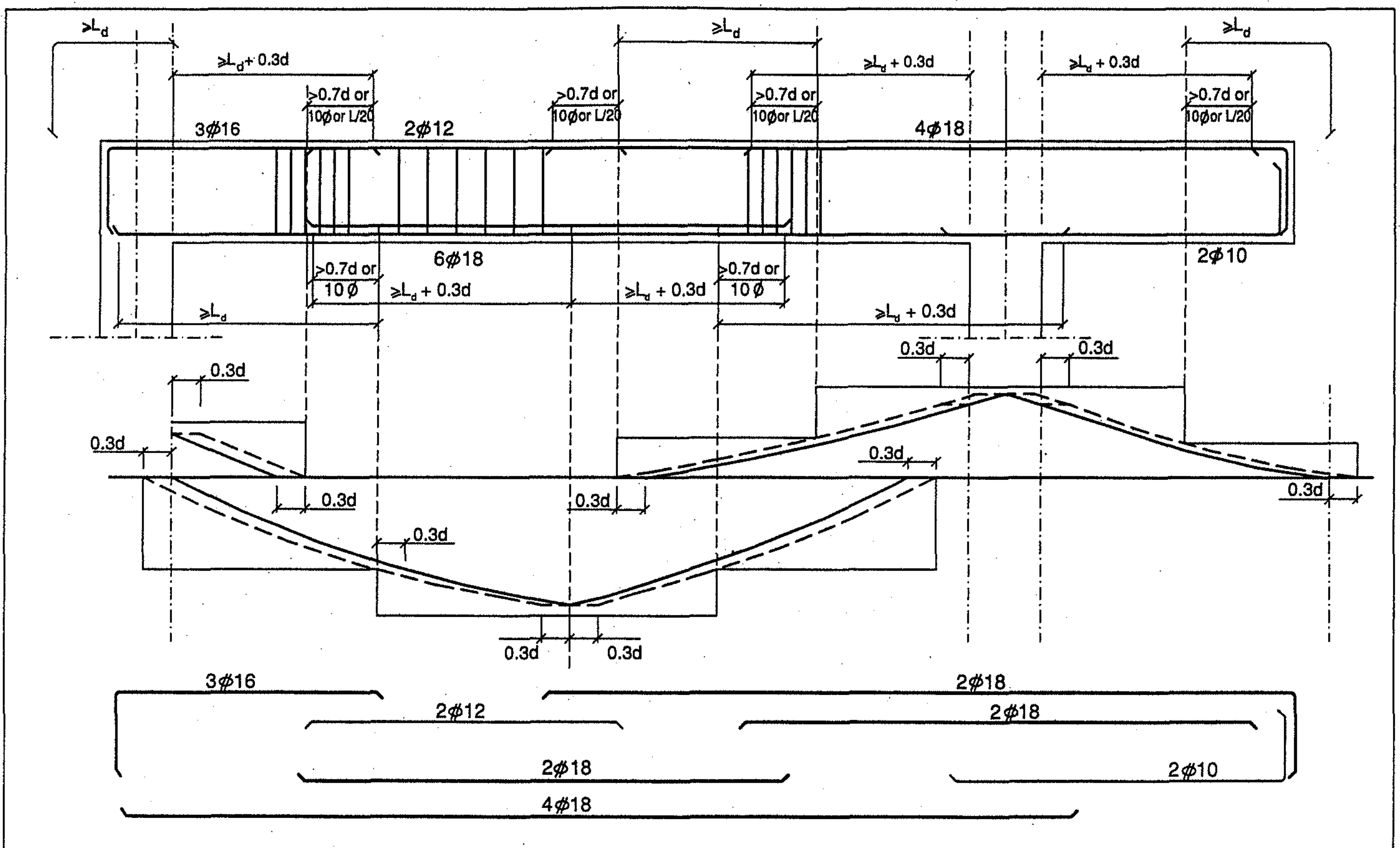


Fig. EX. 6.6b Curtailment of bars for beam B2 (straight bars)

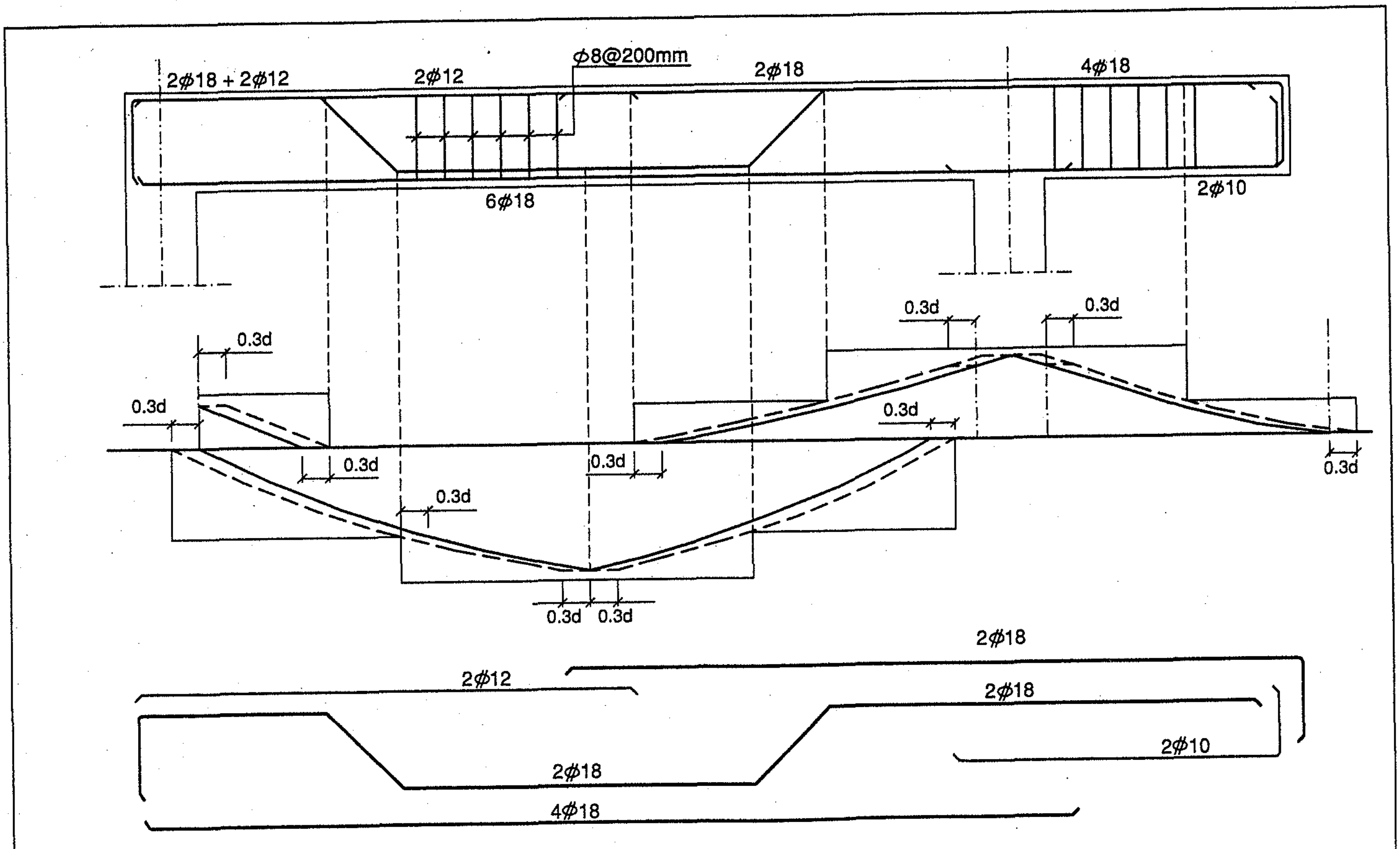


Fig. EX. 6.6c Curtailment of bars for beam B2 (bent-up bars)

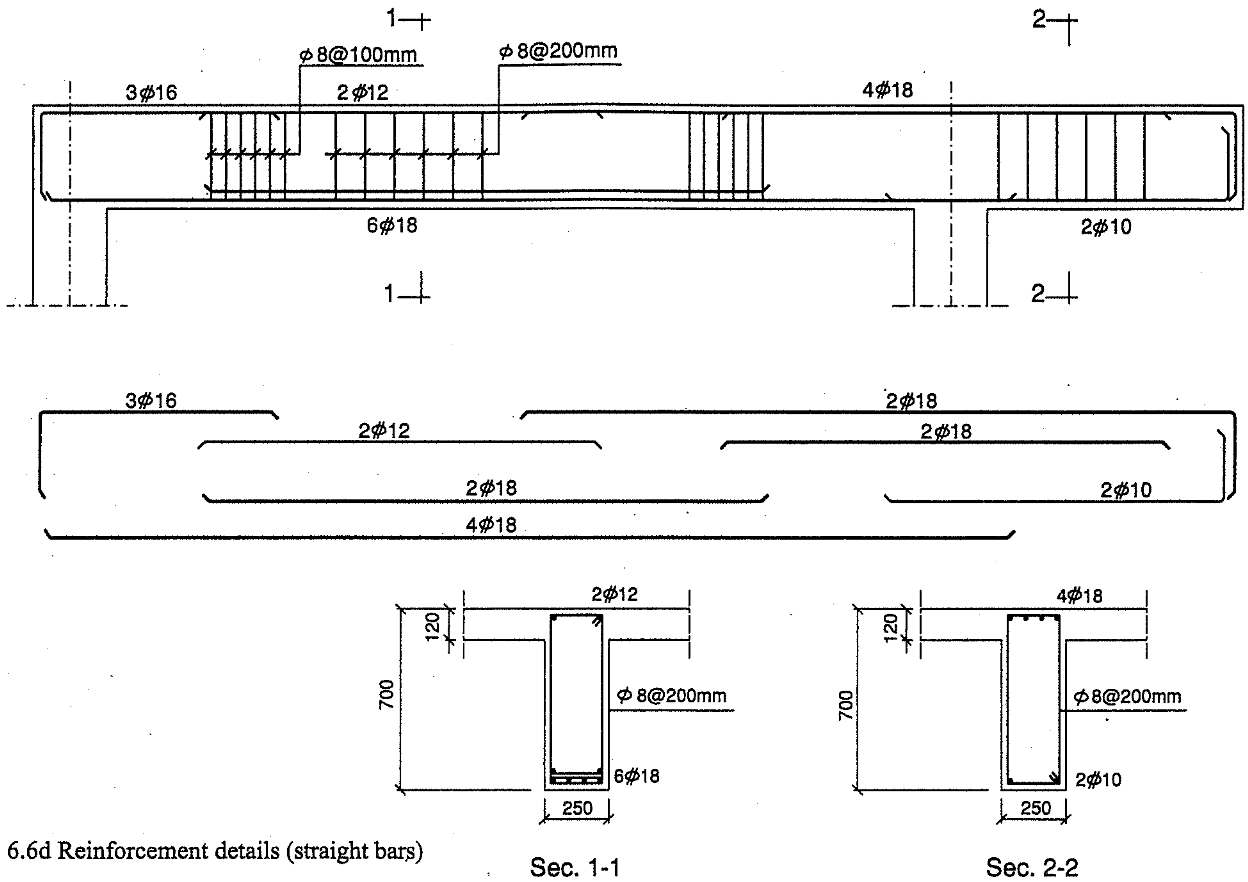


Fig. Ex. 6.6d Reinforcement details (straight bars)

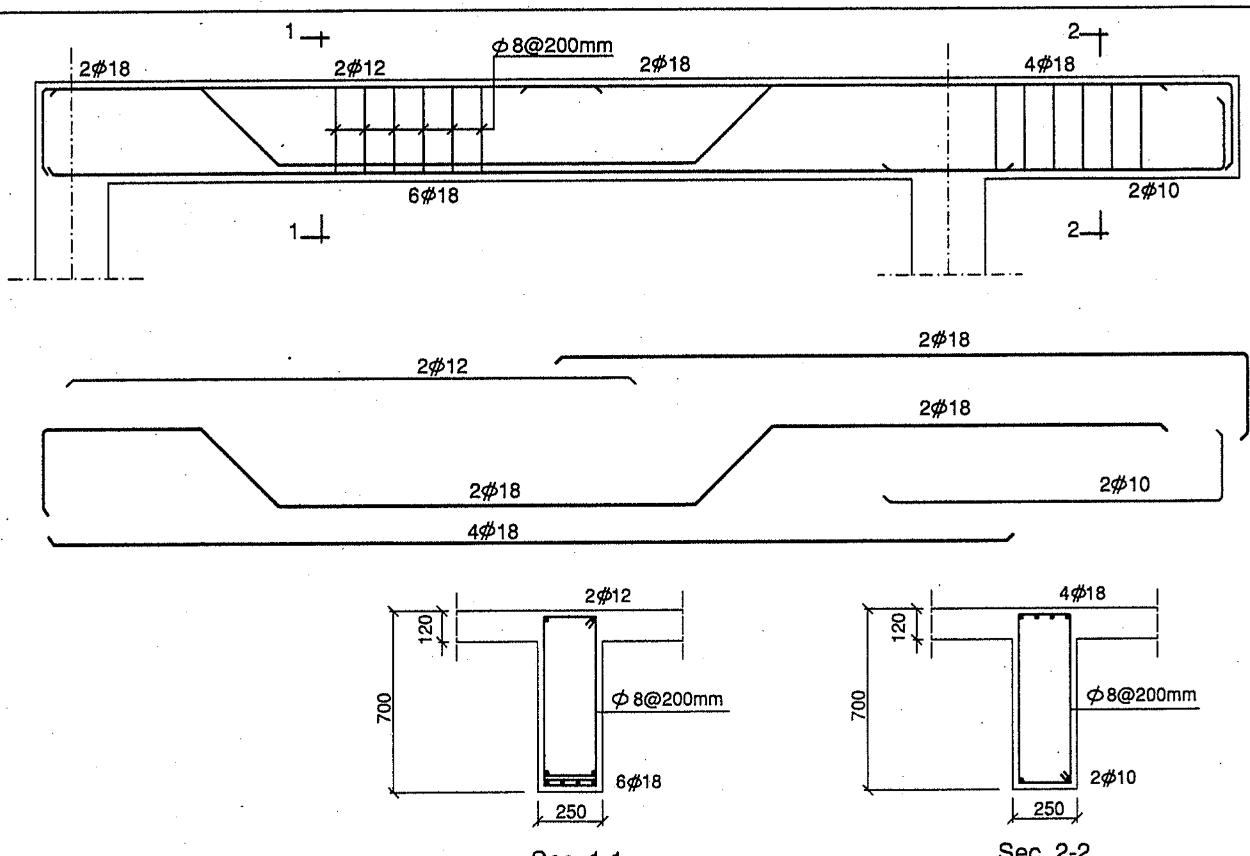


Fig. EX. 6.6e Reinforcement details (bent-up bars)

Example 6.7

It is required to design the continuous beam shown in Fig. EX.6.7a. The characteristics compressive strength of concrete f_{cu} = 30 N/mm^2. The yield strength of the longitudinal steel f_y = 360 N/mm^2 and for the stirrups = 240N/mm^2. The applied unfactored dead and live loads on all spans are 40 kN/m and 15 kN/m, respectively. These loads can be used for designing the beam for bending as well as for shear. Assume also that the spacing between the beams on plan is 4.0 m.

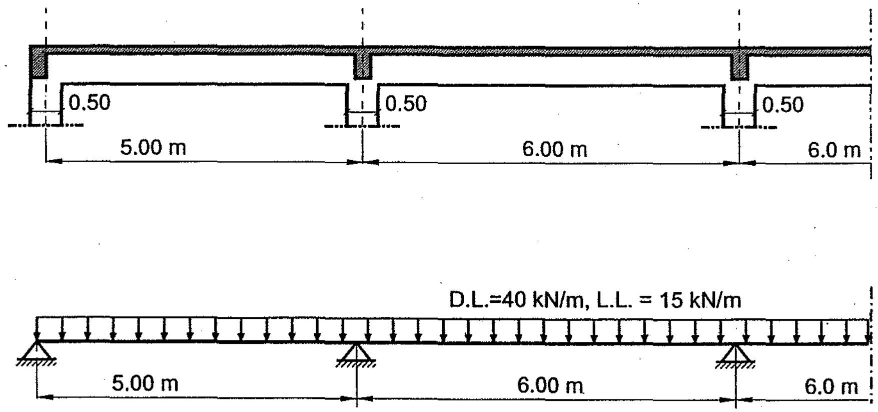


Fig. Ex 6.7a Continuous beam

Solution

Step 1: Flexural Design

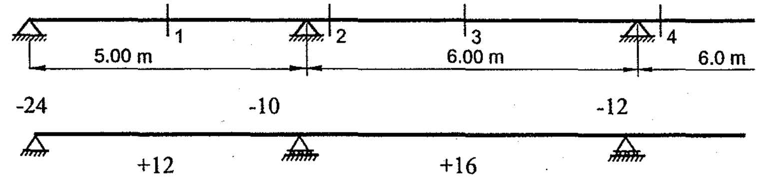
Step 1.1 Calculation of maximum moments

$$w_u = 1.5(w_{D.L.} + w_{L.L.})$$

$$w_u = 1.5(40+15) = 82.5\,kN/m$$

Since spans variations in the continuous beam do not exceed 20 %, the bending moment and shear forces may be estimated using the coefficients given by the Egyptian Code. The critical sections for flexural design are shown in figure below.

Note that the actual span was used to calculate the positive bending and the average length was used to calculate the negative bending over the support.



Sec. 1 $M_u(+ve) = w_u \times \frac{l_1^2}{12} = 82.5 \times \frac{5^2}{12} = 171.9\ kN.m$

Sec. 2 $M_u(-ve) = w_u \times \frac{((l_1+l_2)/2)^2}{10} = 82.5 \times \frac{((5.0+6.0)/2)^2}{10} = 249.7\ kN.m$

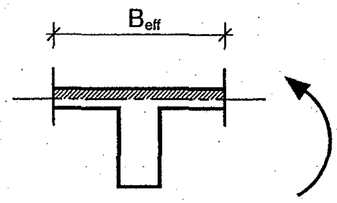
Sec. 3 $M_u(+ve) = w_u \times \frac{l_2^2}{16} = 82.5 \times \frac{6^2}{16} = 185.6\ kN.m$

Sec. 4 $M_u(-ve) = w_u \times \frac{((l_2+l_3)/2)^2}{12} = 82.5 \times \frac{((6.0+6.0)/2)^2}{12} = 247.5\ kN.m$

Step 1.2: Design of critical sections

Assume beam thickness $(t) = \frac{span}{10-12} \cong 600\ mm$

$d = t - \text{cover} = 600 - 50 = 550\ mm$



Section No. 1 T-sec

B_{eff} = the smallest of:

$16t_s + b = 16 \times 120 + 250 = 2170\,mm$

$\frac{L_2}{5} + b = \frac{0.8 \times 5000}{5} + 250 = 1050\,mm$

CL. to C.L. between the beams on plan = 4000 mm

B_{eff} = 1050 mm

$$d = C_1\sqrt{\frac{M_u}{f_{cu}B}}$$

$550 = C_1\sqrt{\frac{171.88 \times 10^6}{30 \times 1050}}$ $\qquad C_1 = 7.44$ & J=0.826

The point is outside the C1-J curve $\therefore$ Use $\frac{c}{d} = (\frac{c}{d})_{min} = 0.125$

$C = 0.125 \times 550 = 68.75\, mm$

$a = 0.80 \times 68.75 = 55\, mm <$ t_s=120 mm

$$A_s = \frac{M_u}{f_y . j . d} = \frac{171.9 \times 10^6}{360 \times 0.826 \times 550} = 1051\ mm^2$$

$(A_s)_{\min}$ = the smaller of:

$$\frac{0.225\sqrt{f_{cu}}}{f_y} \times b \times d = \frac{0.225\sqrt{30}}{360} \times 250 \times 550 = 471\, mm^2$$

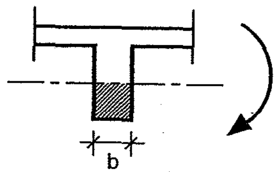
1.3 A_s(required) =1.3x1051=1366mm^2

$(A_s)_{min.}$ = 471 mm^2 < $(A_s)_{required}$

Choose A_s=5Φ18 $A_{s\,(chosen)} = 1272\, mm^2$

<u>Section 2</u> Rectangular Section

b= 250 mm d = 550 mm M_u=249.56 kN.m



$$d = C_1\sqrt{\frac{M_u}{f_{cu}\ b}}$$

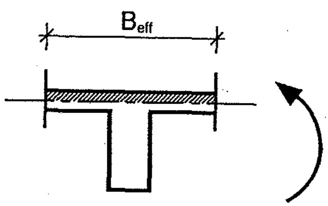
$$550 = C_1\sqrt{\frac{249.56 \times 10^6}{30 \times 250}}$$ $C_1 = 3.0$ & J=0.74

$$A_s = \frac{M_u}{f_y . j . d} = \frac{249.6 \times 10^6}{360 \times 0.74 \times 550} = 1699.0\ mm^2 > A_{s\ minimum}$$

Choose A_s=3Φ18 + 3 Φ 20 $A_{s(chosen)} = 1704\, mm^2$

<u>Section 3</u> Section T-Section

M_u=185.63 kN.m



B_{eff} = smallest of:

$$16t_s + b = 16 \times 120 + 250 = 2170\, mm$$

$$\frac{L_2}{5} + b = \frac{0.7 \times 6000}{5} + 250 = 1090\, mm$$

The factor 0.7 is used because the beam is continuous from both ends.

B_{eff}=1090 mm

$$d = C_1\sqrt{\frac{M_u}{f_{cu} B}}$$

$$550 = C_1\sqrt{\frac{185.63 \times 10^6}{30 \times 1090}}$$ $\therefore C_1 = 7.3$ & J=0.826

The point is outside the C1-J curve ∴ Use $\frac{c}{d} = (\frac{c}{d})_{\min} = 0.125$

$C = 0.125 \times 550 = 68.75\, mm$

$a = 0.80 \times 68.75 = 55\, mm <$ t_s=120 mm

$$A_s = \frac{M_u}{f_y . j . d} = \frac{185.6 \times 10^6}{360 \times 0.826 \times 550} = 1135.0\ mm^2$$

Choose A_s=3Φ20 + 2Φ18 $\rightarrow A_{s(chosen)} = 1451\, mm^2$ (bent bars Fig EX6.7c)

Choose A_s=5Φ18 $\rightarrow A_{s\,(chosen)} = 1272\, mm^2$ (straight bars Fig EX 6.7b)

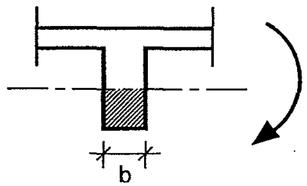
<u>Section No. 4</u> Rectangular Section

b= 250 mm d = 550 mm M_u=247.50 kN.m

$$550 = C_1\sqrt{\frac{247.50 \times 10^6}{30 \times 250}}$$ $C_1 = 3.0$ & J=0.74

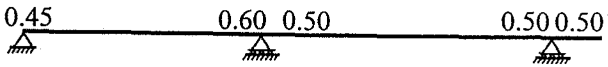
$$A_s = \frac{M_u}{f_y . j . d} = \frac{247.50 \times 10^6}{360 \times 0.74 \times 550} = 1685\, mm^2$$

Choose A_s=3Φ18 + 3Φ20 = 1704 mm^2



Step 2: Check of shear

Shear factors



Maximum Shear force at 1st span $= 0.60 \times w_u \times l_1$

$$= 0.60 \times 82.50 \times 5.0 = 247.5\, KN$$

Maximum Shear force at 2nd span $= 0.50 \times w_u \times l_2$

$$= 0.50 \times 82.50 \times 6.0 = 247.5\, KN$$

For the case of uniform load, the critical section is located at d/2 from the support

$$Q_u = 247.5 - 82.5(0.55/2 + 0.25) = 204.2\, KN$$

$$q_u = \frac{Q_u}{b \times d} = \frac{204.2 \times 10^3}{250 \times 550} = 1.49\ N/mm^2$$

$$q_{u(\max)} = 0.7\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.7\sqrt{\frac{30}{1.5}} = 3.13\ \ N/mm^2 \ < 4.0\, N/mm^2$$

$$q_{u(\max)} = 3.13\, N/mm^2$$

$q_u \le q_{u(\max)}$ the concrete dimensions of the section are adequate for shear.

$$q_{cu} = 0.24\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.24\sqrt{\frac{30}{1.5}} = 1.07\, N/mm^2$$

$q_u > q_{cu}$ web reinforcement is required

$$q_{su} = q_u - 0.5\, q_{cu} = 1.49 - 0.50 \times 1.07 = 0.955\, N/mm^2$$

$$\mu = \frac{A_{st}}{b.s} = \frac{q_{su}}{f_y/\gamma_s}$$

$$\mu = \frac{2 \times A_{st}}{250 \times s} = \frac{0.955}{240/1.15} \qquad A_s = 0.572\ S$$

For $\phi = 10$ mm $A_S = 78.5$ mm^2 s=137 mm

Using ϕ 10 @ 125 mm

Check for minimum stirrups

$$A_{st(\min.)} = \mu_{\min} b\, s = 0.00167 \times 250 \times 125 = 52.0\ mm^2 \qquad < 78.50\ \text{mm}^2 \ldots\text{ok.}$$

Step 3: Reinforcement detailing

Complete reinforcement detailing for the beam is shown in Figs. EX. 6.7.b and EX. 6.7.c for the case of straight and bent bars, receptively.

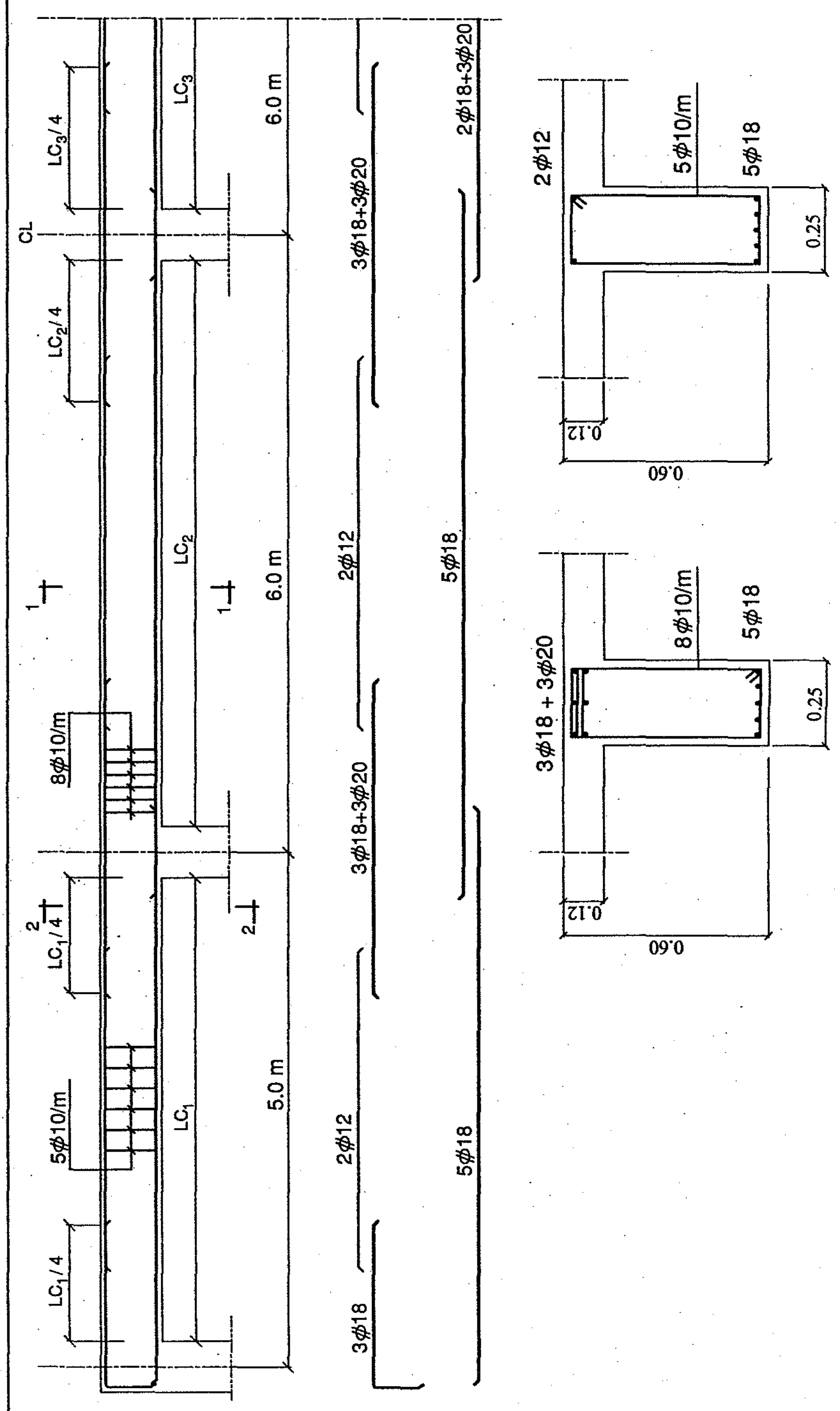


Fig. EX. 6.7b Reinforcement details (straight bars)

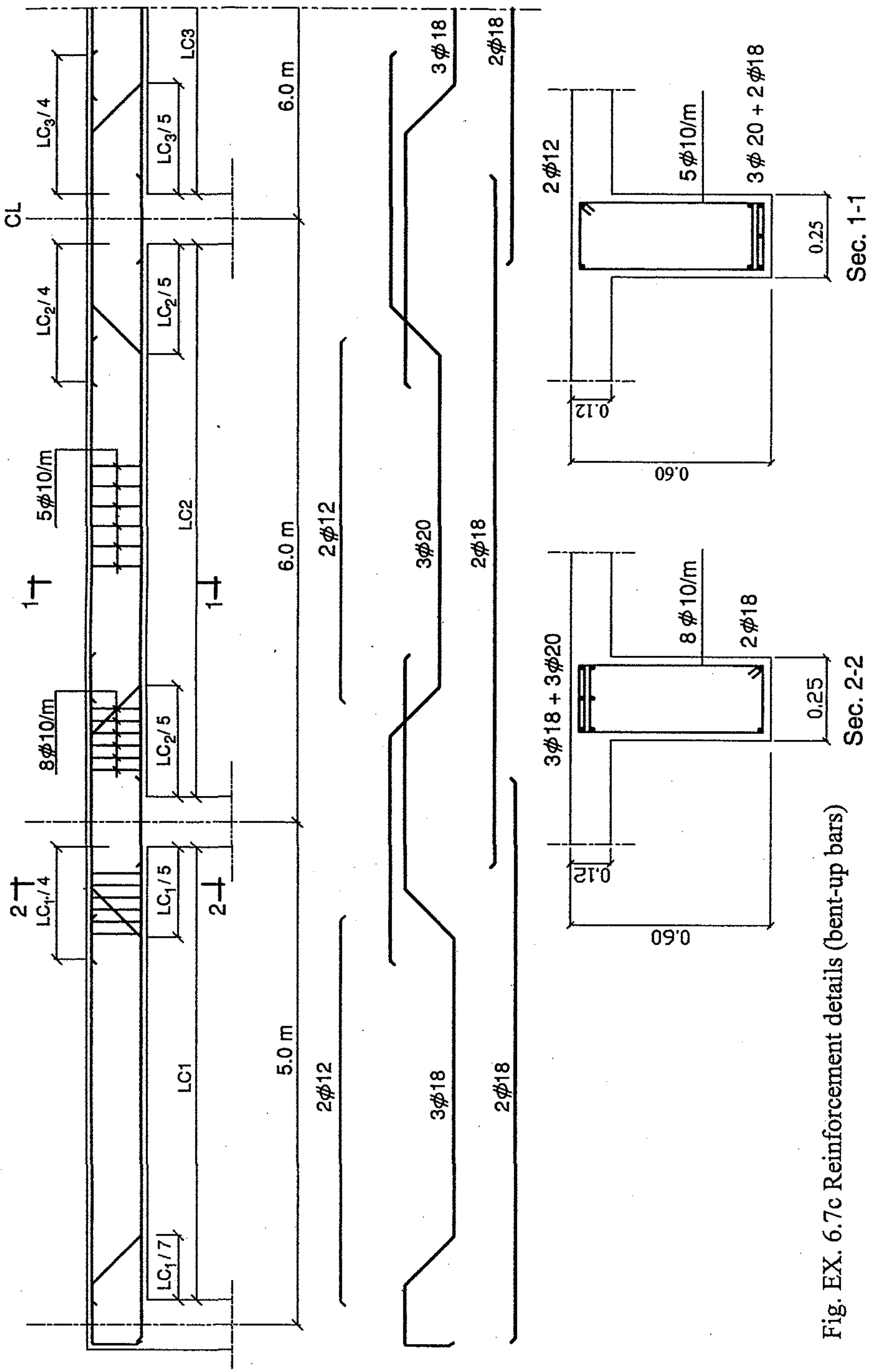


Fig. EX. 6.7c Reinforcement details (bent-up bars)

Example 6.8

It is required to design the continuous beam shown in Fig. EX. 6.8a. The beam is arranged every 3.25 m on plan. The characteristics compressive strength of concrete f_{cu} = 30 N/mm^2. The yield strength of the longitudinal steel f_y = 360 N/mm^2 and for the stirrups = 360 N/mm^2. The applied unfactored dead and live loads are also shown in Fig. EX. 6.8a.

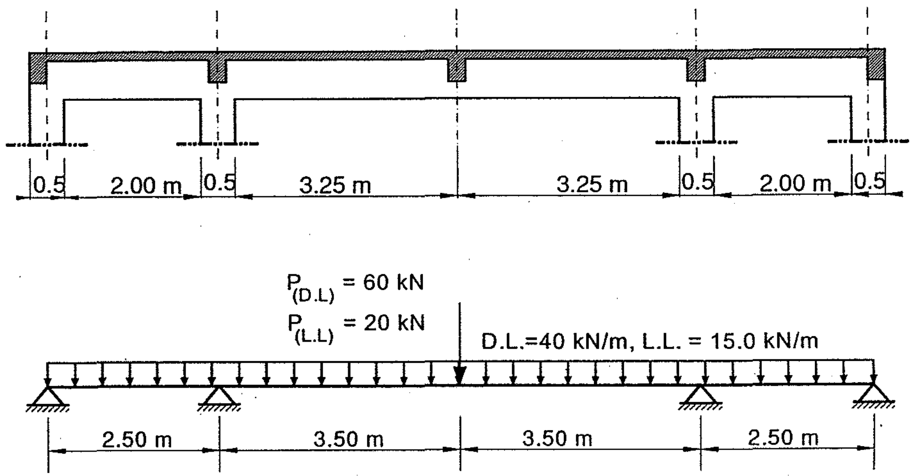


Fig. Ex. 6.8a

Solution

Step 1: Flexural design

Step 1.1: Calculation of maximum moments

In order to get the design bending moments and shear forces, one has to consider the cases of loading that give the maximum straining actions.

<u>Case 1</u>: maximum positive moment at the central span

In order to get the maximum positive bending moment at the central span, one has to use the following loads:

Uniform load at central span $= 1.4\,D.L. + 1.6\,L.L.$

$$= 1.4\times40+1.6\times15=80 \text{ kN/m}$$

Concentrated load at central span $= 1.4\,D.L. + 1.6\,L.L.$

$$= 1.4\times60+1.6\times20=116 \text{ kN/m}$$

Uniform load at end spans $= 0.9\,D.L. = 0.9\times40 = 36$ kN/m

The beam is twice indeterminate, however, because of symmetry only one unknown needs to be determined.

$M_b = M_c$

Applying three moment equation at b

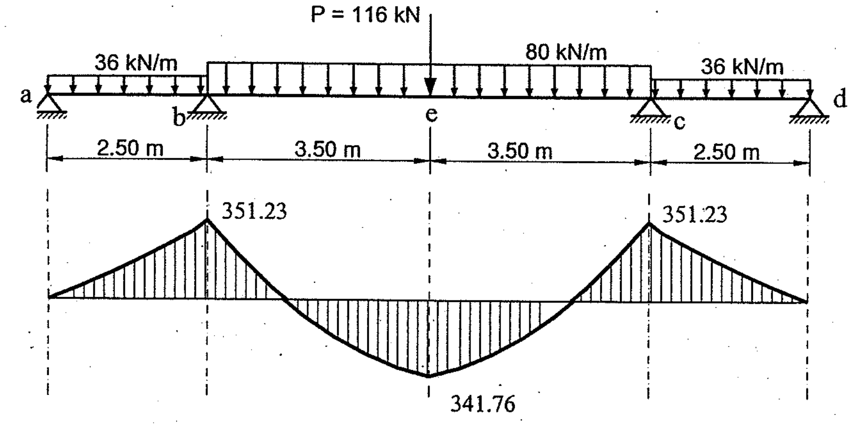
$$0+2(2.5+7)\ M_b + 7\ M_b = -6\left(\frac{w_{ab}\times L^3}{24}+\frac{w_{bc}\times L^3}{24}+\frac{P\times L^2}{16}\right)$$

$$26\ M_b = -6\left(\frac{36\times 2.5^3}{24}+\frac{80\times 7^3}{24}+\frac{116\times 7^2}{16}\right)$$

$M_b = 351.23\ kN.m$

$$M_e = \left(\frac{w_{bc}\times L^2}{8}+\frac{P\times L}{4}\right) - M_b = \left(\frac{80\times 7^2}{8}+\frac{116\times 7}{4}\right) - 351.23 = 341.76\ kN.m$$

The loading and the corresponding bending moment diagram are shown in the following figure.



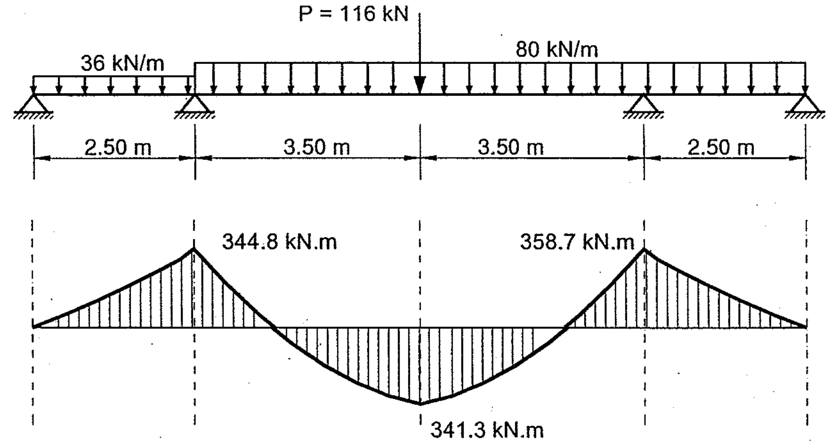
Case 2: Maximum negative moment at the support

In order to get the maximum negative bending moment at the support, one has to use the following loads:

Uniform load at central span and one end span $= 1.4\,D.L. + 1.6\,L.L.$

$= 1.4\times 40 + 1.6\times 15 = 80$ kN/m

Uniform load at other end spans $= 0.9\,D.L. = 0.9\times 40 = 36$ kN/m



The calculations are carried out using a computer program. The loading and the corresponding bending moment diagram are shown in the following figure.

$M_b = -344.8\ kN.m$ $\qquad$ $M_c = -358.7\ kN.m$

Another way for solving the indeterminate beam is to use the 3 moments equation twice (which gives very close solution)

Applying three moment equation at b:

$$0+2(2.5+7)M_b + 7M_c = -6\left(\frac{36\times 2.5^3}{24}+\frac{80\times 7^3}{24}+\frac{116\times 7^2}{16}\right)$$

$19M_b + 7M_c = -9132.12$.........(1)

Applying three moment equation at c:

$$0+2(2.5+7)M_c + 7M_b = -6\left(\frac{80\times 2.5^3}{24}+\frac{80\times 7^3}{24}+\frac{116\times 7^2}{16}\right)$$

$19M_c + 7M_b = -9304$ ....(2)

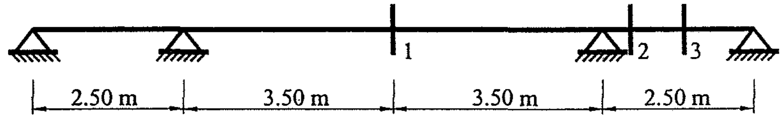
Solving eqs. (1, 2) gives M_b and M_c

$$M_{midspan} = \left(\frac{80\times 7^2}{8}+\frac{116\times 7}{4}\right) - \frac{344.8+358.7}{2} = 341.3\,kN.m$$

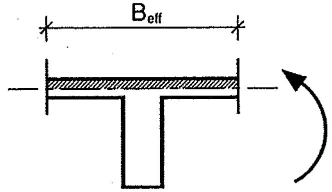
Note: Case 2 will also be used to get the maximum design shear forces at the internal supports.

Step 1.2: Design of critical sections

The critical sections are shown in the figure below



Section No. 1 T-section



B_{eff} = smallest of:

- $16t_s + b = 16\times120 + 250 = 2170\,mm$
- $\frac{L_2}{5} + b = \frac{0.7\times7000}{5} + 250 = 1230\,mm$
- C.L to C.L. between beams = 3250 mm

B_{eff} = 1230 mm

$$Assume\ beam\ thickness\,(t) = \frac{span}{10-12} \cong 700\ mm$$

$$d = t - \text{cover} = 700 - 50 = 650\ mm$$

$$d = C_1\sqrt{\frac{M_u}{f_{cu}B}}$$

$$650 = C_1\sqrt{\frac{341.76\times10^6}{30\times1230}} \qquad \therefore C_1 = 6.75\ \&\ J=0.826$$

The point is outside the C_1-J curve $\therefore \frac{c}{d} < (\frac{c}{d})_{min}$

$$C = 0.125\times650 = 81.25\,mm$$

$$a = 0.80\times81.25 = 65\,mm < t_s=120\ mm$$

$$A_s = \frac{M_u}{f_y.j.d} = \frac{341.76\times10^6}{360\times0.826\times650} = 1768\,mm^2$$

$(A_s)_{min}$ = the smaller of:

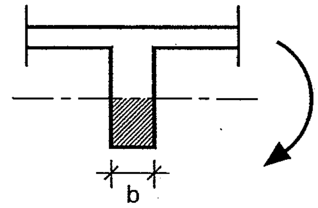
$$\frac{0.225\sqrt{f_{cu}}}{f_y}\times b\times d = \frac{0.225\sqrt{30}}{360}\times250\times650 = 556\,mm^2$$

1.3 A_s(required) = $1.3\times1768 = 2300\,mm^2$

$(A_s)_{min.} = 556\ mm^2 \quad < \quad (A_s)_{required}$

Choose A_s=6Φ20 $\qquad A_{s(chosen)}$ =1885 mm^2

Section No. 2 Rectangular section



b= 250 mm

d = 650 mm

M_u=351.23 kN.m Case 1

M_u=358.7 kN.m Case 2

Use M_u=358.7 kN.m from case 2

$$d = C_1\sqrt{\frac{M_u}{f_{cu}\ b}}$$

$$650 = C_1\sqrt{\frac{358.7\times10^6}{30\times250}} \qquad \therefore C_1 = 2.97\ \&\ J=0.74$$

$$A_s = \frac{M_u}{f_y.j.d} = \frac{358.7\times10^6}{360\times0.74\times650} = 2071\,mm^2 \qquad \text{.......}>A_{smin}$$

Choose A_s= 3Φ22 + 3Φ20 $\qquad A_{s(chosen)} = 2082\,mm^2$

Section No. 3

Section No. 3 is subjected to negative moment less than that of sec. 2. Thus, all the negative reinforcement over section 2 will continue over section 3. Moreover, the code also requires for each exterior panel to carry a positive bending equals to $wL^2/16$.

$$M_u = \frac{w_u\ L^2}{16} = \frac{80\times2.5^2}{16} = 31.25\,kN.m$$

$$B = \frac{L_2}{5} + b = \frac{0.8\times2500}{5} + 250 = 650\,mm$$

C1=15.6 $\quad\rightarrow$J=0.826

$$A_s = \frac{M_u}{f_y.j.d} = \frac{31.25\times10^6}{360\times0.826\times650} = 161\,mm^2$$

$$A_{s\,min} = smaller\ \ of \begin{cases} \frac{0.225\sqrt{30}}{360}250\times650 = 556\ mm^2 \\ 1.3\times161 = 210\ mm^2 \end{cases} = 210\ mm^2$$

But not less than $A_s = \frac{0.15}{100}250\times650 = 244\,mm^2$

A_{smin} =244 > A_s → use A_{smin} $\qquad$ (3Φ12)

Step 1.3: Calculation of development length

$$L_d = \{\frac{\alpha.\beta.\eta.(\frac{f_y}{\gamma_s})}{4 f_{bu}}\}.\phi$$

$$f_{bu} = 0.30\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.30\sqrt{\frac{30}{1.5}} = 1.34 \; N/mm^2$$

$\eta_{bottom}=1.0$ $\quad\quad\quad$ $\eta_{top}=1.3$

For bars in tension:-

$\alpha=1.0$ (Straight Bars) and $\beta=0.75$ (deformed bars)

$$L_{d(bottom)} = \{\frac{1.0\times0.75\times1.0\times(360/1.15)}{4\times1.34}\}.\phi = 44\phi$$

$$L_{d(top)} = \{\frac{1.0\times0.75\times1.3\times(360/1.15)}{4\times1.34}\}.\phi = 57\phi$$

For bars in compression:-

$\alpha=1.0$ (Straight Bars) and $\beta=0.50$ (deformed bars)

$$L_{d(bottom)} = \{\frac{1.0\times0.50\times1.0\times(360/1.15)}{4\times1.34}\}.\phi = 29\phi$$

$$L_{d(top)} = \{\frac{1.0\times0.50\times1.3\times(360/1.15)}{4\times1.34}\}.\phi = 38\phi$$

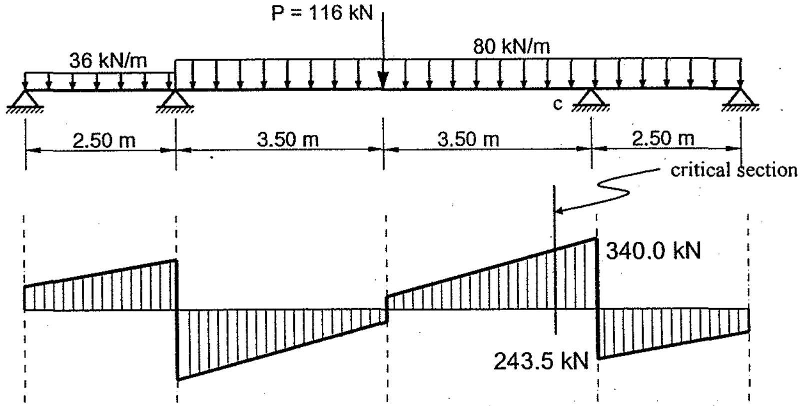
Or we can directly use the coefficients from Table (5.3) with f_{cu}=30 N/mm^2

For Tension: $L_{d(bottom)}=50\Phi$, $\quad$ $L_{d(top)}=65\Phi$

For Compression: $L_{d(bottom)}=40\Phi$, $\quad$ $L_{d(top)}=52\Phi$

Step 2: Check of shear

The loading and shear force diagram (obtained from the computer program) for case 2 is shown below



For the case of uniform load, the critical section is located at d/2 from the support

$$Q_u = Q_c - w_u(d/2 + half\ column\ width)$$

$$Q_u = 340 - 80\times(0.65/2+0.50/2) = 294\,kN$$

$$q_u = \frac{Q_u}{b\times d} = \frac{294\times10^3}{250\times650} = 1.81\,N/mm^2$$

$$q_{u(max)} = 0.7\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.7\sqrt{\frac{30}{1.5}} = 3.13 \; N/mm^2 < 4.0\,N/mm^2$$

$$q_{u(max)} = 3.13\,N/mm^2$$

$q_u \le q_{u(max)}$ the concrete dimensions of the section are adequate for shear.

$$q_{cu} = 0.24\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.24\sqrt{\frac{30}{1.5}} = 1.07\,N/mm^2$$

$q_u > q_{cu}$ web reinforcement is required

$$q_{su} = q_u - 0.5\,q_{cu}$$

$$q_{su} = 1.81 - 0.50\times1.07 = 1.275\,N/mm^2$$

$$\mu = \frac{A_{st}}{b.s} = \frac{q_{su}}{f_y / \gamma_s}$$

$$\mu = \frac{2 \times A_s}{250 \times s} = \frac{1.275}{360/1.15}$$

For Φ = 10 mm A_S=78.5 mm^2 s=154 mm

Using 7 Φ 10/m (s=142 mm …o.k)

Check for minimum stirrups

$$\mu_{\min} = \frac{0.4}{f_y} = \frac{0.4}{360} = 0.0011 \quad \text{(not less than 0.0010 for high grade steel)}$$

$$A_{st(\min.)} = \mu_{\min} \times b \times s$$

$$A_{st(\min.)} = 0.0011 \times 250 \times 142 = 39.7\ mm^2 \quad < (2\text{x } 78.5 \text{ mm}^2) \text{ o.k}$$

Step 3: Reinforcement detailing

Complete reinforcement detailing for the beam is shown in Figs. EX. 6.8b and EX 6.8c for the case of straight and bent bars, respectively.

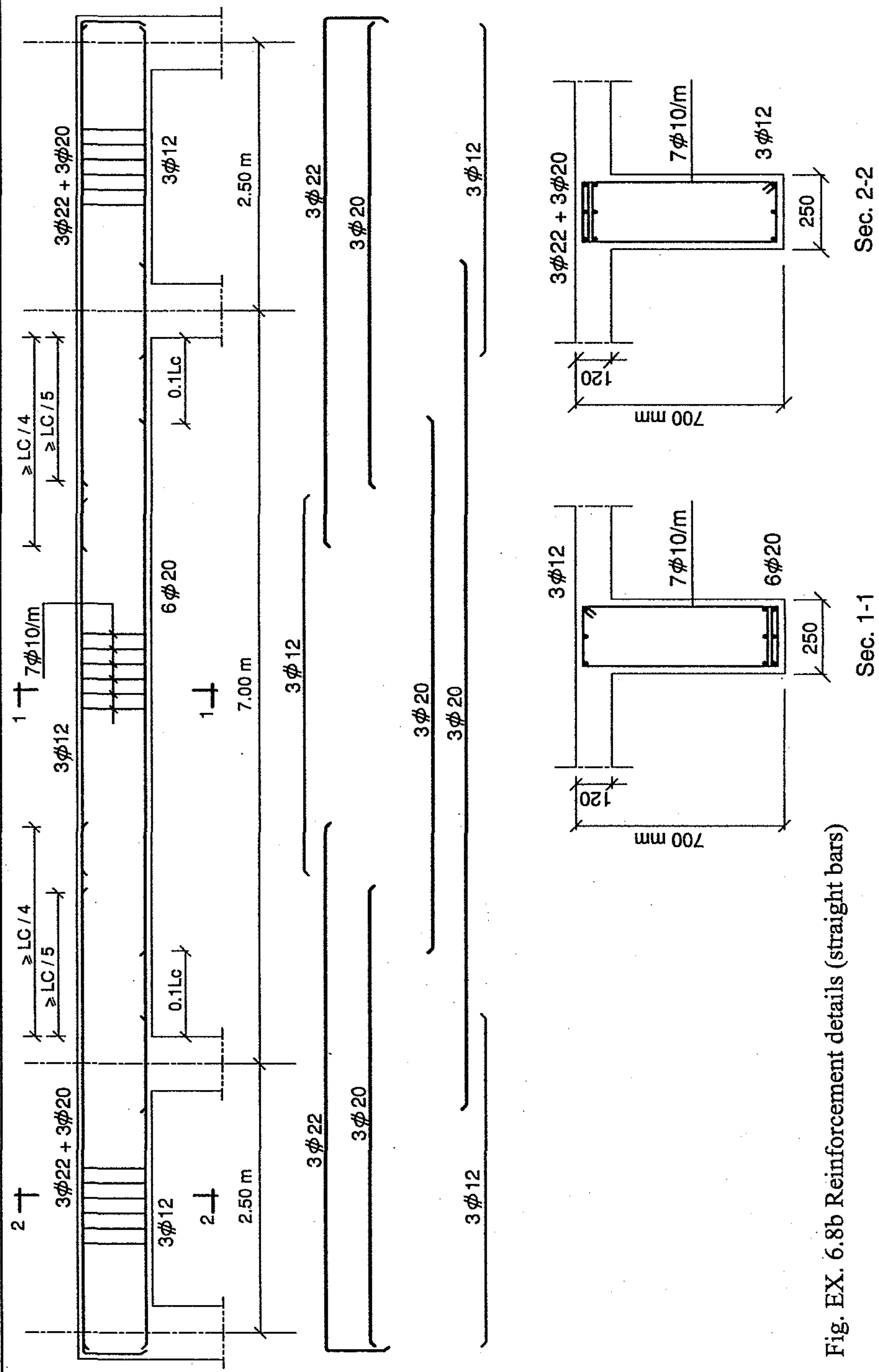


Fig. EX. 6.8b Reinforcement details (straight bars)

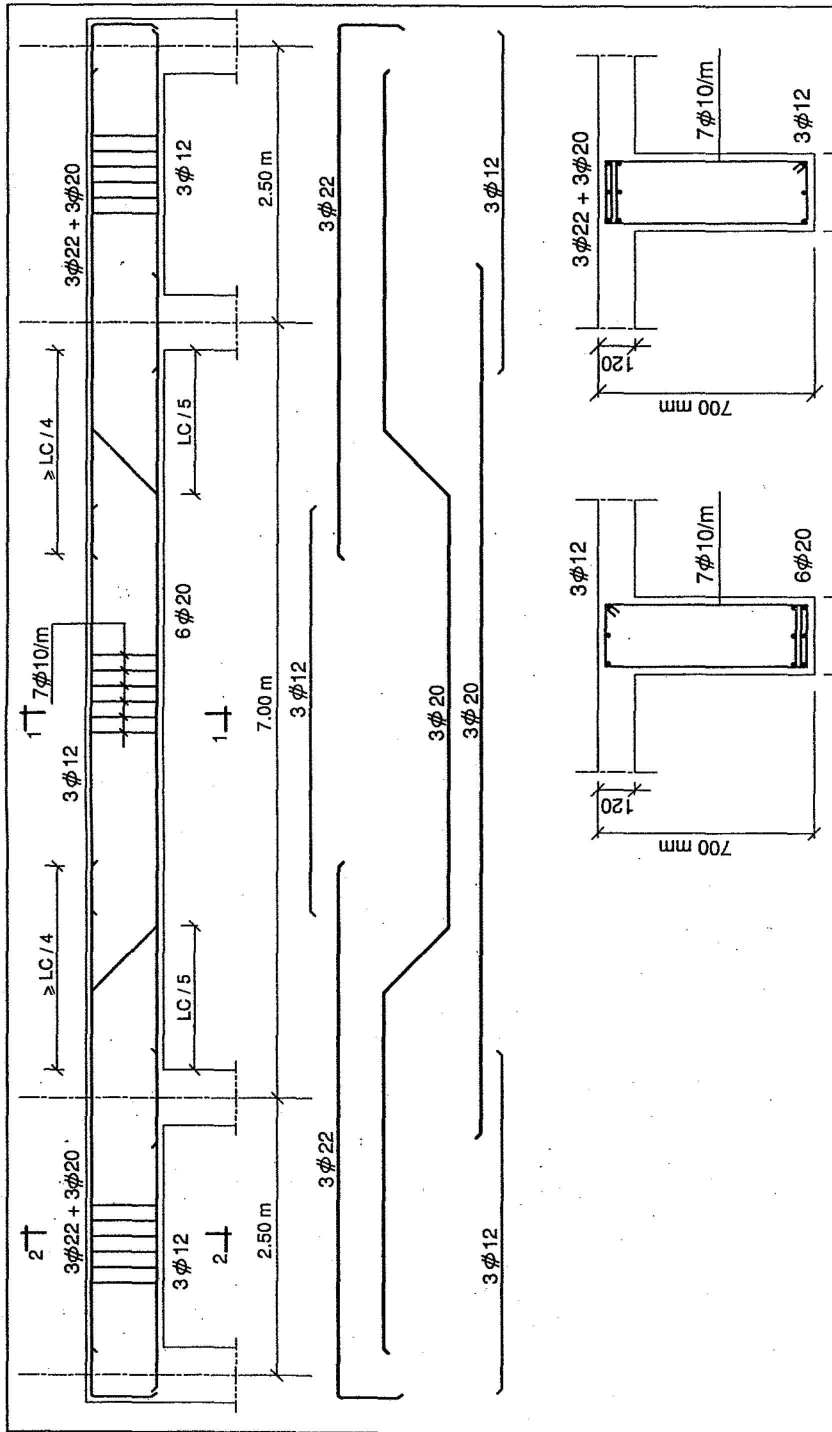
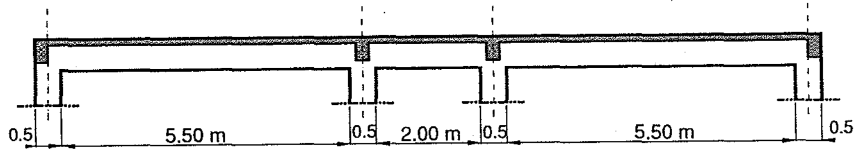


Fig. EX. 6.8c Reinforcement details (bent-up bars)

Example 6.9

It is required to design the continuous beam with unequal spans shown in Fig. EX. 6.9a. The beam is arranged every 2.75 m on plan. The characteristics compressive strength of concrete f_{cu} = 30 N/mm^2. The yield strength of the longitudinal steel f_y = 360 N/mm^2 and for the stirrups = 360 N/mm^2. The applied unfactored dead and live loads are shown in Fig. EX. 6.9a. These loads can be used for designing the beam for bending as well as for shear.



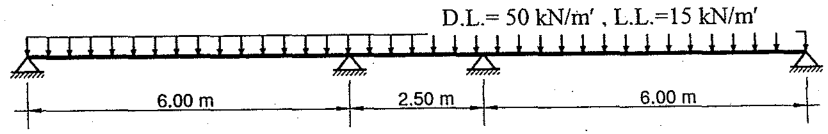


Fig. EX. 6.9a

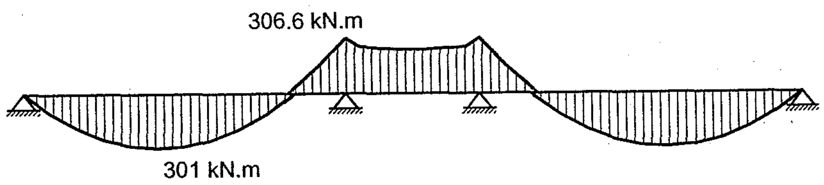
Solution

Step 1: Flexural design

Step 1.1: Calculation of maximum moments

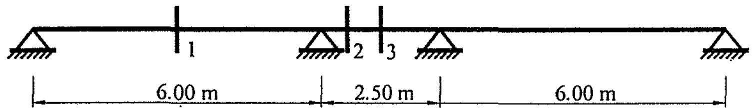
$w_u = 1.4\,DL + 1.6\,LL = 1.4 \times 50 + 1.6 \times 15 = 94\, kN/m'$

The absolute bending moment diagram (the envelope of the bending moment) that is obtained from two cases of loading is obtained using a computer program. The results are shown below.



Step 1.2: Design of critical sections

The critical sections are shown below



For Section 1-1 $M_u(+ve) = 301 kN.m$

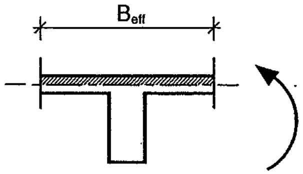
For Section 2-2 $M_u(-ve) = 306.6 kN.m$

Section No. 1 T-section

B_{eff} = smallest of:

- $16t_s + b = 16\times120 + 250 = 2170 mm$
- $\frac{L_2}{5} + b = \frac{0.8\times6000}{5} + 250 = 1210 mm$
- C.L. to C.L. between beams = 2750 mm

B_{eff} = 1210 mm



$$Assume\ beam\ thickness\,(t) = \frac{span}{10-12} \cong 600\ mm$$

$$d = t - cover = 600 - 50 = 550\ mm$$

$$d = C_1\sqrt{\frac{M_u}{f_{cu}B}}$$

$$550 = C_1\sqrt{\frac{301\times10^6}{30\times1210}} \qquad \therefore C_1 = 6.04\ \&\ J=0.826$$

The point is outside the C_1-J curve $\therefore \frac{c}{d} < (\frac{c}{d})_{min}$ Use $\frac{c}{d} = (\frac{c}{d})_{min} = 0.125$

$$c = 0.125\times550 = 68.75 mm$$

$$a = 0.80\times68.75 = 55 mm < t_s = 120\ mm$$

$$A_s = \frac{M_u}{f_y.j\,d} = \frac{301\times10^6}{360\times0.826\times550} = 1841 mm^2$$

$(A_s)_{min}$ = the smaller of:

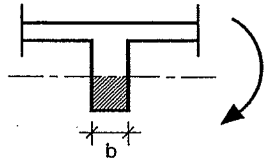
$$\frac{0.225\sqrt{f_{cu}}}{f_y}\times b\times d = \frac{0.225\sqrt{30}}{360}\times250\times550 = 471 mm^2$$

1.3 A_s(required) =1.3x1841=2393 mm^2

$(A_s)_{min.}$ = 471 mm^2 < $(A_s)_{required}$

Choose A_s=3Φ22+3Φ18 $\qquad A_{s(chosen)}$ =1902 mm^2

Section No. 2 Rectangular section



$M_u(-ve) = 306.6$ b= 250 mm d = 550 mm

$$d = C_1\sqrt{\frac{M_u}{f_{cu}\,b}}$$

$$550 = C_1\sqrt{\frac{306.6\times10^6}{30\times250}} \qquad \therefore C_1 = 2.72 \text{ and } J= 0.71$$

$$A_s = \frac{M_u}{f_y.j\,d} = \frac{306.6\times10^6}{360\times0.71\times550} = 2181 mm^2$$

Choose A_s=6Φ22 $\qquad A_{s(chosen)}$ =2281 mm^2

Section No. 3

Section No. 3 is subjected to negative moment less than that of sec. 2. Thus, all the negative reinforcement over section 2 will continue over section 3. Moreover, the code also requires for each interior panel to carry a positive bending equals to $wL^2/24$.

$$M_u = \frac{w_u\,L^2}{24} = \frac{94\times2.5^2}{24} = 24.5 kN.m$$

$$B = \frac{L_2}{5} + b = \frac{0.7\times2500}{5} + 250 = 600 mm$$

C1=14.9 →J=0.826

$$A_s = \frac{M_u}{f_y.j\,d} = \frac{24.5\times10^6}{360\times0.826\times550} = 149.8 mm^2$$

$$A_{s\,min} = smaller\ of \begin{cases} \frac{0.225\sqrt{30}}{360} 250\times 550 = 470.7\ mm^2 \\ 1.3\times161 = 194.8\ mm^2 \end{cases} = 194.8\ mm^2$$

But not less than $A_s = \frac{0.15}{100} 250 \times 550 = 206\, mm^2$

$A_{smin} = 206 > A_s \rightarrow$ use A_{smin} (2Φ12)

Step 1.3: Calculation of the Development Length

$$L_d = \{\frac{\alpha.\beta.\eta.(\frac{f_y}{\gamma_s})}{4 f_{bu}}\}.\phi$$

$$f_{bu} = 0.30\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.30\sqrt{\frac{30}{1.5}} = 1.34\ N/mm^2$$

$\eta_{bottom}=1.0$ $\eta_{top}=1.3$

For bars in tension:-

α=1.0 (Straight Bars) and β=0.75 (deformed bars)

$$L_{d(bottom)} = \{\frac{1.0 \times 0.75 \times 1.0 \times (360/1.15)}{4 \times 1.34}\}.\phi = 44\phi$$

$$L_{d(top)} = \{\frac{1.0 \times 0.75 \times 1.3 \times (360/1.15)}{4 \times 1.34}\}.\phi = 57\phi$$

For bars in compression:-

α=1.0 (Straight Bars) and β=0.50 (deformed bars)

$$L_{d(bottom)} = \{\frac{1.0 \times 0.50 \times 1.0 \times (360/1.15)}{4 \times 1.34}\}.\phi = 29\phi$$

$$L_{d(top)} = \{\frac{1.0 \times 0.50 \times 1.3 \times (360/1.15)}{4 \times 1.34}\}.\phi = 38\phi$$

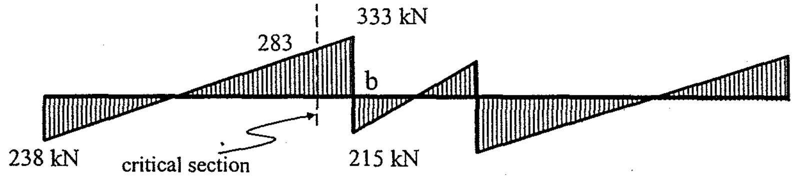
Or we can directly use the coefficients from Table (5.3) with f_{cu}=30 N/mm^2

For Tension: $L_{d(bottom)}=50\Phi$, $L_{d(top)}=65\Phi$

For Compression: $L_{d(bottom)}=40\Phi$, $L_{d(top)}=52\Phi$

Step 2: Shear design

The shear force diagram is shown in the following figure



For the case of uniform load, the critical section at d/2 from the support.

$Q_u = Q_b - w_u(d/2 + \text{half column width})$

$Q_u = 333 - 94 \times (0.55/2 + 0.50/2) = 283.65\, kN$

$$q_u = \frac{Q_u}{bd} = \frac{283.65 \times 10^3}{250 \times 550} = 2.06\, N/mm^2$$

$$q_{u(max)} = 0.7\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.7\sqrt{\frac{30}{1.5}} = 3.13\ N/mm^2 < 4.0\, N/mm^2 \longrightarrow q_{u(max)} = 3.13\, N/mm^2$$

$q_u \le q_{u(max)}$ the concrete dimensions of the section are adequate for shear.

$$q_{cu} = 0.24\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.24\sqrt{\frac{30}{1.5}} = 1.07\, N/mm^2$$

$q_u > q_{cu}$ web reinforcement is required

$q_{su} = q_u - 0.5 q_{cu} = 2.06 - 0.50 \times 1.07 = 1.53\, KN/m^2$

$$\mu = \frac{2 \times A_s}{250 \times s} = \frac{1.53}{360/1.15}$$

For ϕ = 10 mm A_S=78.5 mm^2

s=128 mm →Using Φ 10 @ 125 mm (8 Φ 10 /m′)

Check for minimum stirrups

$$\mu_{min} = \frac{0.4}{f_y} = \frac{0.4}{360} = 0.00111 \quad \text{(not less than 0.0010)}$$

$A_{st(min.)} = \mu_{min}.b.s = 0.00111 \times 250 \times 125 = 34.7\ mm^2$ <(2x78.5 …ok.)

Step 3: Reinforcement detailing

Complete reinforcement detailing for the beam is shown in Figs. EX. 6.9b and EX. 6.9c for the case of straight and bent bars, respectively.

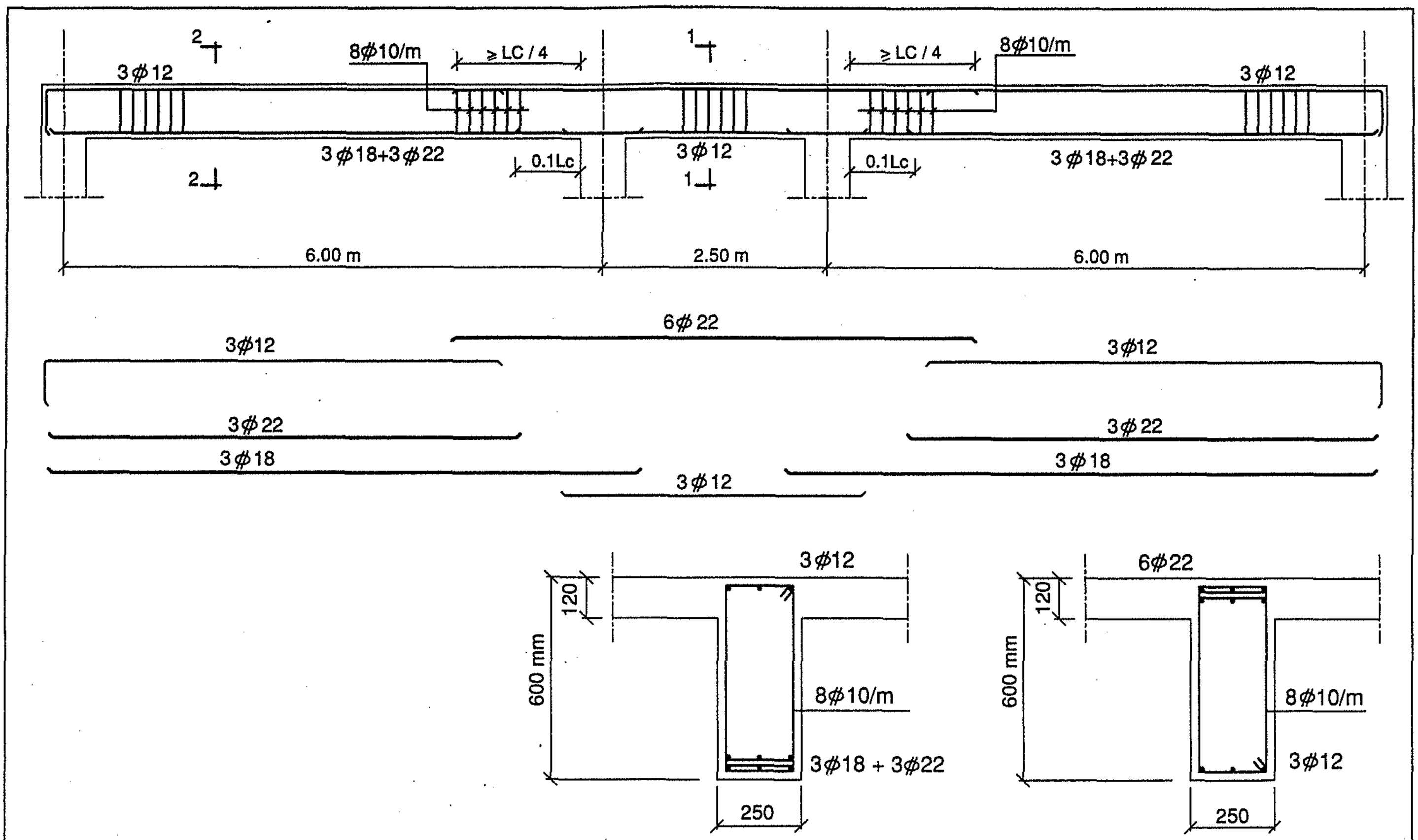


Fig. EX. 6.9b Reinforcement details (straight bars)

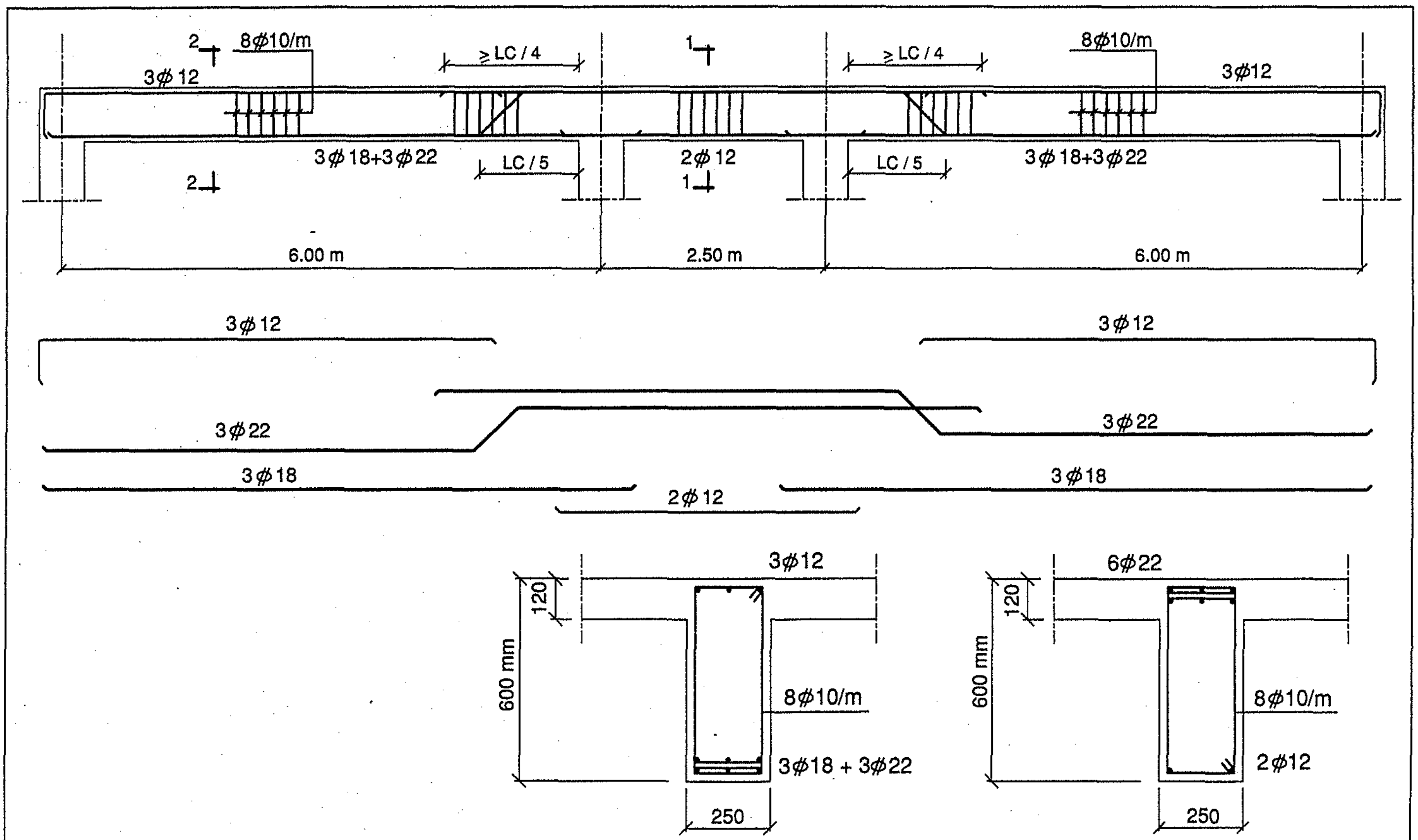


Fig. EX. 6.9c Reinforcement details (bent-up bars)

7

TRUSS MODEL FOR BEAMS FAILING IN SHEAR

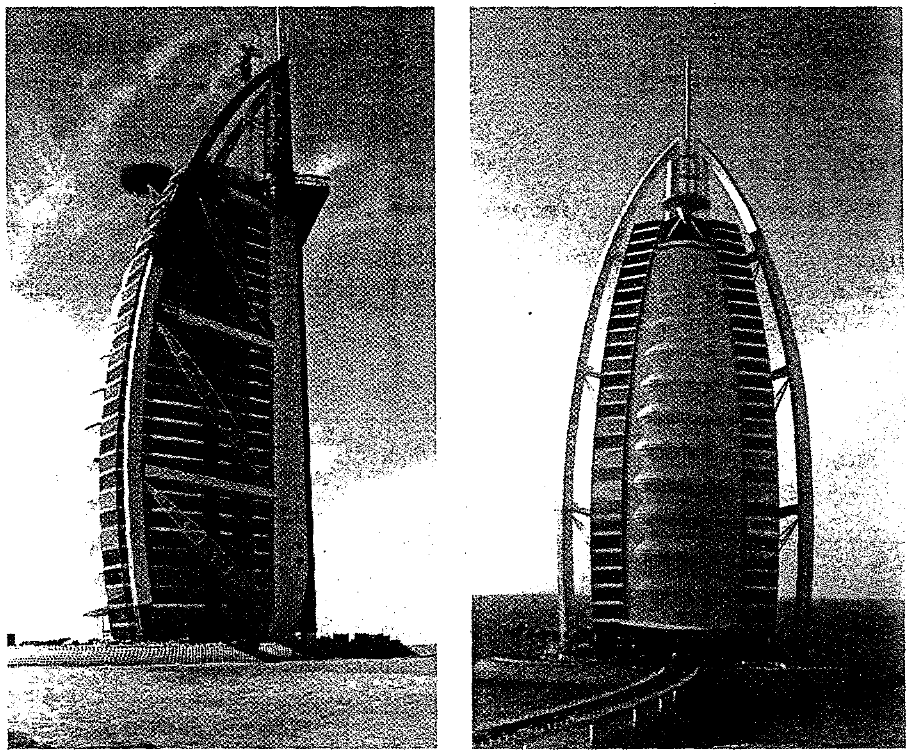


Photo 7.1 Burj Al Arab, Dubai.

7.1 Introduction

In the previous chapters of this book, the design for bending and the design for shear have been treated independently. Shear-flexure interaction in slender R/C beams can be expressed in terms of a mathematical-mechanical model. The best model for slender beams with web reinforcement is the *Truss Model*. It provides an excellent conceptual model to show the forces that exist in cracked R/C beams.

Many international design codes for reinforced concrete structures have used the truss model as the basis for design procedures for shear and flexure. The Egyptian Code for Design and Construction of Concrete Structures does not explicitly mention the truss model as a design tool. However, it includes provisions that satisfy its requirements. The information presented in this chapter is adequate for a practicing engineer to understand and to apply the truss model. Researchers, however, could find more details in scientific papers related to the subject.

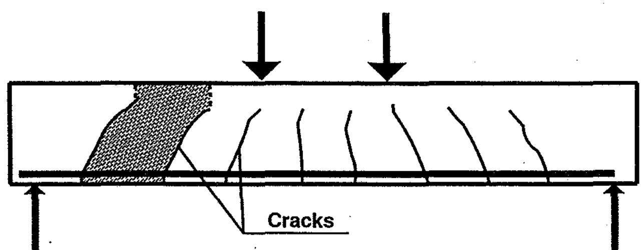
Chapter (7) makes frequent reference to the information presented in the previous chapters of this book and assumes that the reader is familiar with what have been presented.

7.2 Background

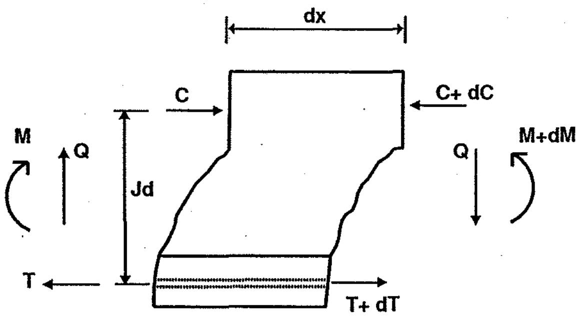
The objective of this section is to provide some basic concepts before introducing the reader to the truss model.

7.2.1 Slender Beams Versus Deep Beams

Figure (7.1a) shows a cracked reinforced concrete beam. The initial stage of cracking generally results in vertical flexural cracking. Increasing the external loads results in formation of diagonal cracks.



a: Cracked beam



b: Portion of the beam between two cracks

Fig. 7.1 Equilibrium of a segment of a cracked beam

Referring to Fig. (7.1b), the equilibrium of a section between two diagonal cracks leads to the following equation:

$$dM = Q\ dx \quad \text{..(7.1a)}$$

$$dT = \frac{dM}{jd} = \frac{Q\ dx}{jd} \quad \text{..(7.1b)}$$

in which (dT) is the gradient of the tension force in the longitudinal steel, Q is the shear force and jd is the lever arm.

Hence, the relationship between the shear force and the tension force in the steel reinforcement can be written as:

$$Q = \frac{d}{dx}(T\ jd) \quad \text{..(7.2)}$$

which can be expanded as:

$$Q = \frac{d(T)}{dx}\ jd + \frac{d(jd)}{dx}\ T \quad \text{..............................(7.3)}$$

Two extreme cases can be identified;

1- If the lever arm jd, remains constant as assumed in the classical beam theory, then:

$$\frac{d(jd)}{dx} = 0 \qquad \text{and} \qquad Q = \frac{d(T)}{dx}\ jd$$

where $d(T)/dx$ is the shear flow across any horizontal plane between the reinforcement and the compression zone as shown in Fig. (7.1). The above equation indicates that shear transfer is accompanied by a change in the tension force in the steel reinforcement and a constant lever arm. This is called a shear transfer by "*beam action*".

2- The other extreme case occurs if the shear flow, $d(T)/dx$ (which is equal to the change in the tension force in the steel reinforcement), equals zero, giving:

$$\frac{dT}{dx} = 0 \qquad \text{and} \qquad Q = T\ \frac{d(jd)}{dx}$$

This occurs if the shear flow cannot be transmitted due to change in the force in the steel reinforcement. In such a case, the shear is transmitted by "*arch action*" rather than by beam action.

Figure (7.2) shows the flow of internal forces in a relatively deep beam. In this member, the compression force, C, in the inclined strut and the tension force, T, are constant. The longitudinal reinforcement provides the tie to the arch.

It should be noted that arch action is not a shear mechanism in the sense that it does not transmit a tangential force to a nearly parallel plane. However, arch action permits the transfer of the applied load directly to the supports and reduces the contribution of the other types of shear transfer.

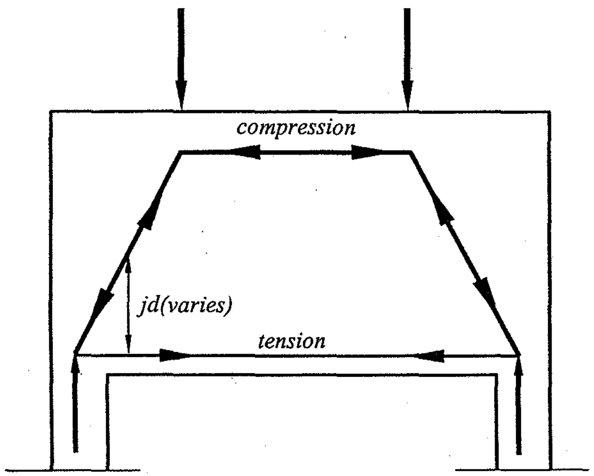


Fig. 7.2 Arch action in deep beams

The behavior of beams failing in shear varies widely depending on the relative contribution of beam action and arch action. In shallow (slender) beams, shear is transmitted by beam action. In deep beams, arch action dominates the behavior.

7.2.2 Analysis of Forces in R/C Slender Beams

The external loads acting on a cylinder reinforced concrete beam are resisted through a system of internal forces. There are two approaches to obtain the internal forces in slender beams. The first approach is to investigate the equilibrium of a free body of the beam (also called sectional analysis). The second approach is to analyze the beam using a conceptual model that represents the flow of forces (for example the truss model).

7.2.2.1 Sectional Analysis

The design of reinforced concrete beams can be based on analyzing the critical section for bending and the critical section for shear independently. Some additional precautions could be taken to account for the effect of their interaction. Consider, for example, the beam shown in Fig. 7.3a. Applying the flexural theory of reinforced concrete presented in Chapter (2), results in designing the critical section for bending (Section 1-1 in Fig. 7.3b). On the other hand, analysis of the forces transferring shear across the inclined crack, as presented in Chapter (4), results in design equations for shear (Fig. 7.3c).

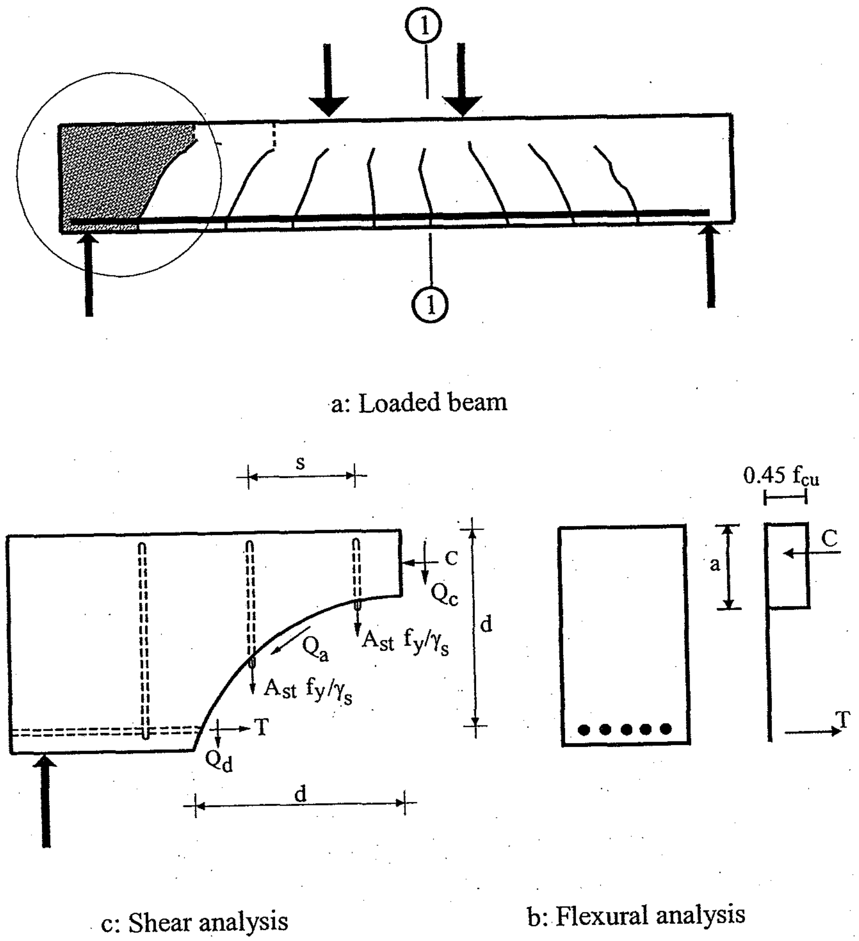


Fig. 7.3 Sectional analysis of R/C beams

7.2.2.2 Mechanical - Mathematical Models

Researchers have developed a number of models to express the behavior of slender reinforced concrete beams after cracking. Among the best of them is the *Truss Model*, which provides an excellent conceptual model to show the forces existing in a cracked concrete beam. A combination of the *sectional analysis* and the *truss model* has led to the recent developments in the national and international codes for design of R/C structures.

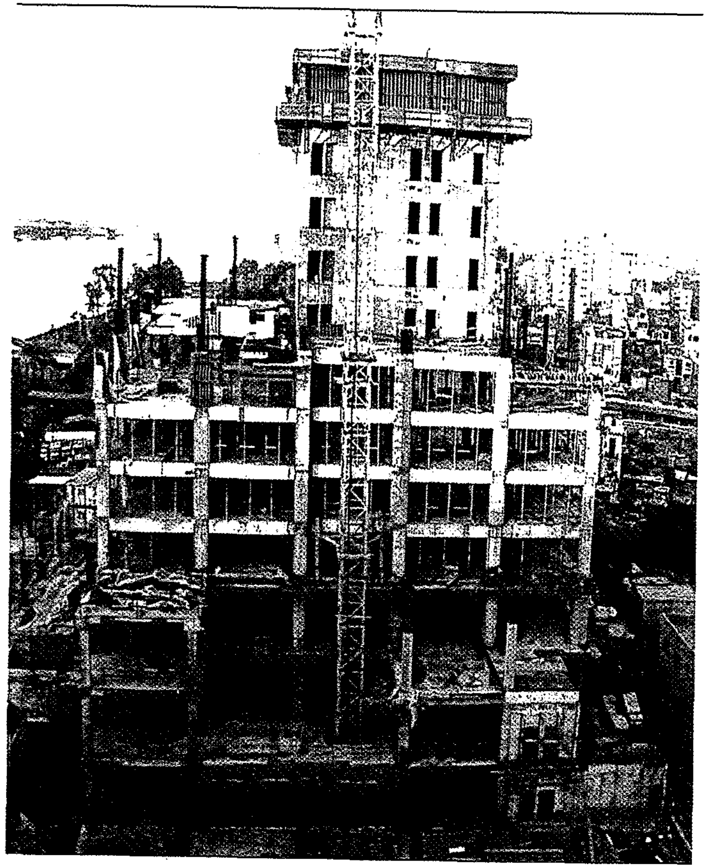
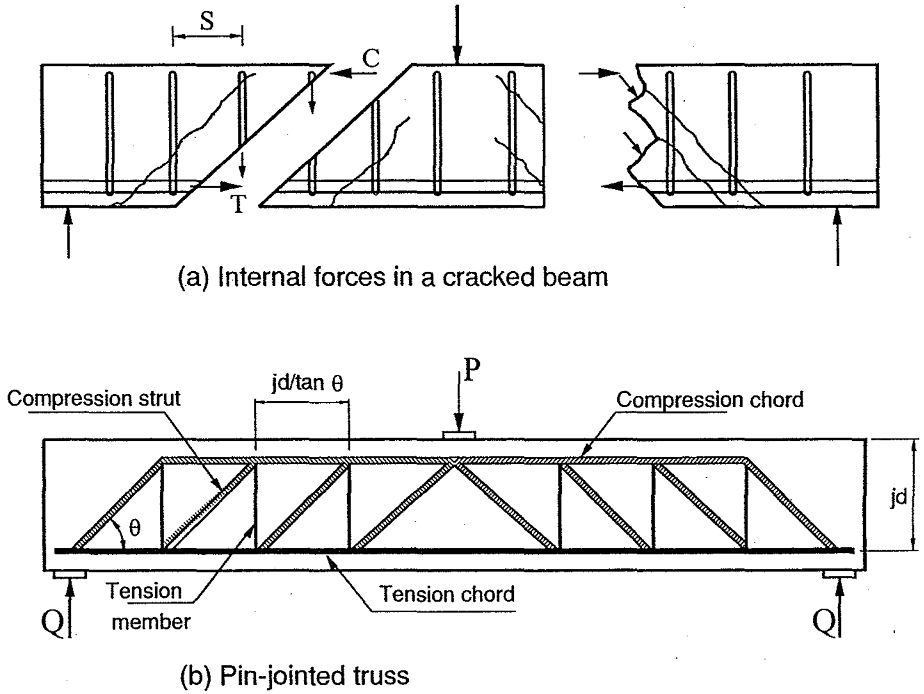


Photo 7.2 Nile City Tower during construction, Cairo, Egypt.

7.3 Truss Model for Slender Beams

As shown in Fig. 7.4, a beam with inclined cracks develops compressive and tensile forces, C and T, in its top and bottom parts "chords", vertical tension in stirrups "vertical member" and inclined compressive forces in the concrete "diagonals" between the inclined cracks. This highly indeterminate system of forces is replaced by an analogous truss.



Components of the Truss Model

- Compression Chord → Concrete in the compression zone
- Tension Chord → Steel in the tension zone
- Vertical tension members → Stirrups
- Diagonals → Concrete in diagonal compression

Fig. 7.4 Truss analogy for beams falling in shear

7.4 Traditional 45-Degree Truss Model

7.4.1 Formation of the 45-Degree Truss

Figure 7.5 shows a truss model for a simple beam in which the directions of the diagonal compression stresses are assumed to remain at 45 degrees. This truss that models a cracked reinforced concrete beam can be formed by:

- Lumping all of the stirrups cut by section A-A into one vertical member.
- Lumping the diagonal concrete members cut by section B-B into one diagonal member with an angle of inclination of 45° with respect to the beam axis. This diagonal member is stressed in compression to resist the shear on section B-B.
- Considering the longitudinal tension reinforcement as the bottom chord of the truss.
- Considering the flexural compression zone of the beam acts as the top chord.

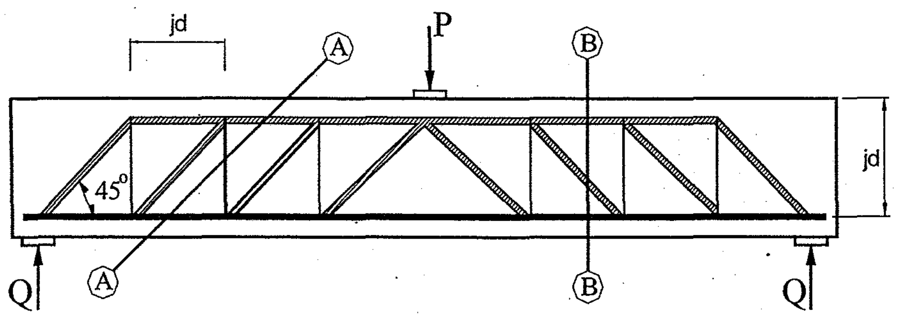


Fig. 7.5 45-Degree Truss Model

7.4.2 Evaluation of the Forces in the Stirrups

Each vertical member represents a number of n stirrups. Hence, the area of a vertical member is given by:

$$\textit{area of a vertical member} = n\ A_{st} = \frac{jd}{s}\ A_{st} \quad \text{.........(7.4)}$$

in which jd is the lever arm, s is the spacing between stirrups and A_{st} is the area of all the branches of one stirrup.

From the free body diagram shown in Fig. 7.6, the shear force is equal to the tension force carried by the stirrups and can be calculated as:

$$Q = \frac{A_{st}(f_y / \gamma_s)\, jd}{s} \quad \text{.........(7.5)}$$

Note: Equation (7.5) is equivalent to equation (4.17) derived from sectional analysis with d is replaced by *jd*.

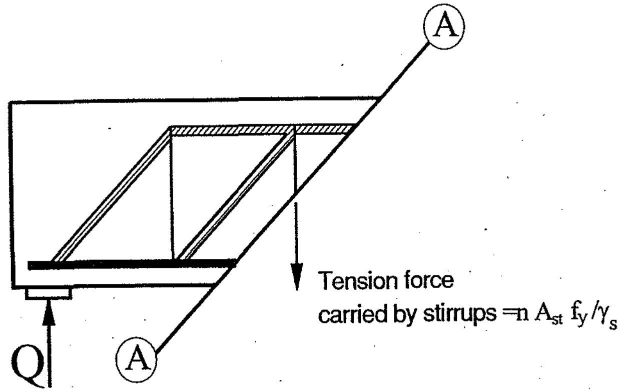


Fig. 7.6 Calculation of forces in stirrups

It should be mentioned that the previously described Truss-Model ignores the concrete contribution to the shear strength of the beam (q_{cu}). This simplification will be discussed briefly in a later section.

7.4.3 The Compression Force in the Diagonals

The free body shown in Fig. 7.7 is a cut by a vertical section B-B. The shear force at section B-B, Q, must be resisted by the vertical component of an inclined compressive force D in the diagonals.

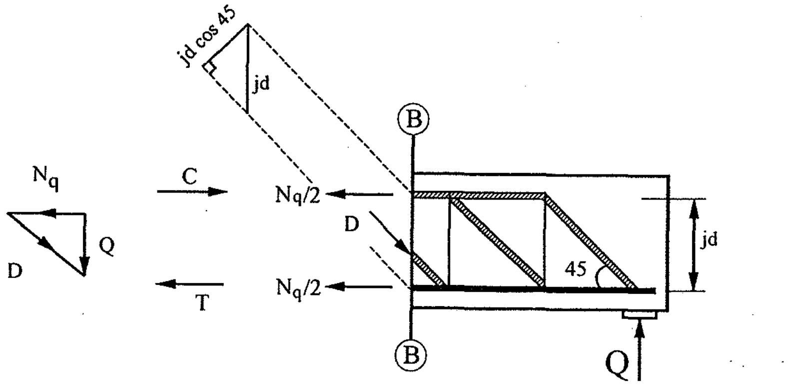


Fig. 7.7 Calculation of the diagonal compression force

Equilibrium of forces at section B-B gives:

$$Q = D \sin 45^o \quad \text{.......................................(7.6)}$$

The width of diagonal member is b and its thickness is $(jd \cos 45^o)$. Hence, the total compression force resisted by the diagonal member is equal to the average compression stress in the diagonal direction, f_{cd}, multiplied by the area of the diagonal member. Hence, the diagonal force D can expressed as:

$$D = f_{cd}\, b \left(jd \cos 45^o\right) \quad \text{.................................(7.7)}$$

Substituting equation (7.7) into equation (7.6) results in:

$$Q = f_{cd}\, b \left(jd \;\cos 45^o\right) \sin 45^o \quad \text{..............................(7.8)}$$

$$Q = 0.5\, f_{cd}\, b \;\; jd \quad \text{......................................(7.9)}$$

Equation 7.9 indicates that the shear strength of a concrete beam reaches its maximum value when the compressive stress in the web reaches the crushing

strength of concrete, f_{ce}, no matter how much web reinforcement is provided, i.e.,

$$Q \le 0.5 f_{ce} b \; jd \quad \text{......................................(7.10)}$$

$$Q = \frac{A_{st} f_y / \gamma_s \; jd}{s} \le 0.5 f_{ce} b \; jd \quad \text{..............................(7.11)}$$

From Eq. (7.9), the compressive stress in the diagonals is given by

$$f_{cd} = \frac{Q}{0.5 b \;\; jd} \quad \text{...(7.12)}$$

The web of the beam will crush if the inclined compressive stress f_{cd}, exceeds the crushing strength of concrete in the compression diagonals, f_{ce}. In otherwards, the compressive stresses in the web of the beam should satisfy:

$$f_{cd} \;\le\; f_{ce} \quad \text{...(7.13)}$$

The compressive strength of concrete in the web is called the *"effective strength of concrete"* and it tends to be less than the cube or the cylinder strength of concrete. This is attributed to the fact that concrete in the web is cracked as shown in Fig. 7.8.

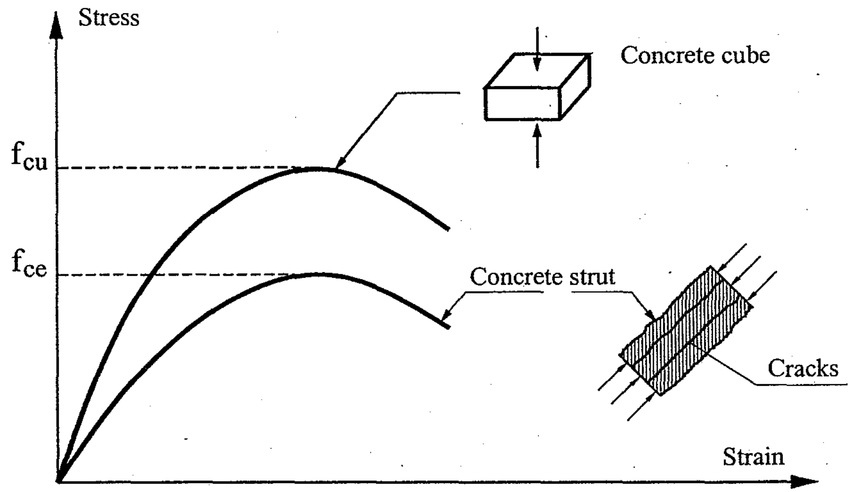


Fig. 7.8 Effective compressive strength of concrete

The effective concrete strength is frequently expressed as:

$$f_{ce} = \beta_s \left(0.67 \frac{f_{cu}}{\gamma_c}\right) \quad \text{.................................(7.14)}$$

where β_s is efficiency factor that takes into account the effect of cracking on the effective compressive strength of the concrete in the strut.
For a strut in normal weight R/C beams that are not subjected to an axial tension force, the value of β_s can be reasonably taken equal to 0.6.

If Eq. 7.13 is not satisfied, then crushing of concrete in the compression diagonals would occur (shear-compression failure).

Note: Equation 7.8 can be written in the form:

$$\frac{Q}{b\,jd} = \frac{f_{cd}\, b(jd \cos 45^{o}) \sin 45^{o}}{b\,jd}$$

The above equation can be further simplified as:

$$q = f_{cd} \sin 45^{o} \cos 45^{o} = 0.5\, f_{cd}$$

Hence, it can be seen that there is a direct relation between the compressive stresses developed in the diagonal struts, f_{cd}, and the applied shear stress, q. Accordingly, in order to prevent crushing of the concrete in the compression struts, one has two options:

1. Limit the compression stresses developed in the web, f_{cd}, to be less than the effective compressive strength of concrete f_{ce}.
2. Limit the applied shear stress, q, to be less than the maximum ultimate shear stress, q_{umax}.

7.4.4 The Axial (Longitudinal) Force Due to Shear

Figure 7.8 re-examines the equilibrium of a free body of the beam resulted from a vertical cut B-B

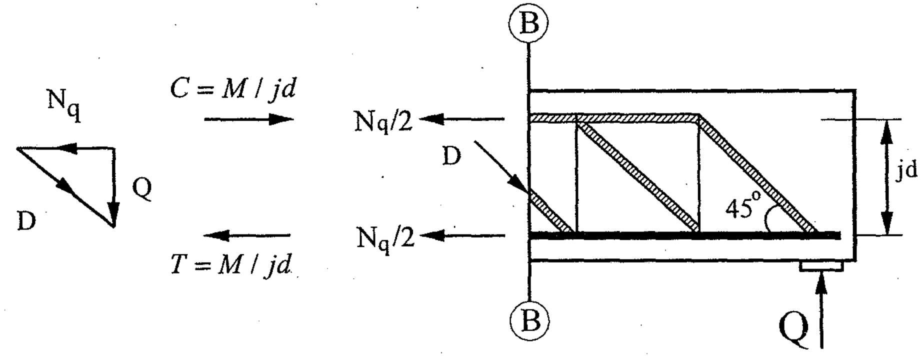


Fig. 7.8 Additional longitudinal force due to shear

As mentioned in Section 7.4.3, the shear force Q at Sec. B-B is resisted by the vertical component of the diagonal compression force D. Force equilibrium indicates that a horizontal tension force N_q must be developed at Sec. B-B. This force is equal to:

$$N_q = \frac{Q}{\tan 45^{o}} \quad \text{...(7.15a)}$$

$$N_q = Q \quad \text{..(7.15b)}$$

Since the shear is assumed uniformly distributed over the depth of the beam, N_q acts at mid-depth and $N_q/2$ will act on both the top and the bottom chords of the truss. These forces will be added to the compression force, C, and the tension force, T, caused by flexure, $C = T = M / jd$. Hence, the forces in the top and bottom chords of the truss at Sec. B-B are as follows:

Compression force in the truss member at Section B-B = $-C + N_q/2$
Tension force in the truss member at Section B-B = $T + N_q/2$

In other wards, the force in the compression chord of the truss will be less than that caused due to the bending moment and the force in the tension chord of the truss will be more that caused due to the bending moment.

7.4.5 Comments on the 45-Degree Truss-Model

1- The 45-degree truss model neglects the shear components Q_{cz}, Q_{ay} and Q_d shown in Fig. (4.4) in Chapter (4). Thus truss models in general, do not assign any shear "to the concrete" and predict that beams without shear reinforcement have zero shear strength.

2- Similar to any conceptual model, there are some simplifications in the truss model. The assumption of 45° angle of inclination of the diagonal cracks along the span is, of course, not correct. However, it gives conservative results in most cases.

7.4.6 Comparison of the Truss Model and ECP 203

1. The ECP 203 uses the truss model Eq. 7.5 to design the stirrups. However, to account for the fact that concrete contributes to the shear strength of the beam, ECP 203 assigns part of the design shear force to be resisted by concrete.

2. The truss model presents Eq. 7.12, that is resulted from the analysis of the forces carried by the diagonals. It indicates that the web of the beam will crush if the inclined compressive stress exceeds the effective compressive strength of concrete. Instead of limiting the compressive stresses in the web, the ECP 203 avoids crushing failure of the web through limiting the shear stresses in the web to an upper limit value ($q_u \le q_{u\max}$).

3. The truss model indicates that there is an additional longitudinal tension force due to shear. This tension force should be added to the tension force resulted from the bending moment when calculating the required longitudinal steel. This fact is partially taken into consideration in the ECP 203 through using the shifted bending moment diagram in detailing the longitudinal reinforcement.

Example 7.1

A simply supported beam of rectangular cross section carries a factored load of 100 kN/m. The distance between the center-lines of the supporting columns is 8.0 m and the width of the support is 0.8 m. The beam is shown below.

Data

$b = 250$ mm

$t = 1000$ mm

$f_{cu} = 25\, N/mm^2$

$f_y = 240\, N/mm^2$ (stirrups), $f_y = 360\, N/mm^2$ (longitudinal steel).

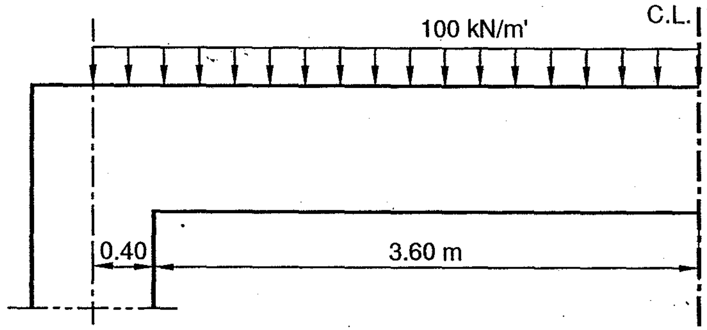
1- Based on reasonable assumptions, draw a possible 45-degree truss that would model the flexure-shear behavior of the beam.

2- Draw the stirrups force diagram.

3- Choose the distribution of stirrups that would result in simultaneous yielding of all vertical members.

4- Draw the tension force diagram obtained from the ordinary flexure theory. On the same drawing, plot the tension force diagram obtained from truss analysis.

5- Is there a possibility of web crushing of this beam (Assume that β_s=0.6).



Simply supported beam subjected to uniform load

Solution

Step 1: Forming the truss model

The development of the 45-degree truss model is shown in Fig. EX 7.1b. The value jd can be reasonably assumed equal to about 0.9d, where d is the effective depth of the beam. Hence, $jd \cong 0.8t = 0.8\,m$. In order to form a 45° truss, one has to lump the stirrups every 0.80 m in one vertical member. Hence, the concentrated load at each joint (P) equals to:

$$P = w \times 0.8 = 100 \times 0.8 = 80\ kN$$

It can be seen that the angle of inclination of all the diagonal compression members located in the clear span of the beam is $45°$. The diagonal member located inside the column transmits the vertical joint load directly to the column support. In spite of the fact that its inclination is not $45°$, the truss is referred to as a $45°$ truss since the inclination of the effective diagonals in the clear span of the beam is $45°$.

Step 2: Drawing the stirrups force diagram

Forces in the vertical members (stirrups)

$$Q = \frac{w \times L}{2} = 100 \times 4 = 400\ kN$$

$$S_1 = Q - P = 400 - 80 = 320\ kN$$

$$S_2 = Q - 2 \times P = 400 - 2 \times 80 = 240\ kN$$

$$S_3 = Q - 3 \times P = 400 - 3 \times 80 = 160\ kN$$

$$S_4 = Q - 4 \times P = 400 - 4 \times 80 = 80\ kN$$

The diagram that shows the variation of the forces in the vertical members (and hence the variation of the forces in the stirrups) is presented in Fig. EX. 7.1c. In this diagram, the force in each vertical member is drawn as a constant number in the tributary length of each vertical member.

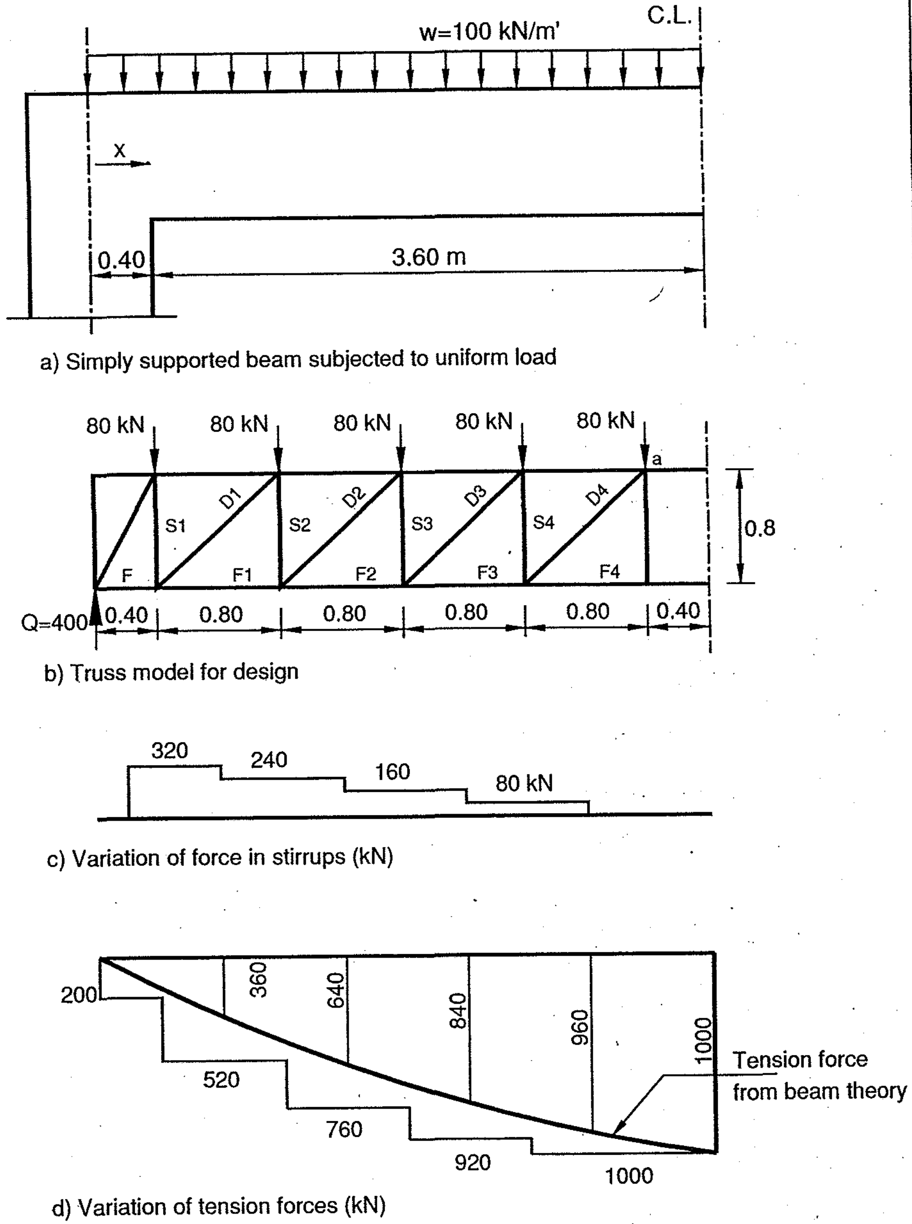


Fig. EX. 7.1 Development of the truss model for design

Step 3: Distribution of the stirrups

The choice of the stirrups that would result in yielding of all vertical members means that one should satisfy the following equation:

$$s = \frac{A_{st}(f_y/\gamma_s)jd}{S_i} = \frac{A_{st} \times 240/1.15 \times 800}{S_i}$$

Member	Force (S_i), kN	Diameter(mm)	A_{st} (mm^2)	$s_{required}$ (mm)	s_{chosen} (mm)
S_1	320	10	157	82	80
S_2	240	10	157	109	105
S_3	160	10	157	164	160
S_4	80	8	100	208	200

Step 4: The tension force diagram

Step 4.1: According to the flexural theory

The bending moment at any point of the beam equals

$$M_i = Q \cdot x - \frac{w \cdot x^2}{2} = 400\,x - 50\,x^2$$

The tension force in the longitudinal steel can be calculated from the flexural theory as follows:

$$T_i = \frac{M_i}{j\,d} = \frac{M_i}{0.8}$$

The calculations are carried out in the following table

x (m)	0.8	1.6	2.4	3.2	4
M_i (kN.m)	288	512	672	768	800
T_i (kN)	360	640	840	960	1000

Step 4.2: According to the truss model

The tension force according to the truss model can be obtained through one of the following two options:

- **Option 1**: Analyzing the forces at the bottom chord of the 45- degree truss.
- **Option 2**: Adding the value of the tension force due to shear to that due to the bending moment.

Option 1

The calculations are carried out by using the method of sections to determine the forces in the bottom chord.

For example, the forces F_4 is calculated as follows

Taking the moment about point (a) gives

$$F_4(0.8) = 400 \times 3.6 - 80 \times (0.8 + 1.6 + 2.4 + 3.2)$$

F_4=1000 kN

Similarly, the rest of the forces in the other members can be obtained as follows:

$F_3(0.8) = 400 \times 2.8 - 80 \times (0.8 + 1.6 + 2.4)$ ....F_3=920 kN

$F_2(0.8) = 400 \times 2.0 - 80 \times (0.8 + 1.6)$ ....F_2=760 kN

$F_1(0.8) = 400 \times 1.2 - 80 \times (0.8)$ ....F_1=520 kN

$F(0.8) = 400 \times 0.4$ ....F=200 kN

Option 2

The axial force developed due to shear at a certain section equals to half the value of the shear force at that section as shown in section 7.4.4. The axial force is calculated at the middle of each member of the bottom chord as follows:

x (m)	0.8	1.6	2.4	3.2
Shear force (Q_i)	320	240	160	80
Longitudinal axial force ($N_{qi}/2$)	160	120	80	40

The value in the tension chord is determined as the sum of the tension obtained from the bending theory and the axial force due to shear

$$F_i = T_i + N_{qi}/2$$

x (m)	0.8	1.6	2.4	3.2
Tension from the bending theory (T_i)	360	640	840	960
Longitudinal axial force (N_{qi} /2)	160	120	80	40
Tension chord force $F_i = T_i + N_{qi}/2$	520	760	920	1000

The variations of the tension force according to the bending theory and to the truss model are shown in Fig. EX. 7.1d. It can be seen that at mid span, where the shear force is equal to zero, the tension force obtained from the bending theory is equal to that obtained from the truss analysis.

Step 5: Check web crushing

To check the possibility of web crushing, one should compare the maximum stresses in the web to the effective concrete strength. The compression force (D_i) in each diagonal member can be obtained through the analysis of the members in the truss. Alternatively, it can be obtained using the following equation:

$$D_i = \frac{Q_i}{\sin 45}$$

where Q_i is the shear force at the center of the panel (or the center of the diagonal).

member	(Q_i), kN	D_i, kN
D_1	320	452.5
D_2	240	339.4
D_3	160	226.3
D_4	80	113.1

The maximum compression force equals to 452.5 kN

$$D = f_{cd}\, b\, (jd \cos 45^\circ)$$

$$452.5 \times 1000 = f_{cd} \times 250 \times (800 \cos 45^\circ)$$

$$f_{cd} = 3.2\, N/mm^2$$

$$f_{ce} = \beta_s \left(0.67 \frac{f_{cu}}{\gamma_c}\right) = 0.6 \times 0.67 \times \frac{25}{1.5} = 6.7\, N/mm^2$$

Since $f_{cd} < f_{ce}$, the beam is considered safe against web crushing.

Note: As shown in Fig. EX. 7.1, the truss analogy predicts that in order to resist shear, beams needs both stirrups and longitudinal reinforcement. This is an important behavioral aspect that can not be noticed using ordinary flexural theory.

Example 7.2

The figure shown below is for a cantilever beam of a rectangular cross section that carries two concentrated loads.

Data

$b = 300$ mm

t = 1100 mm

$f_{cu} = 30\, N/mm^2$

$f_y = 250\, N/mm^2$ (stirrups), $f_y = 360\, N/mm^2$ (longitudinal steel).

It is required to:

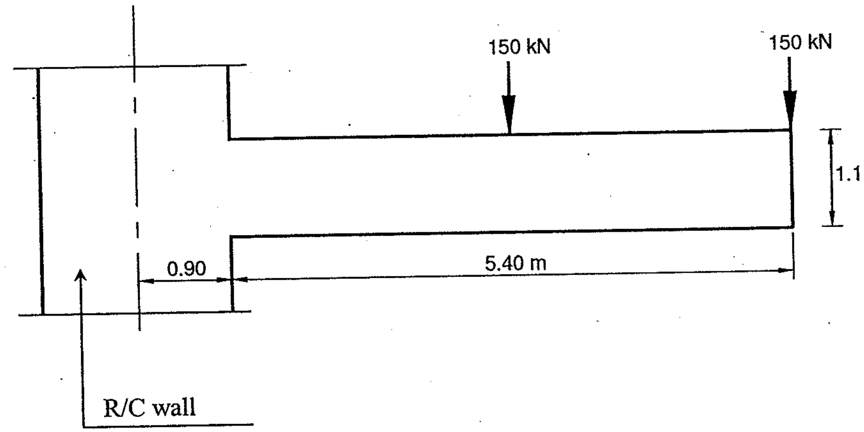
1- Propose a possible 45°-Truss that would model the flexure-shear behavior of the beam.

2- Draw the stirrups force diagram.

3- Choose the distribution of stirrups that would result in simultaneous yielding of all vertical members.

.4- Draw the tension force diagram obtained from the ordinary flexure theory. On the same drawing, plot the tension force diagram obtained from truss analysis.

5- Is there a possibility of web crushing of this beam (Assume β_s=0.6).



Solution

Step 1: Forming the truss model

One of the possible truss models that represent the flexure-shear interaction of the beam is shown in Fig. EX. 7.2. The value jd can be reasonably assumed equal to about 0.9d, where d is the effective depth of the beam. Hence, $jd \cong 0.8t = 0.8 \times 1.1 \cong 0.9\,m$.

The development of the 45-degree truss model is shown in the Fig. EX 7.2b. It can be seen that the angle of inclination of all the diagonal compression members located in the clear span of the beam is 45°. The diagonal member located inside the column transmits the vertical joint load directly to the column support.

Step 2: Drawing the stirrups force diagram

Forces in the vertical members (stirrups)

$Q = 150 + 150 = 300\ kN$

$M = 150 \times 6.3 + 150 \times 3.6 = 1485\ kN$

$S_1 = S_2 = S_3 = 300\ kN$

$S_4 = S_5 = S_6 = 150\ kN$

Step 3: Distribution of the stirrups

The diagram that shows variation of the force in the vertical members (and hence the variation of the force in stirrups) is presented in Fig. EX. 7.2.c. In this diagram, the force in each vertical member is drawn as a constant number in the tributary length of each vertical member. The choice of the stirrups that would result in yielding of all vertical members means that one should satisfy the following equation:

Assume that the stirrups diameter is 10 mm, A_{st}= 157 mm^2 (for two branches)

$$s = \frac{A_{st}(f_y/\gamma_s)jd}{S_i} = \frac{157 \times 250/1.15 \times 900}{S_i}$$

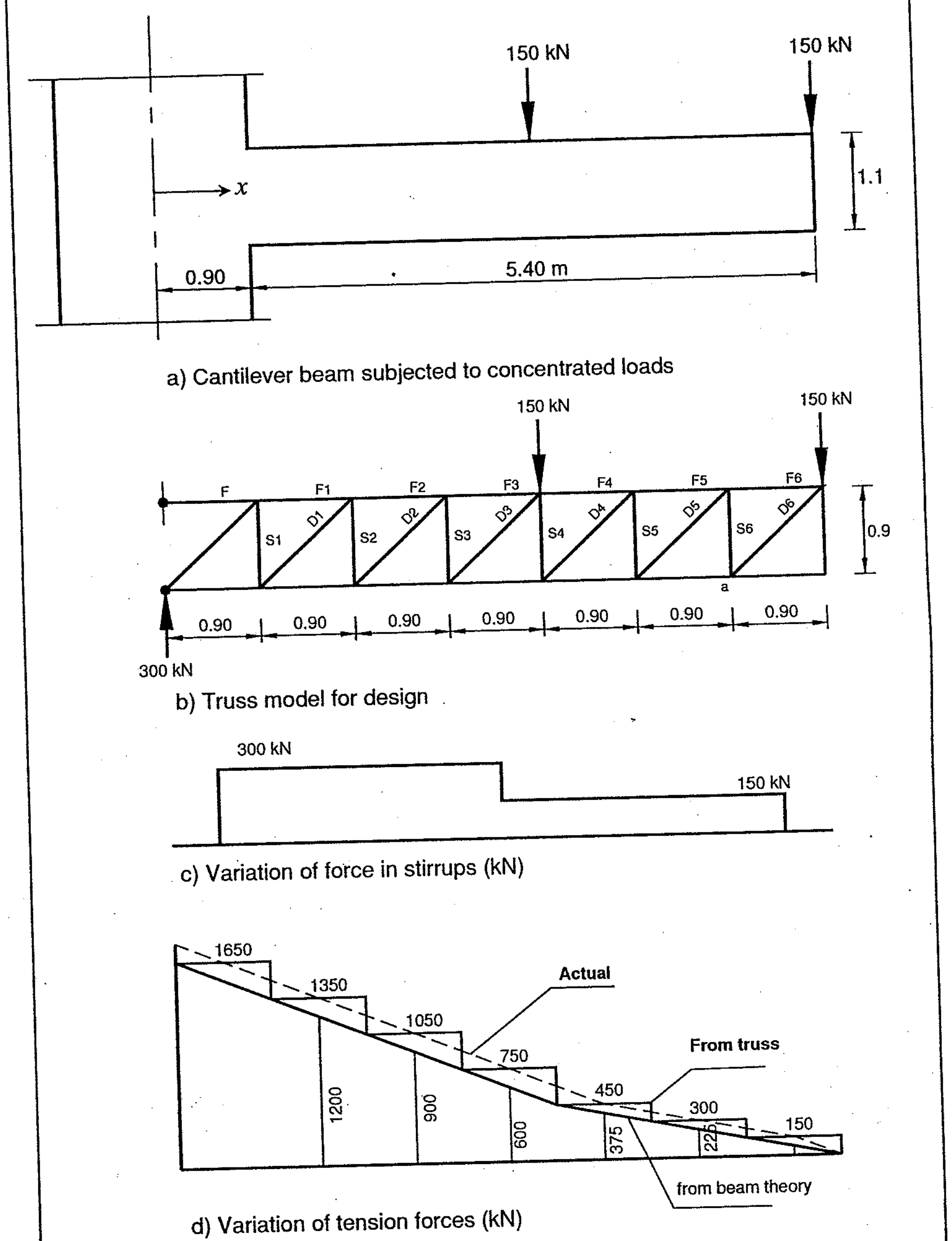


Fig. EX. 7.2 Development of the truss model for design

members	Force (Si), kN	$s_{required}$(mm)	s_{chosen}(mm)
S_1,S_2,S_3	300	102	100 (10ϕ10/m′)
S_4,S_5,S_6	150	204	200 (5ϕ10/m′)

Step 4: The tension force diagram

Step 4.1: According to the flexural theory

The tension force in the longitudinal steel can be calculated from the flexural theory as follows:

$$T_i = \frac{M_i}{j\,d} = \frac{M_i}{0.9}$$

The calculations are carried out in the following table

x (m)	0	0.45	1.35	2.25	3.15	4.05	4.95	5.85
M_i (kN.m)	1485	1350	1080	810	540	337.5	202.5	67.5
T_i (kN)	1650	1500	1200	900	600	375	225	75

Step 4.2: According to the truss model

The tension force due to the truss model can be obtained through one of the following two options:

- **Option 1**: Analyzing the forces at the **top** chord of the 45- degree truss.
- **Option 2**: Adding the value of the tension force due to shear to that due to the bending moment.

Option 1

The calculations are carried out by using the method of sections to determine the forces in the top chord.

For example, the forces F_6 is calculated by taking the moment about point (a):

$F_6(0.9) = 150\times 0.9$ .... F_6=150 kN

Similarly, the force in other members can be calculated as follows:

$F_5(0.9) = 150\times 1.8$ .... F_5=300 kN

$F_4(0.9) = 150\times 2.7$ .... F_4=450 kN

$F_3(0.9) = 150\times 3.6 + 150\times 0.90$ .... F_3=750 kN

$F_2(0.9) = 150\times 4.5 + 150\times 1.8$ .... F_2=1050 kN

$F_1(0.9) = 150\times 5.4 + 150\times 2.7$ .... F_1=1350 kN

$F\,(0.9) = 150\times 6.3 + 150\times 3.6$ .... F=1650 kN

Option 2

The axial force developed at the top chord of the beam due to shear equals half the shear force at that section as shown in section 7.4.4. The axial force is calculated at the middle of each panel as follows:

x (m)	1.35	2.25	3.15	4.05	4.95	5.85
Shear Force	300	300	300	150	150	150
Longitudinal Axial Force (Nq/2)	150	150	150	75	75	75

The value in the tension chord is determined as the sum of the tension obtained from the bending theory and the axial force due to shear

$$F_i = T_i + N_{qi}/2$$

x (m)	1.35	2.25	3.15	4.05	4.95	5.85
Tension from the bending theory (T_i)	1200	900	600	375	225	75
Longitudinal axial force (N_{qi}/2)	150	150	150	75	75	75
Tension chord force $F_i = T_i + N_{qi}/2$	1350	1050	750	450	300	150

Step 5: Check web crushing

To check the possibility of web crushing, one should compare the maximum stresses in the web to the effective concrete strength. The compression force (D_i) in each diagonal member can be obtained through the analysis of the members in the truss. Alternatively, it can be obtained using the following equation:

$$D_i = \frac{Q_i}{\sin 45}$$

where Q_i is the shear force at the center of the panel (or the center of the diagonal).

members	(Q_i), kN	D_i, kN
$D_1=D_2=D_3$	300	424.3
$D_4=D_5=D_6$	150	212.1

The maximum compression force in the diagonal equals 424.3 kN

$$D = f_{cd}\, b\, (jd \cos 45^\circ)$$

$$424.3 \times 1000 = f_{cd} \times 300 \times (900 \cos 45^\circ)$$

$$f_{cd} = 2.2\ N/mm^2$$

$$f_{ce} = \beta_s \left(0.67 \frac{f_{cu}}{\gamma_c}\right) = 0.6 \times 0.67 \times \frac{30}{1.5} = 8.04 N/mm^2$$

Since $f_{cd} < f_{ce}$, the beam is considered safe against web crushing.

Note: As shown in Fig. EX. 7.2, the truss analogy predicts that in order to resist shear, beams needs both stirrups and longitudinal reinforcement. This is an important behavioral aspect that can not be noticed using ordinary flexural theory.

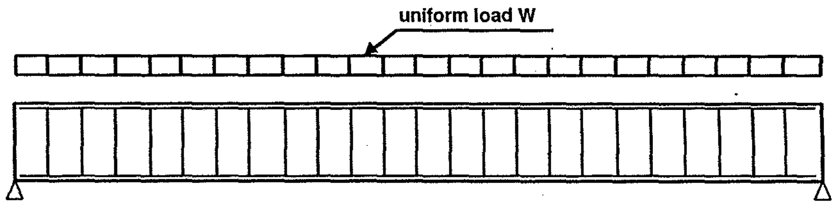
Photo 7.3 Convent of La Tourette, by Le Corbusier, France.

7.6 The Variable-Angle Truss Model

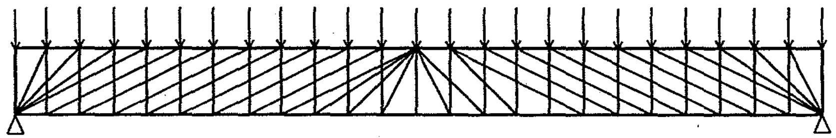
7.6.1 General

In the variable-angle truss model, the angle of inclination of the cracks is not equal to 45°and it might vary in value along the span of the beam.

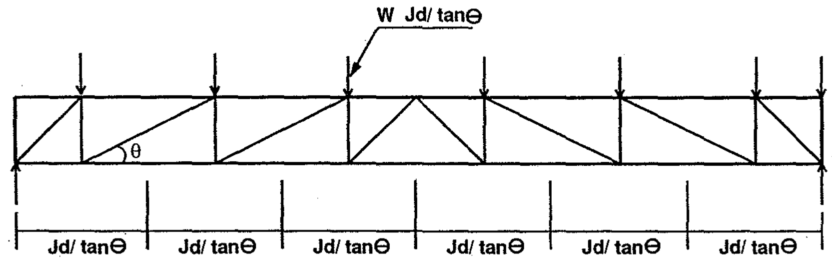
Figure 7.9a shows a beam carrying a uniformly distributed load. The *detailed truss* corresponding to such a beam is shown in Fig.7.9b. In reality, each stirrup represents a vertical truss member. The tensile members are the vertical stirrups and the longitudinal steel at the bottom. The compressive members in the truss are really forces in the concrete, not separate truss members. The truss shown in Fig. 7.9b is statically indeterminate. If all the stirrups yield at failure, the truss is called a *plastic truss model* and it becomes statically determinate since the force in each vertical member will be known (equals to $A_{st} f_y/1.15$). The truss shown in Fig. 7.9b can be simplified for design purposes, *design truss*, as shown in Fig. 7.9c. This simplification is made through assuming a suitable angle of



a) Simply supported beam subjected to uniform load



b) Detailed truss model of beam



c) Truss model used in design

Fig. 7.9 The variable angle truss model

7.6.2 Analysis of the Variable Angle Truss Model

The internal forces of the truss model in which the angle of inclination of the cracks is assumed to be equal θ can be derived in a similar way to that followed in the 45-degree truss model.

Referring to Fig. 7.10, the vertical component of the shear force is resisted by tension forces in the stirrups crossing this section. The horizontal projection of section A-A is $(jd/\tan\theta)$ and the number of stirrups it cuts is $(jd/\tan\theta)/s$. The force in one stirrup is $A_{st}\, f_y / \gamma_s$, which can be calculated from:

$$A_{st}\, f_y / \gamma_s = \frac{Q\,s}{jd/\tan\theta} \quad \text{.................................(7.16)}$$

All the variables in equation (7.16) are previously defined.

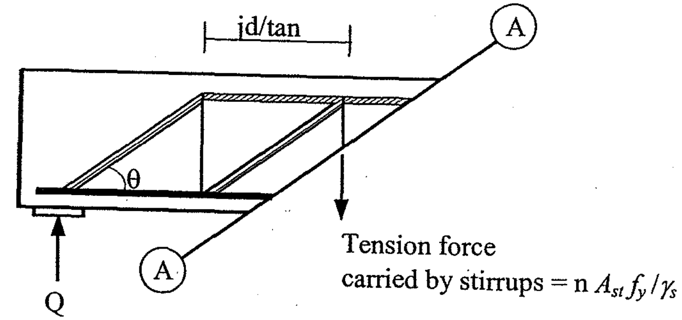


Fig. 7.10 Calculation of forces in stirrups

Figure 7.11 shows that the shear force Q at section B-B is resisted by the vertical components of the diagonal compression force D.

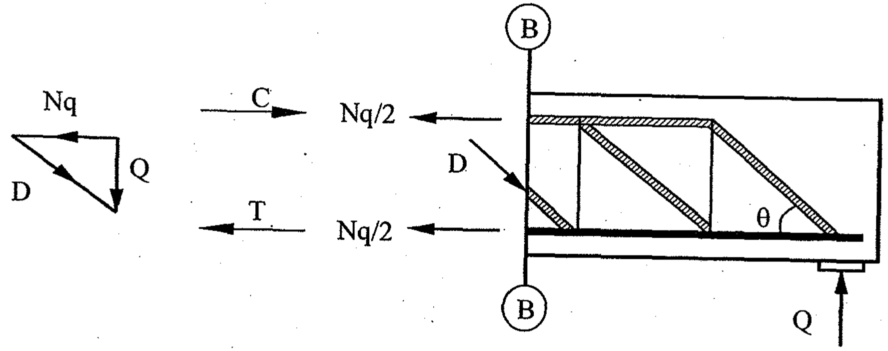


Fig. 7.11 Equilibrium of forces at section B-B.

The diagonal member has a width b and a thickness $jd\cos\theta$. Hence, the total compression force resisted by the diagonal member is equal to the average compression stress is the diagonal direction, f_{cd}, multiplied by the area of the diagonal member. Hence, the diagonal force D can expressed as:

$$D = f_{cd}\; b\; jd \cos\theta \quad \text{.....................................(7.17)}$$

The shear force Q at section B-B and the diagonal compression force D are related by:

$$D = \frac{Q}{\sin\theta} \quad \text{...(7.18)}$$

Hence, Eq. 7.17 can be rewritten as:

$$Q = f_{cd}\; b\; jd \sin\theta\cos\theta \quad \text{..................................(7.19)}$$

Equation 7.19 indicates that the shear strength of a concrete beam reaches its maximum value when the compressive stress in the web reaches the crushing strength of concrete, f_{ce}, no matter how much web reinforcement is provided.

From Fig. 7.11, the shear force Q is resisted by the vertical component of the diagonal compression force D. Force equilibrium indicates that a horizontal tension force N_q results. This force is equal to:

$$N_q = \frac{Q}{\tan\theta} \quad \text{..(7.20)}$$

This force acts a mid-depth of the beam. Since the shear is assumed uniformly distributed over the depth of the beam, N_q acts at mid-height and $N_q/2$ will act on both the top and the bottom chord of the truss. These forces will be added to those caused by flexure.

With the variable angle truss model, the designer can choose any reasonable angle of inclination of the inclined cracks. The value of the angle θ should be in the range $30^\circ \le \theta \le 60^\circ$ in order to ensure satisfactory serviceability performance (to limit the crack width at service load).

The choice of small value of θ reduces the number of stirrups required (see Eq. 7.16) but increases the compression stresses in the web (Eq. 7.19) and increases the tension force N_q. The opposite is true for large angles.

It should be noted that the choice of the crack angle means that the designer "*tells the beam what to do*". This is acceptable for the values mentioned.

Many design codes adopt the variable angle truss model as the basis for shear design. The Canadian Code and the European Code are examples for such codes.

Example 7.3

The figure given below shows a simply supported beam that is monolithically cast with a slab. The beam carries 2 concentrated factored loads each having a value of 300 kN. Data:

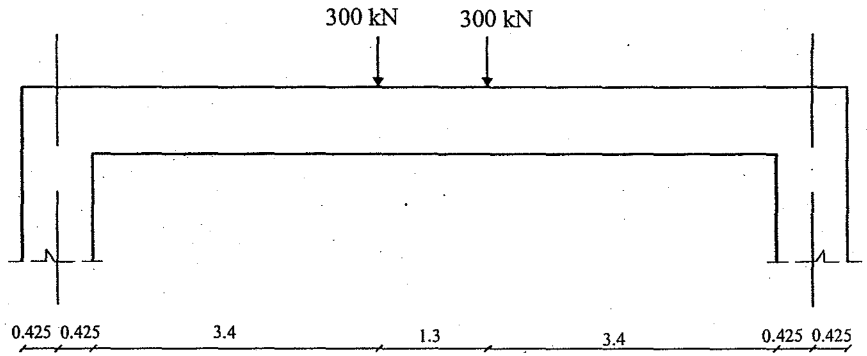
$b = 250$ mm

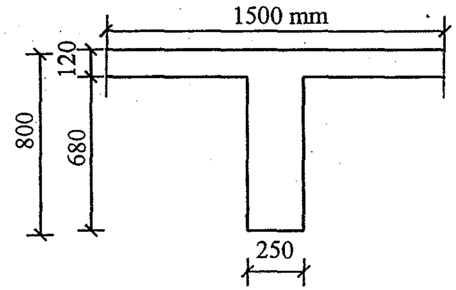
$t = 800$ mm

$f_{cu} = 25\,N/mm^2$

$f_y = 280\,N/mm^2$ (stirrups), $f_y = 360\,N/mm^2$ (longitudinal steel).

1- Based on reasonable assumptions, draw a possible variable angle truss model that would model the flexure-shear behavior of the beam.
2- Draw the stirrups force diagram.
3- Choose the distribution of stirrups that would result in simultaneous yielding of all vertical members.
4- Draw the tension force diagram obtained from the ordinary flexure theory. On the same drawing, plot the tension force diagram obtained from truss analysis.
5- Is there a possibility of web crushing of this beam (Assume that β_s=0.6).





Solution

Step 1: Forming the truss model

The development of the variable angle truss model is shown in the Fig. EX 7.3b. The value jd can be reasonably assumed equal to about 0.9d, where d is the effective depth of the beam. Hence, $jd \cong 0.8t \cong 0.65m$. To form the truss, it is assumed that the vertical members, representing the forces in the stirrups, are arranged every 0.85 m. Such an arrangement gives an angle of inclination of the diagonals in the effective shear span of about 37.4 degrees. This inclination is quite acceptable.

Step 2: Drawing the stirrups force diagram

The forces in the vertical members (stirrups) are all equal to 300 kN as shown in Fig. EX. 7.3c. In this diagram, the force in each vertical member is drawn as a constant number in the tributary length of each vertical member.

Step 3: Distribution of the stirrups

The choice of the stirrups that would result in yielding of all vertical members means that one should satisfy the following equation:

$$s = \frac{A_{st}(f_y/\gamma_s)jd/\tan\theta}{S_i} = \frac{A_{st}\times 280/1.15\times 650/\tan 37.4}{S_i}$$

Member	Force (S_i), kN	Diameter	A_{st} (mm^2)	s $_{required}$ (mm)	s $_{chosen}$ (mm)
S_1	300	10	157	108	100
S_2	300	10	157	108	100
S_3	300	10	157	108	100
S_4	300	10	157	108	100
S_5	300	10	157	108	100

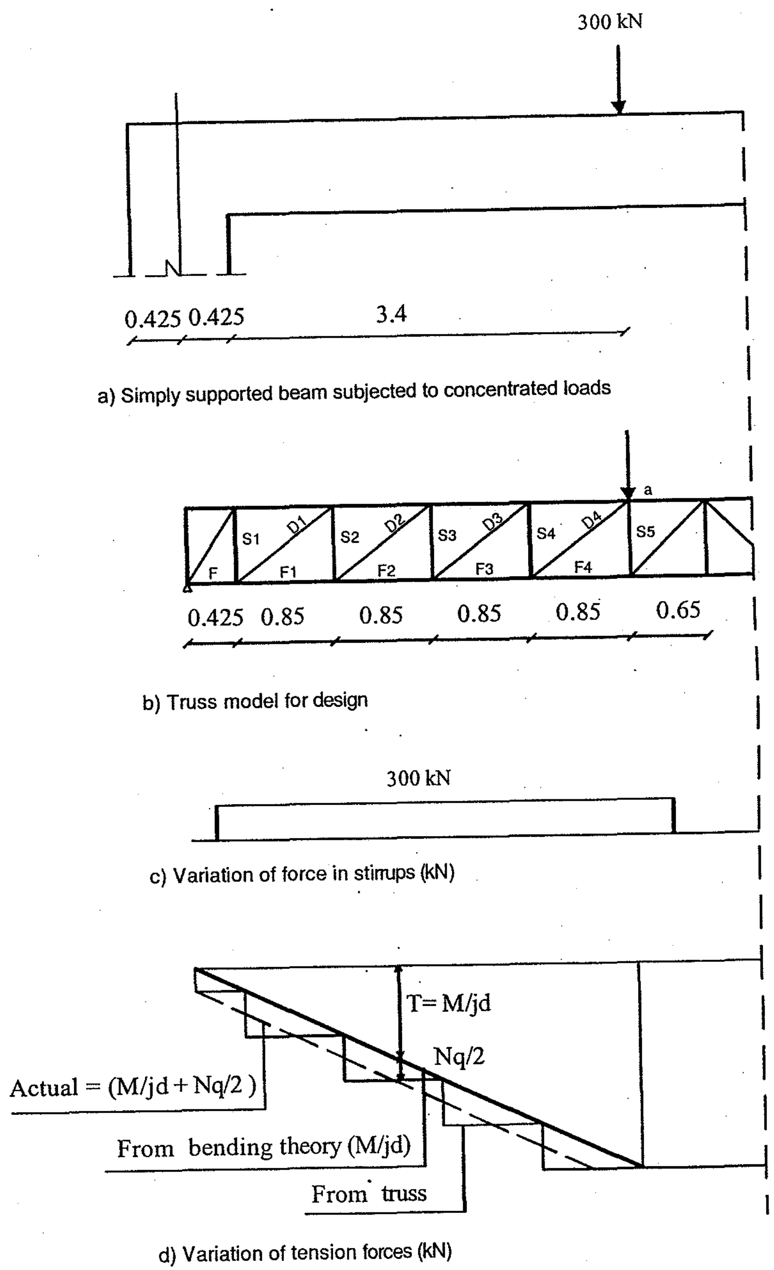


Fig. EX. 7.3 Development of the truss model for design

Step 4: The tension force diagram

Step 4.1: According to the flexural theory

The bending moment at any point of the beam equals

$M_i = Q \cdot x = 300\ x$ .... (x<3.825)

$M_i = 300 \times 3.825 = 1147.5$ .... (x>3.825)

The tension force in the longitudinal steel at the middle of each member can be calculated from the flexural theory as follows:

$$T_i = \frac{M_i}{j\ d} = \frac{M_i}{0.65}$$

The calculations are carried out in the following table

X (m)	0.85	1.7	2.55	3.4	4.15
M_i (kN.m)	255.0	510.0	765.0	1020.0	1147.5
T_i (kN)	392.3	784.6	1176.9	1569.2	1765.4

Step 4.2: According to the truss model

The tension force due to the truss model can be obtained through one of the following two options:

- **Option 1**: Analyzing the forces at the bottom chord of the truss.
- **Option 2**: Adding the value of the tension force due to shear to that due to the bending moment.

Option 1

The calculations are carried out by using the method of sections to determine the forces in the bottom chord.

For example, the force F_4 is calculated by taking the moment about point (a):

$F_4(0.65) = 300 \times 3.825$

F_4=1765.4 kN

Similarly, the rest of the forces can be obtained as follows:

$F_3(0.65) = 300 \times 2.975$ .... F_3=1373.1 kN

$F_2(0.65) = 300 \times 2.125$ .... F_2=980.8 kN

$F_1(0.65) = 300 \times 1.275$ .... F_1=588.5 kN

$F(0.65) = 300 \times 0.425$ .... F=196.2 kN

Option 2

The axial force developed due to shear at a certain section equals to half the value of the shear force at that section as shown in section 7.4.4. The axial force is calculated at the middle of each member of the bottom chord as follows:

$$N_q / 2 = \frac{Q/2}{\tan\theta} = \frac{Q/2}{\tan 37.4}$$

x (m)	0.85	1.7	2.55	3.4
Shear Force	300	300	300	300
Longitudinal Axial Force (Nq/2)	196.2	196.2	196.2	196.2

The value in the tension chord is determined as the sum of the tension obtained from the bending theory and the axial force due to shear

$F_i = T_i + N_{qi}/2$

x (m)	0.85	1.7	2.55	3.4
Tension from the bending theory (T_i)	392.3	784.6	1176.9	1569.2
Longitudinal Axial Force ($N_{qi}/2$)	196.2	196.2	196.2	196.2
Tension chord force $F_i = T_i + N_{qi}/2$	588.5	980.8	1373.1	1765.4

The variations of the tension force according to the bending theory and the truss model are shown in Fig. EX. 7.3d.

Step 5: Check web crushing

To check the possibility of web crushing, one should compare the maximum stresses in the web to the effective concrete strength. The compression force (D_i) in each diagonal member can be obtained through the analysis of the members in the truss. Alternatively, it can be obtained using the following equation:

$$D = \frac{Q}{\sin\theta} = \frac{300}{\sin 37.4} = 493.9\ kN$$

where Q is the shear force at the center of the panel (or the center of the diagonal).
The maximum compression force in the diagonal equals 493.9 kN

$$D = f_{cd}\, b\ \left(jd\cos 37.4°\right)$$

$$493.9 \times 1000 = f_{cd} \times 250 \times \left(650\ \cos 37.4°\right)$$

$$f_{cd} = 3.82\ N/mm^2$$

$$f_{ce} = \beta_s\left(0.67\frac{f_{cu}}{\gamma_c}\right) = 0.6 \times 0.67 \times \frac{30}{1.5} = 8.04\ N/mm^2$$

Since $f_{cd} < f_{ce}$, the beam is considered safe against web crushing.

It can be seen that the force in the tension chord halfway between the truss joints is larger than ($T=M/jd$) by the amount ($N_q/2$) as shown by the dashed line. It should be mentioned that the increase in the tension force due to shear is equivalent to computing the tension force from bending moment diagram that is shifted away from the point of maximum moment by an amount $jd/(2\times\tan\theta)$

8

DESIGN FOR TORSION

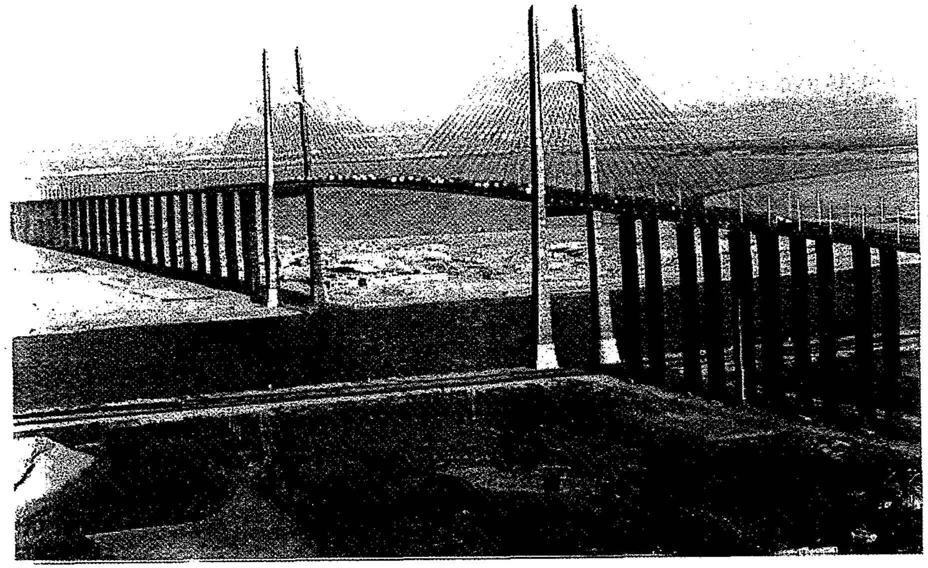


Photo 8.1 The bridge over Suez Canal

8.1 Introduction

Curved bridge girders, edge beams of slabs and shells, spiral stair-cases, and eccentrically loaded box beams constitute examples for members subjected to high twisting moments accompanied by bending moments and shear forces.

Torsion design provisions in the Egyptian Code have gone through major changes based on the results of many researches. This Chapter explains the causes of torsion in reinforced concrete members, introduces the Space Truss Model for torsion and presents the ECP 203 torsion design provisions. It also includes many examples that illustrate the application of the ECP 203 torsion design procedure.

8.2 Equilibrium Torsion and Compatibility Torsion

8.2.1 General

Torsional loading can be separated into two basic categories:

equilibrium torsion where the torsional moment is required for equilibrium of the structure,

compatibility torsion where the torsional moment results from the compatibility of deformations between members meeting at a joint.

8.2.2 Equilibrium Torsion

Figure (8.1a) shows a cantilever beam supporting an eccentrically applied load P at point C, which causes torsion. This torsion must be resisted by beam AB to remain in equilibrium. If the applied torsion is not resisted, the beam will rotate about its axis until the structure collapses.

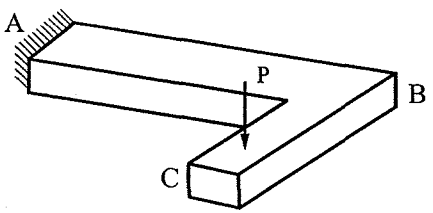


Fig. 8.1a Cantilever beam subjected to equilibrium

Similarly, the canopy shown in Fig. (8.1b) applies a torsional moment to the beam AB. The beam has to be designed to resist the total external factored torsional moment due to the cantilever slab. Otherwise, the structure will collapse. Failure is caused by the beam not satisfying condition of equilibrium of forces and moments resulting from the large external torque.

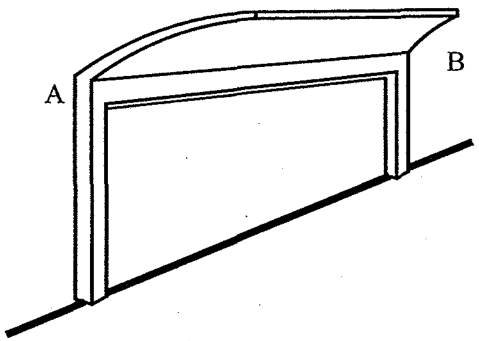


Fig. 8.1b Cantilever canopy

Figure 8.1c shows the girder of a box-girder bridge, in which the truck loading causes torsional moments. Figure 8.1d shows a precast L-shaped beam that supports a system of concentrated loads that result in torsional moments.

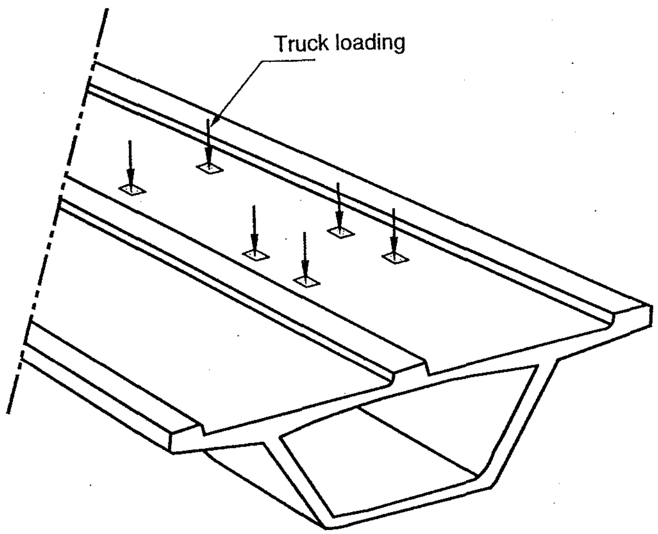


Fig.8.1c Box-girder bridge

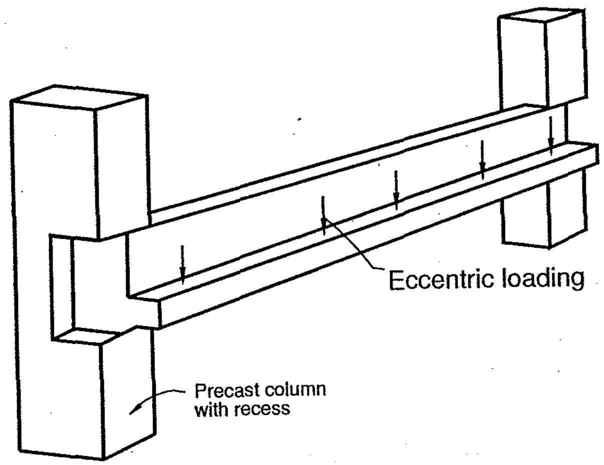


Fig.8.d L-shaped girder

8.2.3 Compatibility Torsion

Figure 8.2 shows an example of compatibility torsion. The beam AB has an end "A" that is built monolithically with beam CD. Beam AB can develop a –ve moment at the end "A" only if beam CD can resist the resulting torsional moment. From compatibility of deformations at joint A, the negative bending moment (M_A) at the end A of beam AB is essentially equal to the torsional moment acting on beam CD. The magnitude of these moments depends on the relative magnitudes of the torsional stiffness of CD and the flexural stiffness of AB. The moment M_A and the torsion M_t result from the need for the end slope of beam AB at "A" to be compatible with the angle of twist of beam CD at point "A". When a beam cracks in torsion, its torsional stiffness drops significantly. Hence, the torsion, M_t, drops and accordingly M_A.

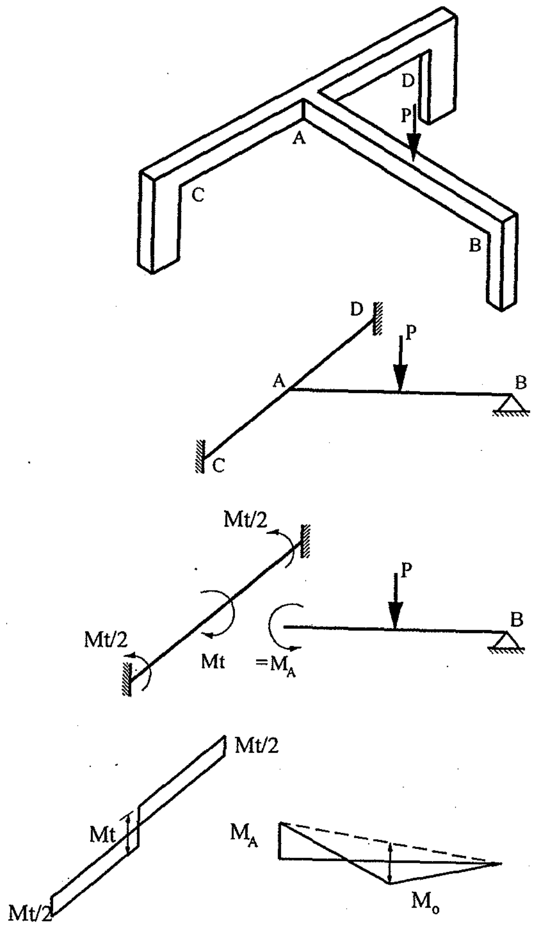
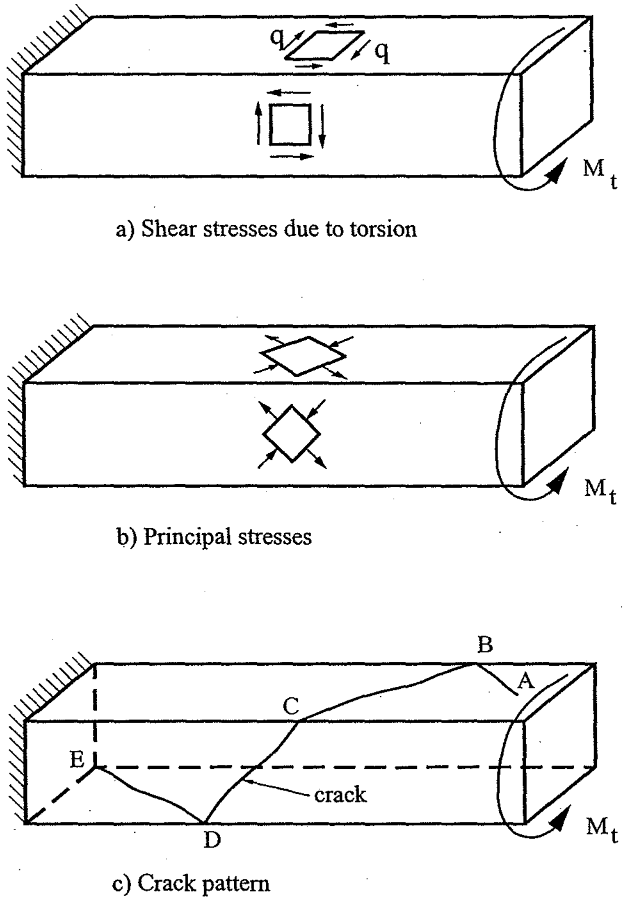


Fig. 8.2 Compatibility torsion

8.3 Principal Stresses due to Torsion

When a beam is subjected to torsion, M_t, shearing stresses are developed on the top and front faces as shown by the elements in Fig (8.3a). The principal stresses on these elements are shown in Fig. (8.3b). The principal tensile stress equal the principal compressive stress and both are equal to the shear stress if M_t is the only loading. The principal tensile stresses eventually cause cracking which spirals around the beam as shown by the line A-B-C-D-E in Fig. (8.3c). Such a crack would cause failure unless it was crossed by reinforcement. This generally takes the form of longitudinal bars in the corners and stirrups. Since the crack spirals around the body, four sided (closed) stirrups are required.



a) Shear stresses due to torsion

b) Principal stresses

c) Crack pattern

Fig. 8.3 Principle stresses and cracking due to torsion

8.4 Thin-Walled Tube in Torsion

As background information to the Egyptian Code torsion provisions, the behavior of thin-walled tubes in torsion will be reviewed. Figure (8.4) shows a thin-walled tube subjected to pure torsion. The only stress component in the wall is the in-plane shear stress, which exhibits as a circulating shear flow F on the cross section. The shear flow F is the resultant of the shear stresses, q, in the wall thickness and is located on the dotted loop. The dotted loop is defined as the centerline of the shear flow.

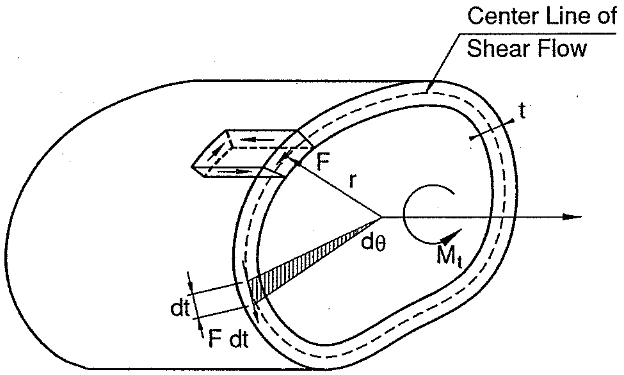


Fig. 8.4 Thin-walled tube subjected to torsion

The relationship between the torque, M_t, and the shear flow, F, can be derived from the equilibrium of moments about the longitudinal axis of the member as follows:

$$M_t = F \oint r \, dt \quad \text{.......(8.1)}$$

It can be seen that the integral is equal to twice the area of shaded triangular.

$$\text{Thus } \oint r \, dt = 2A_o \quad \text{.......(8.2)}$$

$$F = \frac{M_t}{2A_o} \quad \text{.......(8.3)}$$

where A_o is the cross-sectional area bounded by the centerline of the shear flow. The parameter A_o is a measure of the lever arm of the circulating shear flow and will be called *"the lever arm area"*.

For example, in case of a circular cross-section the integral equals

$$2A_o = \oint r \, dt = \int_0^{2\pi} r \, dt = \int_0^{2\pi} r\,(r\,d\theta) = 2\pi r^2 \quad \text{.......(8.4)}$$

The shear stress due to torsion, q, is:

$$q = \frac{M_t}{2\,A_o\,t} \quad \text{.......(8.5)}$$

where t is the thickness of the thin walled tube.

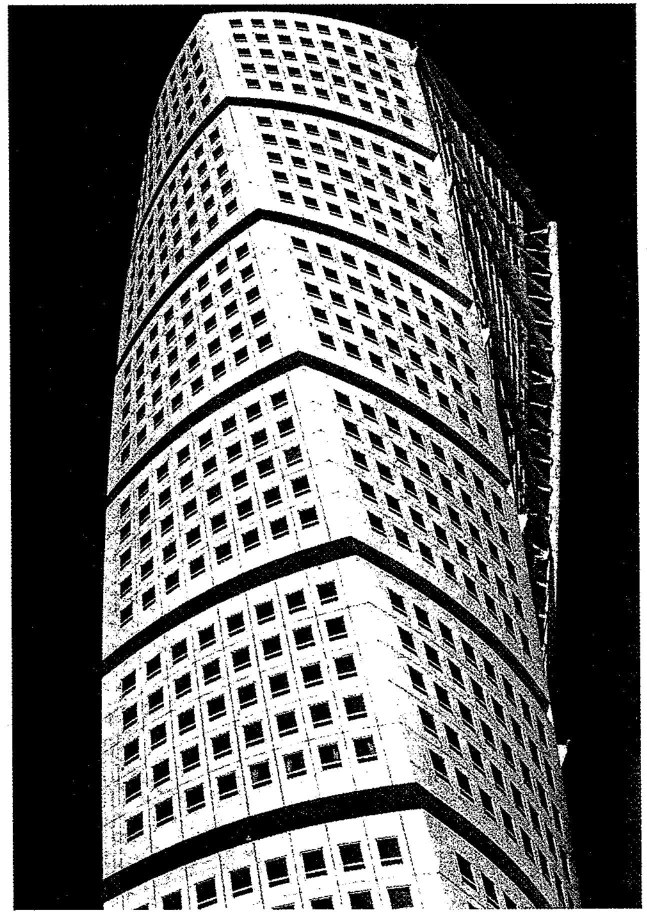


Photo 8.2 HSB Turning Torso 190 metres623 feet 57 stories Completed 2005 tallest building of Sweden

8.5 Space-Truss Model for Torsion

8.5.1 Components of the Space Truss

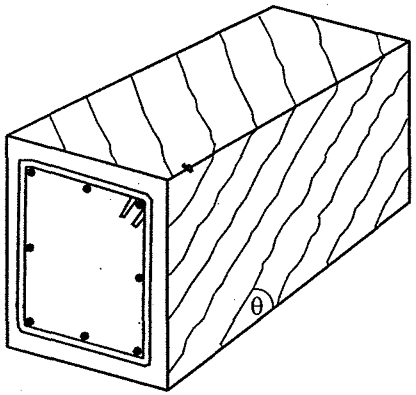
Test results indicated that the torsional strength of a solid reinforced concrete section is very similar to the torsional strength of a hollow section that has the same overall dimensions as long as the thickness of the hollow section is not less than a certain limit. This conclusion led to the development of what so called the *"Space Truss Model"* that explains the behavior of reinforced concrete beams subjected to torsion.

The space truss theory considers all sections, hollow or solid, to be hollow sections. It assumes that the reinforced concrete beam behaves in torsion similar to a thin-walled box with a constant shear flow in the wall cross section. This theory forms the basis of the torsion provisions in the ECP 203.

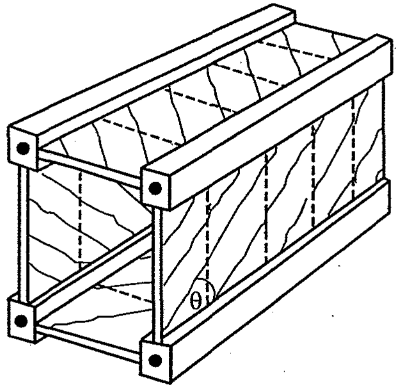
When subjected to torsion, a cracked reinforced concrete beam as the one shown in Fig. (8.5a) can be idealized as shown in Fig. (8.5b). The cracked beam resists the applied torsional moment through acting as a space-truss as shown in Fig. 8.6. The space truss consists of:

- Longitudinal reinforcement concentrated at the corners.
- Closed stirrups
- Diagonal concrete compression members between the cracks which spiral around the beam.

The angle of the inclination of the compression diagonals with respect to the beam axis, θ, depends on the ratio of the force carried by the longitudinal reinforcement to that carried by the stirrups.



a) Section of the actual beam



b) Idealized section of the truss

Fig. 8.5 Idealized cross-section for torsion

The height and the width of the truss are y_1 and x_1, respectively, and are defined as the shorter and the longer center-to-center dimensions of the closed stirrups. The shear flow in the idealized cross section of the truss is given by Eq. 8.3. The total shear force acting on each wall due to torsion is equal to the shear flow (shear force per unit length) times the length of the wall. Hence, the total shear force along each of the top and bottom walls is given by:

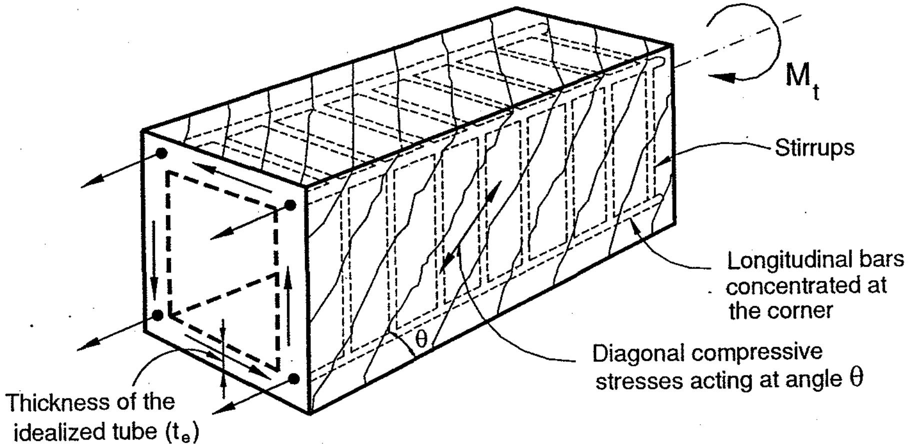
$$Q_1 = F\ x_1 \qquad (8.6)$$

Substituting for the value of F from Eq. 8.3, one gets:

$$Q_1 = \frac{M_t}{2A_o} x_1 \qquad (8.7)$$

Similarly, the shear force along each side wall due to torsion is:

$$Q_2 = \frac{M_t}{2A_o} y_1 \qquad (8.8)$$



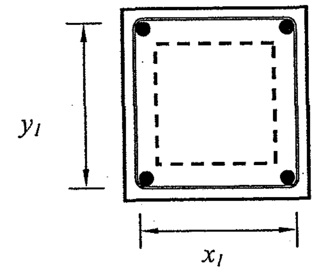


Fig. 8.6 Space truss model for torsion

8.5.2 Diagonal Compressive Stresses

Figure (8.7) shows one of the four walls of the space truss. The shear force Q_2 has been resolved into a diagonal compressive force $D_2 = Q_2/\sin\theta$, parallel to the concrete diagonals, and an axial tensile force N_2. The force D_2 is resisted by inclined compressive stresses, f_{cd}, acting on the compression struts between the cracks.

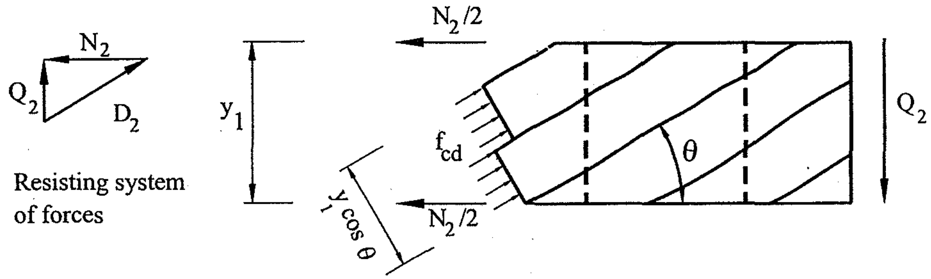


Fig. 8.7 Wall of the space truss-diagonal compressive stresses

The thickness and the width on which D_2 acts are $(y_1 \cos\theta)$ and t_e respectively. The resulting compressive stress is:

$$f_{cd} = \frac{D_2}{y_1 \cos\theta \times t_e} \quad \text{.......... (8.9a)}$$

$$f_{cd} = \frac{Q_2/\sin\theta}{(y_1 \cos\theta) \times t_e} \quad \text{.......... (8.9b)}$$

$$f_{cd} = \frac{Q_2}{t_e \; y_1 \sin\theta \cos\theta} \quad \text{.......... (8.10)}$$

Substation of Eq. 8.8 in Eq. 8.11

$$f_{cd} = \frac{M_t}{2 A_o t_e \sin\theta \cos\theta} \quad \text{.......... (8.11)}$$

Similar stresses act in all four walls. These stresses must not exceed the crushing strength of concrete in the diagonals, f_{ce}. As mentioned in Chapter (7), the crushing strength of the cracked concrete in the struts is less than the cube or the cylinder compressive strength of concrete (See Fig.8.8). The major reason of that is the existence of a transverse tensile strain. The reduction of the compressive strength of cracked concrete is sometimes referred to as "*concrete softening in compression*".

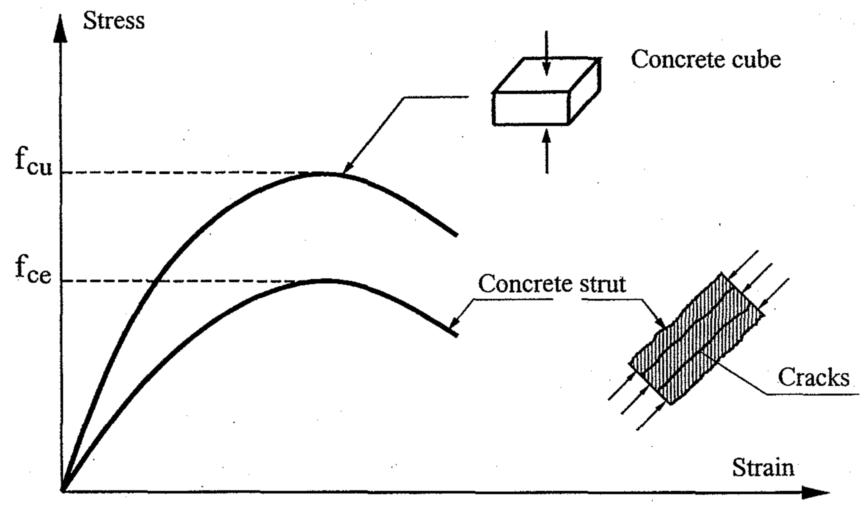


Fig. 8.8 Reduction of compressive strength of the strut

Subsisting of Eq. (8.5) into Eq. (8.11), one gets:

$$f_{cd} = \frac{q}{\sin\theta \cos\theta} \quad \text{.......... (8.12)}$$

Equation (8.12) indicates that there is a relationship between the shear stress due to torsion and the compressive stresses in the concrete diagonals. Hence, limiting the compressive stresses in the wall can be achieved by limiting the shear stresses that is resulted from torsion.

8.5.3 Forces in Stirrups

A section cut along the crack is shown in Fig. (8.9). The crack intercepts n_2 stirrups, where $n_2 = y_1 \cot\theta / s$. These stirrups must equilibrate the force Q_2. Thus, assuming that all the stirrups yield,

$$n_2\ A_{str}\ f_{yst}/\gamma_s = Q_2 \quad \text{.......................................(8.13)}$$

Substituting with the value of n_2 in Eq.8.13 gives

$$\frac{(A_{str}\, f_{yst}/\gamma_s)\ y_1 \cot\theta}{s} = Q_2 \quad \text{.................................(8.14)}$$

Replacing the value of Q_2 given by Eq. (8.8) in Eq. (8.14) gives:

$$A_{str} = \frac{M_t\, s}{2 A_o\, f_{yst}/\gamma_s\ \cot\theta} \quad \text{......................................(8.15)}$$

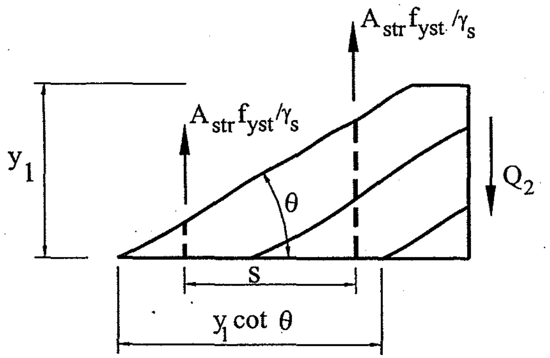


Fig. 8.9 Forces in stirrups

8.5.4 Longitudinal Force

The shear force Q_2 has been resolved into a diagonal compressive force D_2 and, for equilibrium, a tensile force $N_2 = Q_2 \cot\theta$. Because the shear flow is constant along side 2, the force N_2 acts through the centroid of the side. Half of the tension, $N_2/2$, is resisted by each of the longitudinal bars at the top and the bottom of the side wall. A similar resolution of forces occurs in each wall of the space truss. The total longitudinal tension force equals:

$$N = 2(N_1 + N_2) \quad \text{...(8.16)}$$

$$N = 2\,(Q_1 \cot\theta + Q_2 \cot\theta) \quad \text{..............................(8.17)}$$

Subsisting with the value of Q_1 and Q_2 from Eqs. 8.7 and 8.8 gives

$$N = 2\,\left(\frac{M_t\, x_1}{2\, A_o} \cot\theta + \frac{M_t\, y_1}{2\, A_o} \cot\theta\right) \quad \text{..................(8.18)}$$

$$N = \frac{M_t \cot\theta}{2\, A_o}\, 2\,(x_1 + y_1) \quad \text{.................................(8.19)}$$

$$N = \frac{M_t}{2 A_o} P_h \cot\theta \quad \text{...(8.20)}$$

where P_h is the perimeter of the tube; $P_h = 2(x_1 + y_1)$

Longitudinal reinforcement must be provided to resist the entire tension force N. If it is assumed that this steel yields at failure, the required area of longitudinal steel is given by:

$$N = A_{sl} \times \frac{f_y}{\gamma_s} \quad \text{...(8.21)}$$

$$A_{sl} = \frac{M_t\, P_h \cot\theta}{2 A_o f_y/\gamma_s} \quad \text{..(8.22)}$$

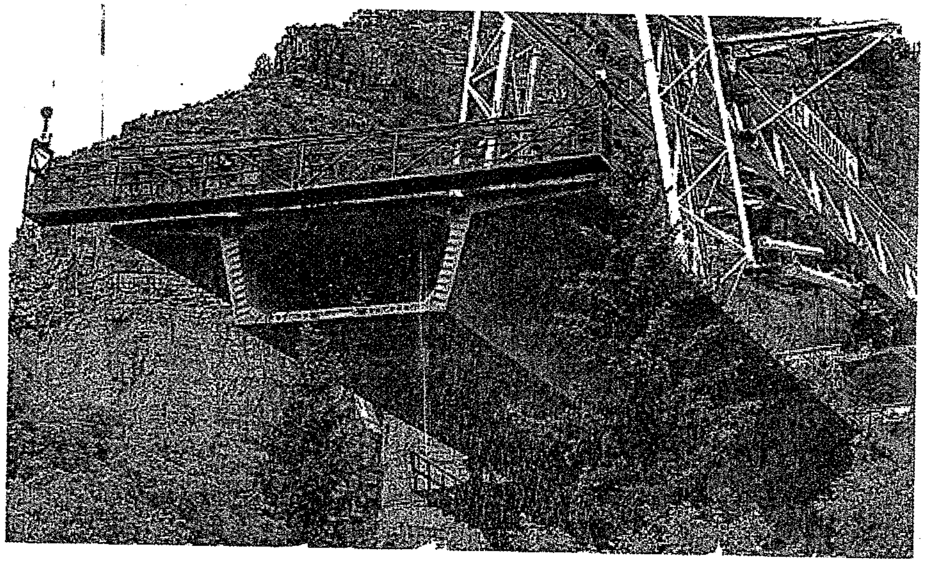


Photo 8.3 A box girder bridge during construction

8.6 The Design for Torsion in the Egyptian Code

8.6.1 General

The ECP 203 torsion design procedure is based on the space truss model with some simplifying assumptions. These assumptions are summarized as follows:

- The angle of inclination of the compression diagonals (which is the angle of inclination of the cracks) is set equal to 45°.
- Simple, but reasonably accurate, expressions are given for calculating the thickness of the walls of the truss model, t_e, and the area enclosed by the shear flow, A_o.
- A limiting value for the allowed shear stresses developed due to torsion is given to ensure prevention of crushing failure of concrete in the struts.

In the ECP 203 torsion design procedure, the following three strength criteria are considered:

- First, a limitation on the shear stress developed due torsion is established such that the stirrups and the longitudinal reinforcement will yield before the crushing of the concrete struts.
- Second, closed stirrups are provided to resist the applied torsional moment.
- Third, the longitudinal steel distributed around the perimeter of the stirrups should be adequate to resist the longitudinal force due to torsion.

8.6.2 Calculation of the Shear Stress due to Torsion

The ECP 203 uses Eq. 8.5, derived for a thin-walled section, to predict the shear stress due to torsion in hollow as well as in solid sections.

The ultimate shear stress developed due to the ultimate torque is given by:

$$q_{tu} = \frac{M_{tu}}{2\,A_o\,t_e} \le q_{tu\,max} \qquad (8.23)$$

For simplicity, the following expressions are suggested by the code for the area enclosed by the shear flow path, A_o, and the equivalent thickness of the shear flow zone, t_e:

$$A_o = 0.85 A_{oh} \qquad (8.24)$$

$$t_e = A_{oh}/P_h \qquad (8.25)$$

where

A_{oh} is the gross area bounded by the centerline of the outermost closed stirrups.

P_h is equal to the perimeter of the stirrups.

The area A_{oh} is shown in Fig. (8.10) for cross-sections of various shapes.

For hollow sections, the actual thickness of the walls of the section should be used in Eq. 8.23 if it is less than t_e .

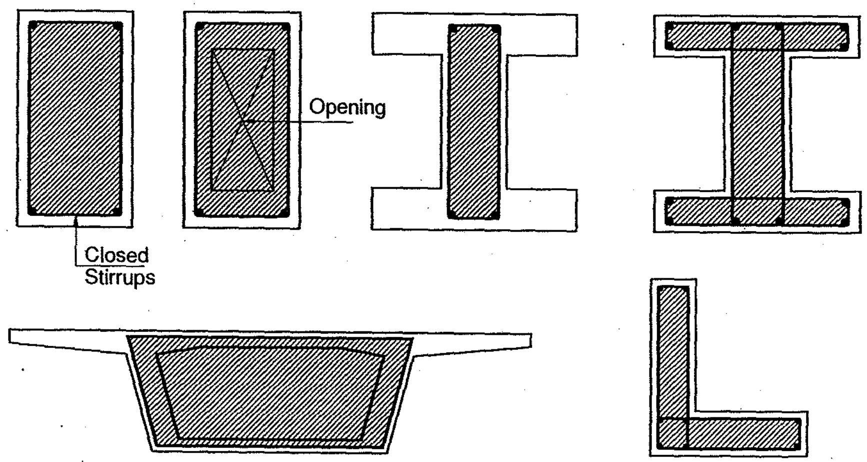


Fig. 8.10 Definition of A_{oh}

8.6.3 Consideration of Torsion

The Egyptian code ECP 203 requires that torsional moments should be considered in design if the factored torsional stresses calculated from Eq. (8.23) exceed $q_{tu\,min}$, given by:

$$q_{tu\,min} = 0.06\sqrt{\frac{f_{cu}}{\gamma_c}} \quad \text{.......(8.26)}$$

8.6.4 Adequacy of the Concrete Cross-Section

The concrete compression diagonals carry the diagonal forces necessary for the equilibrium of the space truss model explained in Section 8.5. Preventing crushing failure of the compression diagonals can be achieved either by limiting the compressive stresses in the concrete struts or by limiting the maximum shear stress. The ECP 203 limits the shear stress calculated by Eq. (8.23) to the value given by:

$$q_{tu\,max} = 0.7\sqrt{\frac{f_{cu}}{\gamma_c}} \le 4.0\ N/mm^2 \quad \text{.......(8.27)}$$

Otherwise, the concrete dimensions of the cross-section must be increased.

8.6.5 Design of Torsional Reinforcement

8.6.5.1 Closed Stirrups

The ECP 203 uses Eq. (8.15) derived from the space truss model with the angle θ set equal to 45°. Hence, the area of one branch of closed stirrups A_{str} is given by:

$$A_{str} = \frac{M_{tu} \cdot s}{2A_o\left(\frac{f_{yst}}{\gamma_s}\right)} \quad \text{.......(8.28)}$$

In case of rectangular sections, Eq. (8.28) takes the form:

$$A_{str} = \frac{M_{tu} \cdot s}{1.7\,(x_1 \cdot y_1)\left(\frac{f_{yst}}{\gamma_s}\right)} \quad \text{.......(8.29)}$$

where x_1 and y_1 are the shorter and the longer center-to-center dimensions of closed stirrups.

8.6.5.2 Longitudinal Reinforcement

The ECP 203 uses Eq. (8.22) derived from the space truss model with the angle θ set equal to 45°. Hence, the area of longitudinal reinforcement A_{sl} is given by:

$$A_{sl} = \frac{M_{tu}\,P_h}{2A_o\,\frac{f_y}{\gamma_s}} \quad \text{.......(8.30)}$$

Substituting the value of M_{tu} from Eq. (8.28), the area of the longitudinal reinforcement can be expressed in terms of A_{str} as follows:

$$A_{sl} = \frac{A_{str}\,P_h}{s}\,\frac{f_{yst}}{f_y} \quad \text{.......(8.31)}$$

where f_y and f_{yst} are the yield strength of the longitudinal reinforcement and the yield strength of the stirrups, respectively.

The area of the longitudinal reinforcement should not be less than:

$$A_{sl\,min} = \frac{0.4\sqrt{\frac{f_{cu}}{\gamma_c}}\,A_{cp}}{f_y/\gamma_s} - \left(\frac{A_{str}}{s}\right)P_h\left(\frac{f_{yst}}{f_y}\right) \quad \text{.......(8.32)}$$

where A_{cp} is the area enclosed by outside perimeter of the section including area of openings.

In the previous equation $\frac{A_{str}}{s}$ should not be less than $\frac{b}{6 \times f_{yst}}$

8.6.6 Code Requirements for Reinforcement Arrangement

The Egyptian Code sets the following requirements with respect to arrangements and detailing of reinforcement for torsion:

1- Stirrups must be closely spaced with maximum spacing (s) such that
$s \le 200\ mm$
$\le \frac{x_1 + y_1}{4}$ which is less

2- Only the outer two legs are proportional for torsion plus shear, and the interior legs are proportional for vertical shear only.

3- Stirrups proportioned for torsion must be closed as shown in Fig. (8.11).

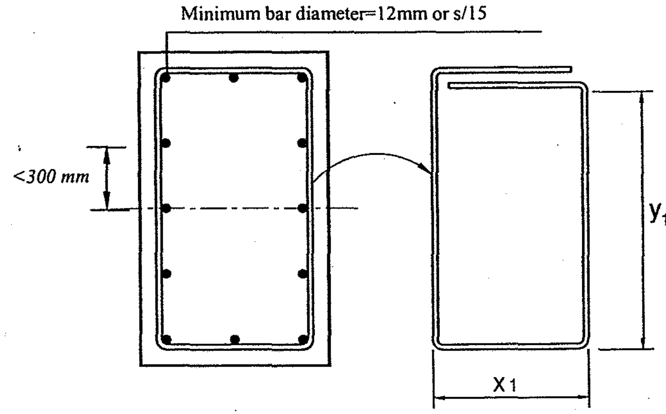


Fig. 8.11 Torsion stirrup and longitudinal reinforcement details

4- For box sections, transversal and longitudinal reinforcement arranged along the outside and the inside perimeter of the section may be considered effective in resisting torsion provided that the wall thickness t_w is less or equal to b/6 where b is the shorter side length of the section. If the wall thickness is thicker, torsion shall be resisted by reinforcement arranged along the outside perimeter only.

5- It is permitted to neglect the effective part of the slab in T and L sections when calculating the nominal shear stresses due to torsion.

6- The spacing of the longitudinal bars should not exceed 300 mm and they should be uniformly distributed along the perimeter. At least one bar must be placed in each corner of the section (i.e. in each corner of stirrup). The minimum bar diameter shall be 12 mm or 1/15 of the spacing between stirrups whichever is larger.

7- Enough anchorage of longitudinal torsional reinforcement should be provided at the face of the supporting columns, where torsional moments are often reach maximum value.

8- In case of considering the effective part of the slab in T and L sections when calculating the nominal shear stresses due to torsion, the following measures are taken refer to Fig. (8.12):

- The effective part of the slab in T and L sections measured from the outer face of the beam should not be more than 3 times the slab thickness.
- The effective part of the slab should be provided with web reinforcement.

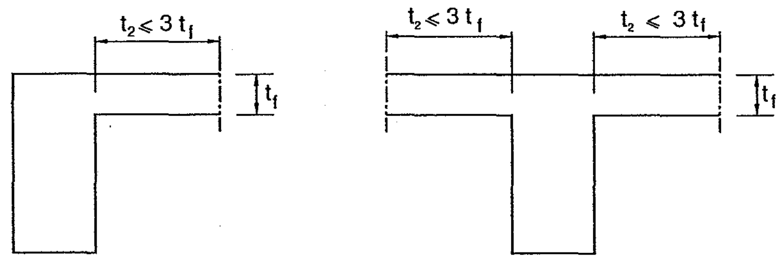


Fig. 8.12 Effective flange width for torsion

Photo 8.4 A circular beam during construction

8.6.7 Summary of Torsion Design According to ECP 203

Step 1: Determine cross-sectional parameters

A_{oh} = area enclosed by the centerline of the closed stirrups.

P_h = Perimeter of the centerline of the closed stirrups.

Step 2: Calculate the shear stress due to the ultimate torsion

$$q_{tu} = \frac{M_{tu}}{2A_o t_e}$$

$$A_o = 0.85\ A_{oh} \qquad t_e = \frac{A_{oh}}{P_h}$$

Note: If the actual thickness of the wall of the hollow section is less than A_{oh}/P_h , then the actual wall thickness should be used.

Step 3: Check the need for considering torsion

$$q_{tu\min} = 0.06\sqrt{\frac{f_{cu}}{\gamma_c}}$$

If $q_{tu} > q_{tu\min}$, one has to consider the shear stresses due to torsion.

Step 4: Check that section size is adequate

$$q_{tu\max} = 0.70\sqrt{\frac{f_{cu}}{\gamma_c}} \le 4.0\ N/mm^2$$

If $q_{tu} \langle\ q_{tu\max}$, the concrete dimensions of the section are adequate.

If $q_{tu} \rangle\ q_{tu\max}$, one has to increase concrete dimensions.

Step 5: Design the closed stirrups

The amount of closed stirrups required to resist the torsion is:

$$A_{str} = \frac{M_{tu} \cdot s}{2A_{oh}\left(\frac{f_{yst}}{\gamma_s}\right)}$$

Check that the provided area of stirrups is more than $A_{str\min}$

$$2A_{str} \ge A_{str\min}$$

$$A_{str\min} = \frac{0.40}{f_{yst}} b \times s$$

Check that the provided spacing is less than the code requirement.

Step 6: Design longitudinal reinforcement

$$A_{sl} = A_{str}\left(\frac{P_h}{s}\right)\left(\frac{f_y}{f_{yst}}\right)$$

Check that the provided longitudinal reinforcement is not less than $A_{sl\min}$

$$A_{sl\min} = \frac{0.4\sqrt{\frac{f_{cu}}{\gamma_c}}\ A_{cp}}{f_y/\gamma_s} - \left(\frac{A_{str}}{s}\right) P_h \left(\frac{f_{yst}}{f_y}\right)$$

In the previous equation $\frac{A_{str}}{s}$ should not be less than $\frac{b}{6 \times f_{yst}}$

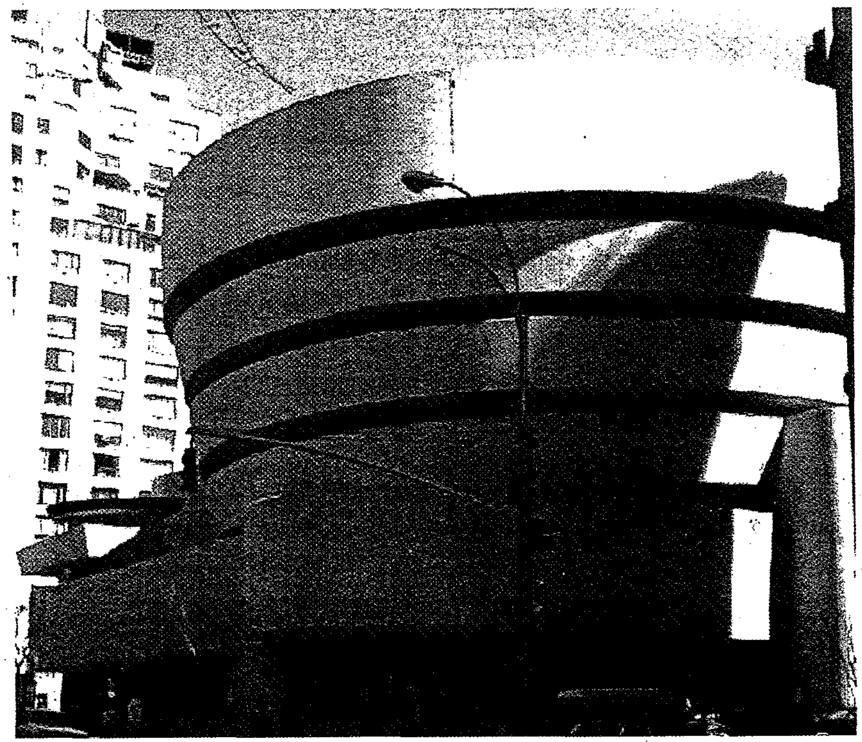


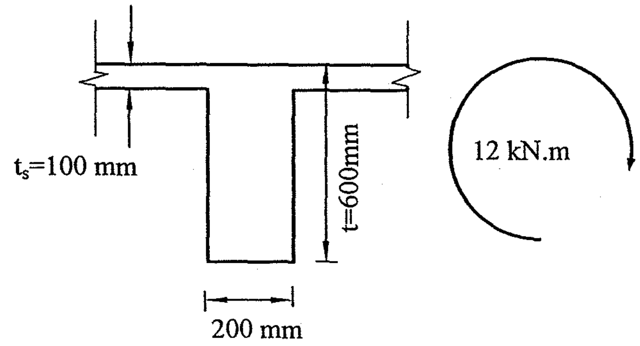
Photo 8.5 Guggenheim Museum, designed by Wright, New York, USA

Example 8.1

Design the T-beam shown in the figure below for torsion if it is subjected to a torsional moment M_{tu} = 12 kN.m

f_{cu} = 20 N/mm^2

f_{yst} = 280 N/mm^2, f_y=280 N/mm^2



Solution

To design a T-section for torsion, one has two options:

1- **Consider** the slab in the calculations and reinforce **both** the slab and the beam for torsion

2- **Do not** consider the slab contribution in torsion design, and provide stirrups and longitudinal reinforcement in the **web only** (easier and more practical for thin slabs).

In this example, the contribution of the slab shall be neglected *(option 2)* as permitted by the code in section 4-2-3-2-b

Step 1: Section properties

Assume a concrete cover of 30 mm to the centerline of the stirrups.

x_1 = 200 –2 x 30 = 140 mm

y_1 = 600 –2 x 30 = 540 mm

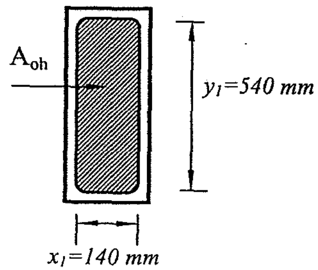
The section properties for the design for torsion are A_{oh} and p_h

$$p_h = 2\times(x_1 + y_1) = 2\times(140+540) = 1360\ mm$$

$$A_{oh} = x_1 .\ y_1 = 140\times 540 = 75600\ mm^2$$

$$A_o = 0.85 A_{oh} = 0.85\times 75600 = 64260\ mm^2$$

$$t_e = \frac{A_{oh}}{p_h} = \frac{75600}{1360} = 55.6\ mm$$



Step 2: Calculations of shear stress due to torsion

$$q_{tu} = \frac{M_{tu}}{2\times A_o \times t_e} = \frac{12\times 10^6}{2\times 64260\times 55.6} = 1.68\ N/mm^2$$

$$q_{tu,\min} = 0.06\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.06\sqrt{\frac{20}{1.5}} = 0.22\ \ N/mm^2$$

Since q_{tu}(1.68)>q_{tumin} (0.22), then torsion has to be considered

Step 3: Check the adequacy of the cross-section dimensions

$$q_{tu,\max} = 0.70\ \sqrt{\frac{f_{cu}}{\gamma_c}} \le 4.0\ N\ /mm^2$$

$$q_{tu,\max} = 0.70\ \sqrt{\frac{20}{1.5}} = 2.56\ N/mm^2$$

Since q_{tu}(1.68) < $q_{tu,max}$(2.56), the cross section dimensions are adequate.

Step 4: Reinforcement for torsion

A- Stirrups reinforcement

According to clause (4-2-3-5-b) in the code, the spacing of the stirrups should be the smaller of

$p_h/8$ (170) mm or 200 mm, try spacing of 150 mm

$$A_{str}=\frac{M_{tu}\times s}{2\times A_o\times f_{yst}/\gamma_s}=\frac{12\times10^6\times150}{2\times64260\times280/1.15}=57.52\,mm^2$$

The area of one branch A_{str}=57.52 mm^2, choose ϕ10 (78 mm^2)

Choose ϕ10/150 mm

$$A_{st,\min}=\frac{0.35}{f_y/1.15}b\times s=\frac{0.4}{f_y}b\times s=\frac{0.4}{280}200\times150=42.85\,mm^2$$

The minimum area of steel for torsion is given by :

$2A_{str}\geq A_{st,\min}$ (two branches)

Noting that A_{st}=0, thus $2A_{str}\geq A_{st,\min}$

$2\times78>42.85$ *ok*

B-Longitudinal Reinforcement

The area of the longitudinal steel is given by: (use calculated A_{str})

$$A_{sl}=\frac{A_{str}\times p_h}{s}\left(\frac{f_{yst}}{f_y}\right)=\frac{57.52\times1360}{150}\left(\frac{280}{280}\right)=521.54\,mm^2$$

Calculate the minimum area for longitudinal reinforcement $A_{sl,min}$

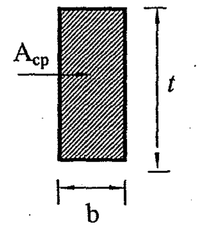
$$A_{sl,\min}=\frac{0.4\sqrt{\frac{f_{cu}}{\gamma_c}}A_{cp}}{f_y/\gamma_s}-\frac{A_{str}\times p_h}{s}\left(\frac{f_{yst}}{f_y}\right)$$

There is a condition on this equation that $\frac{A_{str}}{s}\geq\frac{b}{6\times f_{yst}}$

$$\frac{57.52}{150}\geq\frac{200}{6\times280}...o.k$$

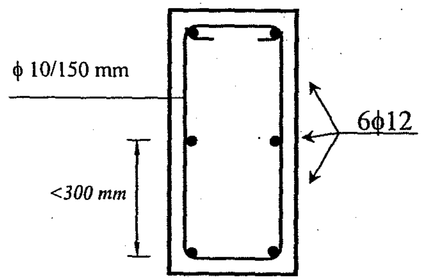
$$A_{sl,\min}=\frac{0.4\sqrt{\frac{20}{1.5}}\times200\times600}{280/1.15}-\frac{57.52\times1360}{150}\left(\frac{280}{280}\right)=198\,mm^2$$

Since $A_{sl}>A_{sl,min}$ ...ok

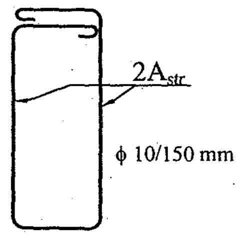


The bar diameter chosen should be greater than 12mm or s/15(10mm)
Also, the maximum spacing between longitudinal steel should be less than 300 mm.

Choose 6ϕ12 (677 mm^2)



Torsional reinforcement



Stirrup detail

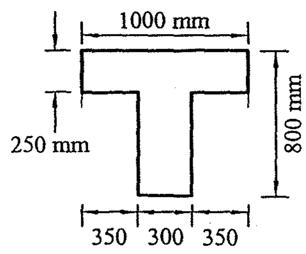
Example 8.2

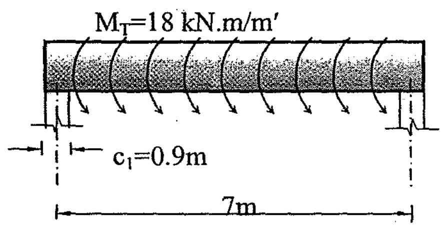
The beam shown in figure is subjected to distributed torsional moment of a value of 18 kN.m′ along its span. Design the beam for torsion considering the contribution of the flanges.

Data

f_{cu} = 30 N/mm^2

f_{yst} = 240 N/mm^2, f_y=360 N/mm^2

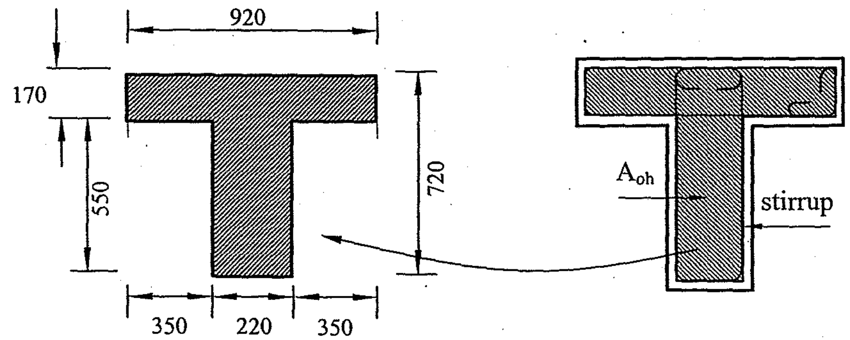




Solution

Step 1: Section properties

Assume concrete cover of 40 mm to the <u>*centerline*</u> of the stirrup all around the cross section



A_{oh} definition

$$p_h = 170 + 550 + 350 + 220 + 350 + 550 + 170 + 920 = 3280\ mm$$

or directly $p_h = 2 \times (720 + 920) = 3280\ mm$

$$A_{oh} = 220 \times 550 + 170 \times 920 = 277400\ mm^2$$

$$A_o = 0.85 A_{oh} = 0.85 \times 277400 = 235790\ mm^2$$

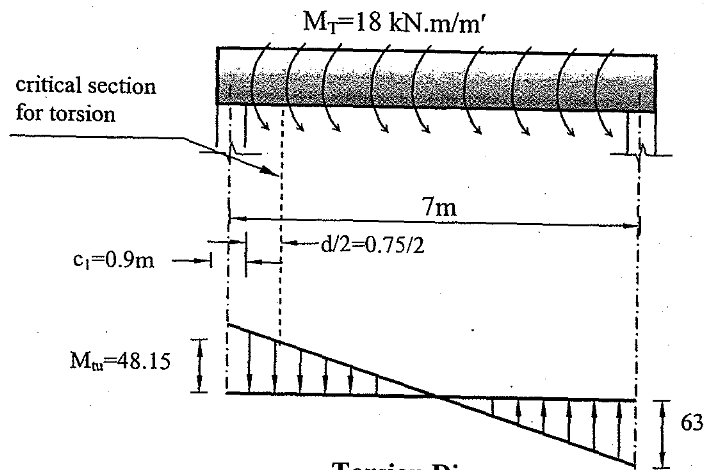
$$t_e = \frac{A_{oh}}{p_h} = \frac{277400}{3280} = 84.6\ mm$$

Step 2: Calculations of shear stress due to torsion

Critical section for torsion is at d/2

$$M_{tu} = \frac{M_T \times L}{2} - M_T\left(\frac{c_1}{2} + \frac{d}{2}\right)$$

$$M_{tu} = \frac{18 \times 7}{2} - 18 \times \left(\frac{0.9}{2} + \frac{0.75}{2}\right) = 48.15\ kN.m$$



Torsion Diagram

$$q_{tu} = \frac{M_{tu}}{2 \times A_o \times t_e} = \frac{48.15 \times 10^6}{2 \times 235790 \times 84.6} = 1.21\ N/mm^2$$

$$q_{tu,\min} = 0.06\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.06\sqrt{\frac{30}{1.5}} = 0.27\ \ N/mm^2$$

Since q_{tu}(1.21)>q_{tumin} (0.27) then torsion should be considered

Step 3: Check the adequacy of the cross-section dimensions

$$q_{tu,\max} = 0.70\sqrt{\frac{f_{cu}}{\gamma_c}} \le 4.0\ N/mm^2$$

$$q_{tu,\max} = 0.70\sqrt{\frac{30}{1.5}} = 3.13\ N/mm^2 < 4.0N/mm^2$$

$q_{tu,max}$=3.13 N/mm^2

Since q_{tu}(1.21)<$q_{tu,max}$(3.13), the cross section dimensions are adequate.

Step 4: Reinforcement for torsion

A- Stirrups reinforcement

According to clause (4-2-3-5-b) in the code, the spacing of the stirrups should be smaller of p_h/8 (410) mm or 200 mm, try spacing of 200 mm

$$A_{str} = \frac{M_{tu} \times s}{2 \times A_o \times f_{yst}/\gamma_s} = \frac{48.15 \times 10^6 \times 200}{2 \times 235790 \times 240/1.15} = 97.84\ mm^2$$

The area of one branch A_{str}=97.84 mm^2 choose ϕ12 (113 mm^2)

Choose ϕ12/200 mm

$$A_{st,\min} = \frac{0.4}{f_y} b \times s = \frac{0.4}{240} 300 \times 200 = 100\ mm^2$$

The minimum area of steel for torsion is given by:

$$2A_{str})_{chosen} \ge A_{st,\min}$$

$$2 \times 113(226) > 100 \quad ok$$

B-Longitudinal Reinforcement

The area of the longitudinal steel is given by:

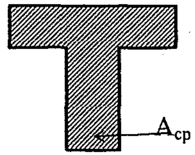
$$A_{sl} = \frac{A_{str} \times p_h}{s}\left(\frac{f_{yst}}{f_y}\right) = \frac{97.84 \times 3280}{200}\left(\frac{240}{360}\right) = 1069.82\ mm^2 \quad \textit{(use the calculated } A_{str}\textit{)}$$

Calculate the minimum area for longitudinal reinforcement $A_{sl,min}$

$$A_{sl,\min} = \frac{0.4\sqrt{\frac{f_{cu}}{\gamma_c}} A_{cp}}{f_y/\gamma_s} - \frac{A_{str} \times p_h}{s}\left(\frac{f_{yst}}{f_y}\right)$$

Check $\frac{A_{str}}{s} \ge \frac{b}{6 \times f_{yst}}$ $\qquad \frac{97.84}{200} \ge \frac{300}{6 \times 240} ...o.k$

$$A_{cp} = 300 \times 550 + 250 \times 1000 = 415000\ mm^2$$



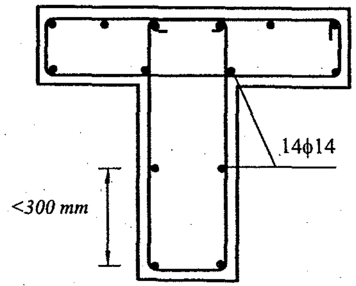
$$A_{sl,\min} = \frac{0.4\sqrt{\frac{30}{1.5}} \times 415000}{360/1.15} - \frac{97.84 \times 3280}{200}\left(\frac{240}{360}\right) = 1301\ mm^2$$

Since A_{sl} <$A_{sl,min}$ ...use A_{sl}=$A_{sl,min}$=1301 mm^2

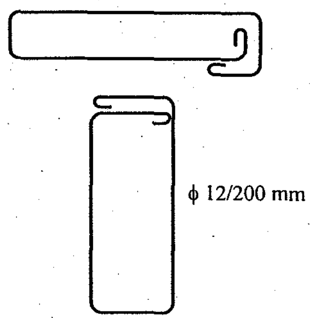
The bar diameter chosen should be greater than 12mm or s/15(13.3 mm)

The maximum spacing between longitudinal steel should be less than 300 mm.

Choose 14ϕ14.



Torsional reinforcement details



Stirrup detail

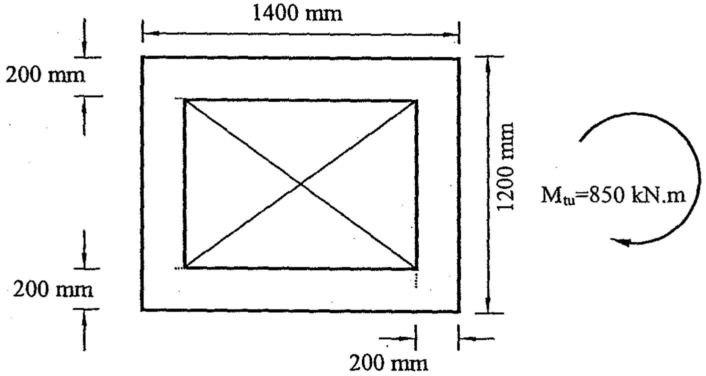
Example 8.3

The figure shown below is for the cross section of a main girder that is subjected to a factored torsional moment of a value of 850 kN.m. It is required to design the girder for torsion.

Data

f_{cu} = 25 N/mm^2

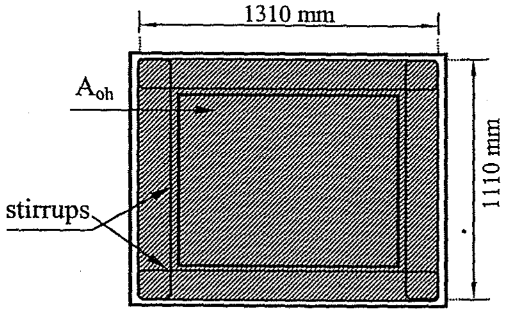
f_{yst} = 360 N/mm^2, f_y=360 N/mm^2



Solution

Step 1: Section properties

Assume concrete cover of 45 mm to the *centerline* of the stirrup all around the cross section



$$p_h = 2(1310+1110) = 4840\ mm$$

$$A_{oh} = 1310\times1110 = 1454100\ mm^2$$

$$A_o = 0.85 A_{oh} = 0.85\times1454100 = 1235985\ mm^2$$

$$t_e = \frac{A_{oh}}{p_h} = \frac{1454100}{4840} = 300.4\ mm > t_{actual}\ (200\ mm)$$

Use t_e=t_{actual}=200 mm

Step 2: Calculations of shear stress due to torsion

$$q_{tu} = \frac{M_{tu}}{2\times A_o \times t_e} = \frac{850\times10^6}{2\times1235985\times200} = 1.72\ N/mm^2$$

$$q_{tu,\min} = 0.06\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.06\sqrt{\frac{25}{1.5}} = 0.24\ N/mm^2$$

Since q_{tu}(1.72)>q_{tumin} (0.24) then torsion should be considered.

Step 3: Check the adequacy of the cross-section dimensions

$$q_{tu,\max} = 0.70\sqrt{\frac{f_{cu}}{\gamma_c}} \le 4.0\ N/mm^2$$

$$q_{tu,\max} = 0.70\sqrt{\frac{25}{1.5}} = 2.86\ N/mm^2 < 4.0 N/mm^2$$

Since q_{tu}(1.72)<$q_{tu,max}$(2.86), , the cross section dimensions are adequate.

Step 4: Reinforcement for torsion

A- Stirrups reinforcement

According to clause (4-2-3-5-b) in the code, the spacing of the stirrups should be smaller of: P_h/8 (605) mm or 200 mm, <u>try spacing of 200 mm</u>

$$A_{str} = \frac{M_{tu}\times s}{2\times A_o\times f_{yst}/\gamma_s} = \frac{850\times10^6\times200}{2\times1235985\times360/1.15} = 219.68\ mm^2$$

For box sections having a wall width less than b/6, the code permits dividing the obtained area of stirrups between the two sides of the wall.

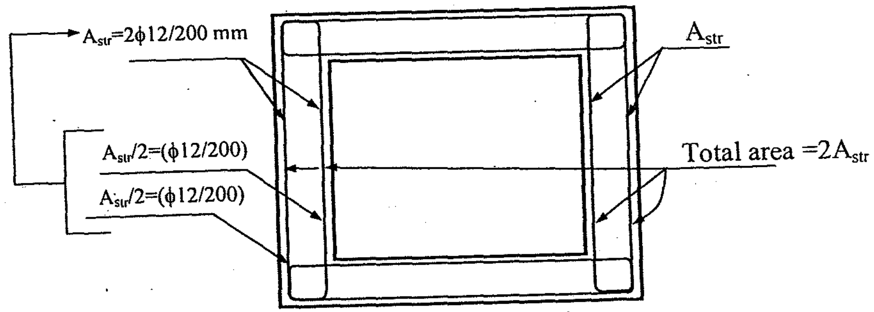
For the two vertical walls (webs) $t_w(200) \le \frac{1200}{6}$

For the two horizontal walls (flanges) $t_w(200) < \frac{1400}{6}$

Hence, area of the cross section of the stirrup on each side of the wall will be equal to 219.68/2=109.8 mm^2.

Thus choose ϕ12/200 mm

i.e. A_{str}=2x113=266 mm^2 >$A_{str,required}$ (219.68 mm^2)



$$A_{st,\min}=\frac{0.4}{f_y}b\times s=\frac{0.4}{360}(2\times 200)\times 200=89\ mm^2$$

The minimum area of steel for torsion is: $2A_{str}\geq A_{st,\min}$

$4\times 112>A_{st,\min}(89)\quad ok$

Final design ϕ12/200 mm

B-Longitudinal Reinforcement

The area of the longitudinal steel is given by:

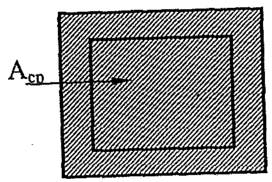
$$A_{sl}=\frac{A_{str}\times p_h}{s}\left(\frac{f_{yst}}{f_y}\right)=\frac{219.7\times 4840}{200}\left(\frac{360}{360}\right)=5316\ mm^2$$

Calculate the minimum area for longitudinal reinforcement $A_{sl,min}$, (use the chosen A_{str})

$$A_{sl,\min}=\frac{0.4\sqrt{\frac{f_{cu}}{\gamma_c}}A_{cp}}{f_y/1.15}-\frac{A_{str}\times p_h}{s}\left(\frac{f_{yst}}{f_y}\right)$$

There is a condition on this equation that $\frac{A_{str}}{s}\geq\frac{b}{6\times f_{yst}}$ (code 4-2-3-5-c)

$$\frac{219.7}{200}\geq\frac{1400}{6\times 360}\ldots o.k$$



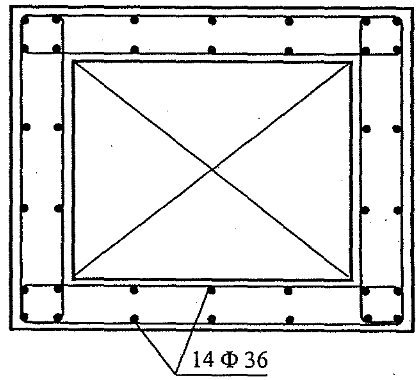
$$A_{cp}=1400\times 1200=1680000\ mm^2$$

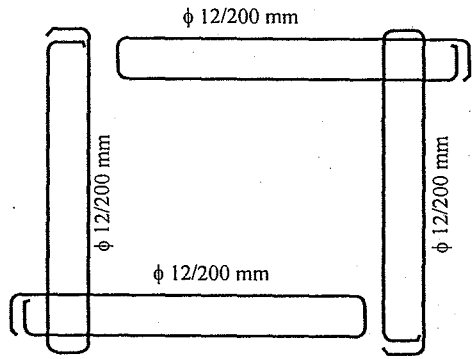
$$A_{sl,\min}=\frac{0.4\sqrt{\frac{25}{1.5}}\times 1680000}{360/1.15}-\frac{219.7\times 4840}{200}\left(\frac{360}{360}\right)=3447\ mm^2$$

Since $A_{sl}>A_{sl,min}$ ...o.k

The bar diameter chosen should be greater than 12mm or s/15(13.3 mm)

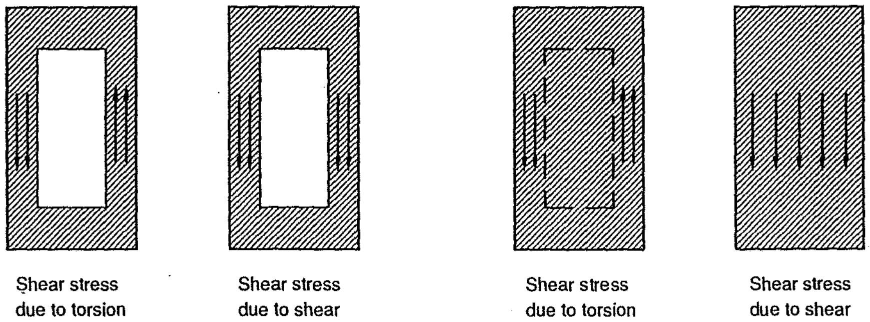
Choose 36 Φ 14(5541 mm²).





8.7 Combined Shear and Torsion

When a hollow section is subjected to a direct shear force and a torsional moment, the shear stresses on one side of the cross section are additive and on the other side are subtractive as shown in Figs. 8.13a.
When a solid section is subjected to combined shear and torsion, the shear stresses due to shear are resisted by the entire section, while the shear stresses due torsion are resisted by the idealized hollow section as shown in Fig. 8.13b.



a) Hollow section b) Solid section

Fig. 8.13 Addition of torsional and shear stresses

8.8 The Design for Shear and Torsion in ECP 203

8.8.1 Consideration of Torsion

The Egyptian code ECP 203 requires that torsional moments should be considered in design if the factored torsional stresses calculated from Eq. (8.23) exceed $q_{tu\,min}$, given by:

$$q_{tu\,min} = 0.06\sqrt{\frac{f_{cu}}{\gamma_c}} \qquad \text{.......(8.33)}$$

8.8.2 Adequacy of the Concrete Cross-Section

The shear stresses due to direct shear q_u and due to torsional moment q_{tu} are given by:

$$q_u = \frac{Q_u}{b_w\, d} \qquad\qquad q_{tu} = \frac{M_{tu}}{2A_o t_e}$$

The Egyptian Code concentrates on the side of the hollow section where the shear and torsional stresses are additive. On that side:

$$q_u + q_{tu} \le 0.7\sqrt{\frac{f_{cu}}{\gamma_c}} \le 4.0\ N/mm^2 \qquad \text{.......(8.34)}$$

In a solid section, the shear stresses due to direct shear are assumed to be uniformly distributed across the width of the section, while the torsional shears only exist in the walls of the assumed thin-walled tube, as shown in Fig. (8.13b). The direct summation of the two terms tends to be conservative and a root-square summation is used

$$\sqrt{(q_u)^2 + (q_{tu})^2} \le 0.7\sqrt{\frac{f_{cu}}{\gamma_c}} \le 4.0\ N/mm^2 \qquad \text{.......(8.35)}$$

If the above equations (8.34) and (8.35) are not satisfied, then one has to increase the concrete dimensions of the cross section.

8.8.3 Design of Transverse Reinforcement

For members under combined shear and torsion, the Egyptian Code requires that the transverse steel due to torsion should be added to that due to shear. Concrete is assumed to contribute to the shear strength of the beam. It does not, however, contribute to the torsional strength of the beam. The transverse reinforcement for combined shear and torsion is obtained according to Table (8.1).

Table (8.1) Transverse reinforcement requirements according to ECP 203

	$q_{tu} \le 0.06\sqrt{f_{cu}/\gamma_c}$	$q_{tu} > 0.06\sqrt{f_{cu}/\gamma_c}$
$q_u \le q_{cu}$	Provide minimum reinforcement as given by Eq. 8.43	Provide reinforcement to resist q_{tu}, given by Eq. 8.28
$q_u > q_{cu}$	Provide reinforcement to resist $q_u - q_{cu}/2$	Provide reinforcement to resist $q_u - q_{cu}/2$ and q_{tu}

In Table (8.1), q_{cu} is the concrete contribution to the shear strength and is obtained from:

$$q_{cu} = 0.24\sqrt{f_{cu}/\gamma_c} \quad N/mm^2 \quad \text{.......(8.36)}$$

The total amount of stirrups needed for shear and torsion should satisfy the following equation:

$$2A_{str} + A_{st})_{min} \ge \frac{0.40}{f_{yst}} b \times s \quad \text{.......(8.37)}$$

8.8.4 Design of Longitudinal Reinforcement.

Longitudinal steel for torsion should be obtained using Eqs. 8.31 and 8.32. No longitudinal steel is required for shear. As mentioned before, the use of the shifted bending moment diagram takes care of the additional tension force due to shear.

8.8.5 Summary of the Design for Shear and Torsion

Step 1: Determine cross-sectional parameters

The cross-sectional parameters for combined shear and torsion design are b, d, A_{oh} and, P_h.

Step 2: Calculate the ultimate shear stresses due to Q_u and M_t

$$q_u = \frac{Q_u}{b\,d}$$

$$q_{tu} = \frac{M_{tu}}{2A_o t_e}$$

$$A_o = 0.85\ A_{oh} \qquad t_e = \frac{A_{oh}}{P_h}$$

Note: If the actual thickness of the wall of the hollow section is less than A_{oh}/P_h, then the actual wall thickness should be used.

Step 3: Check the need for considering torsion

Calculate the minimum shear stress below which torsion can be neglected.

$$q_{tu\,min} = 0.06\sqrt{\frac{f_{cu}}{\gamma_c}}$$

If $q_{tu} > q_{tu\,min}$, we have to consider the shear stresses due to torsion

Step 4: Check that section size is adequate

To check the adequacy of the concrete dimensions of the cross section, the following equations must be satisfied:

For Hollow sections

$$q_u + q_{tu} \le 0.7\sqrt{\frac{f_{cu}}{\gamma_c}} \le 4.0\ N/mm^2$$

For solid sections

$$\sqrt{(q_u)^2 + (q_{tu})^2} \le 0.7\sqrt{\frac{f_{cu}}{\gamma_c}} \le 4.0\ N/mm^2$$

If $q_{tu} < q_{tu\,max}$ and $q_u < q_{u\,max}$, the concrete dimensions of the section are adequate.
If the above condition is not satisfied, one has to increase the dimensions.

Step 5: Design the closed stirrups

Calculate the concrete contribution to the shear resistance, q_{cu}

$$q_{cu} = 0.24\sqrt{\frac{f_{cu}}{\gamma_c}}$$

Check the requirements for transverse reinforcement from Table (8.1)
If $q_u > q_{cu}$, calculate the stirrups needed for shear

$$q_{su} = q_u - 0.5\, q_{cu}$$

$$\frac{A_{st}}{s} = \frac{q_{su}\, b}{f_{yst}/\gamma_s}$$

The area of one branch of stirrups needed for torsion is obtained from:

$$\frac{A_{str}}{s} = \frac{M_{tu}}{1.7 A_{oh}(f_{yst}/\gamma_s)}$$

The area of one branch of stirrups needed for resisting shear and torsion = $A_{str} + \frac{A_{st}}{n}$

where n is the number of branches determined from shear calculations as shown in Fig. 8.14.

Check that the chosen area of stirrups satisfies the minimum requirements

$$(A_{st} + 2A_{str})_{chosen} > \frac{0.4}{f_{yst}} b \times s$$

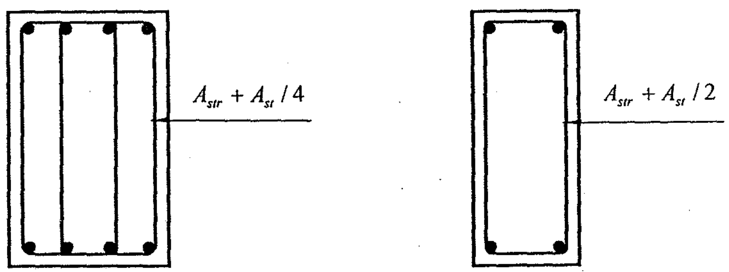


Fig. 8.14 Stirrups for shear and torsion

Step 6: Design longitudinal reinforcement

$$A_{sl} = A_{str}\left(\frac{P_h}{s}\right)\left(\frac{f_y}{f_{yst}}\right)$$

Check that the provided longitudinal torsional reinforcement is more than the minimum requirement, where:

$$A_{sl\,min} = \frac{0.40\sqrt{\frac{f_{cu}}{\gamma_c}}\, A_{cp}}{f_y/\gamma_s} - \left(\frac{A_{str}}{s}\right) P_h \left(\frac{f_{yst}}{f_y}\right)$$

In the previous equation $\frac{A_{str}}{s}$ should not be less than $\frac{b}{6 \times f_{yst}}$

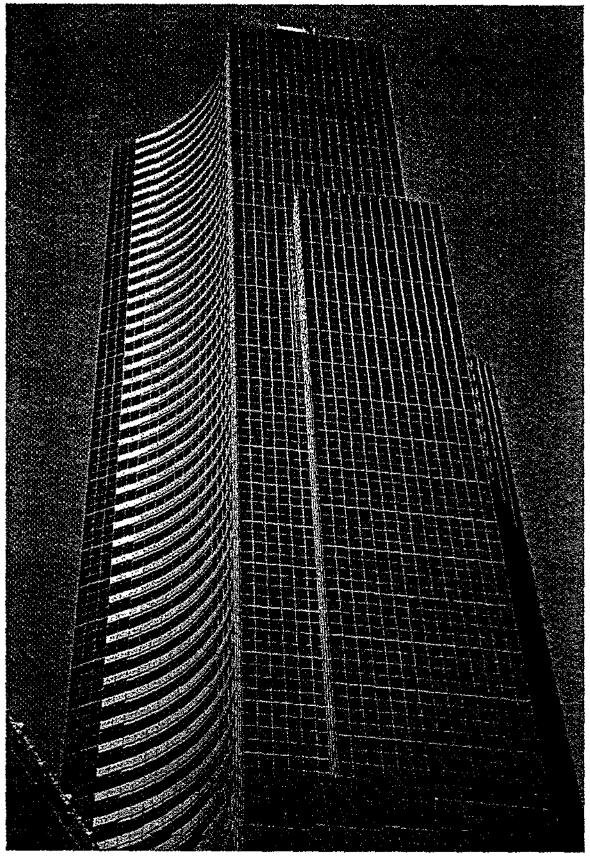


Fig. 8.5 High-rise building in Seattle, USA

8.9 Compatibility Torsion

In statically indeterminate structures where the torsional moment results from compatibility of deformations between members meeting at a joint, the torsional moment can be reduced by redistribution of internal forces after cracking. The ECP 203 assumes that in case of compatibility torsion, design can be based on an ultimate torque equal to:

$$M_{tu} = 0.316 \frac{A_{cp}^2}{P_{cp}} \sqrt{f_{cu}/\gamma_c} \qquad f_{cu} \text{ in } N/mm^2 \text{ ....(8.44)}$$

in which p_{cp} is the perimeter of the gross concrete cross section

8.10 Torsional Rigidity

The torsional rigidity is defined as the torsional moment required to cause a unit angle of twist.

For an uncracked elastic member, the theory of elasticity gives the torsional rigidity as (C x G); where C is the torsional constant of the cross section and G is the shear modulus of elasticity. At torsional cracking there is a sudden increase in the angle of twist and hence a sudden drop in the effective value of CG.

The Egyptian Code recommends the value of G to be equal to o.42 E_c where E_c is the modulus of elasticity of concrete as given Chapter (1).

The ECP 203 gives the following equation for calculating the torsional constant of rectangular sections:

$$C = \beta\, b^3 t\, \eta \text{ ....(8.45)}$$

in which

η= 0.7 for rectangular sections in which the shear stresses due to torsion do no exceed $0.316\sqrt{f_{cu}/\gamma_c}$.

η= 0.2 for rectangular sections after cracking

β is a factor that depends on the ratio t/b as given in Table 8.2.

Table 8.2 Values of the factor β for calculating the torsional rigidity

t/b	1	1.5	2	3	5	〉5
β	0.14	0.2	0.23	0.26	0.29	0.33

For calculating the torsional rigidity of a cross-section having T, L or box shapes, one can divide the cross section into rectangles, each of short side x and long side y and the value C will be given as

$$C = \sum \beta\, x^3 y \text{ ....(8.46)}$$

The arrangement of rectangles that maximize the sum is used.
It should be mentioned that the drastic drop in the torsional rigidity allows a significant redistribution of torsion in indeterminate beam systems (compatibility torsion).

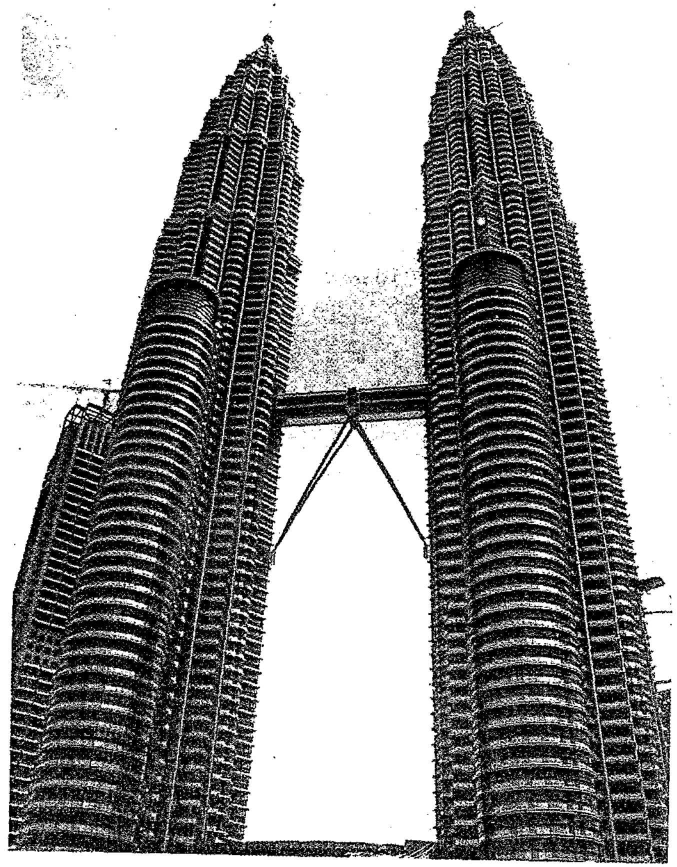


Fig. 8.6 Petronas Towers, by Cesar Pelli, at Kuala Lumpur, Malaysia (1998)

Example 8.4

The figure below gives the concrete dimensions for a cross section that is subjected to the following straining actions at the critical section near the support:

M_u = 42 kN.m (negative bending)

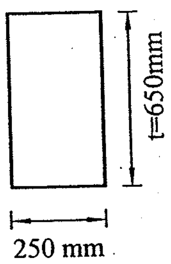
Q_u = 172.5 kN

M_{tu} = 16.1 kN.m

It is required to perform a complete design for the cross section knowing that the material properties are:

f_{cu} = 25 N/mm^2

f_{yst} = 280 N/mm^2, f_y=360 N/mm^2



Solution

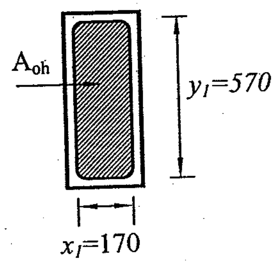
Step 1: Design stresses

A. Shear Stress:

$$q_u = \frac{Q_u}{b \times d} = \frac{172.5 \times 1000}{250 \times 600} = 1.15\ N/mm^2$$

B. Torsional stresses

Assume concrete cover of 40 mm



x_1 = 250 – 2 x 40 = 170 mm

y_1 = 650 – 2 x 40 = 570 mm

$$p_h = 2 \times (x_1 + y_1) = 2 \times (170 + 570) = 1480\ mm$$

$$A_{oh} = x_1 \cdot y_1 = 170 \times 570 = 96900\ mm^2$$

$$A_o = 0.85 A_{oh} = 0.85 \times 96900 = 82365\ mm^2$$

$$t_e = \frac{A_{oh}}{p_h} = \frac{96900}{1480} = 65.47\ mm$$

$$q_{tu} = \frac{M_{tu}}{2 \times A_o \times t_e} = \frac{16.1 \times 10^6}{2 \times 82365 \times 65.47} = 1.49\ N/mm^2$$

$$q_{tu,\min} = 0.06\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.06\sqrt{\frac{25}{1.5}} = 0.2449\ N/mm^2$$

Since $q_{tu} > q_{tumin}$ then torsion should be considered

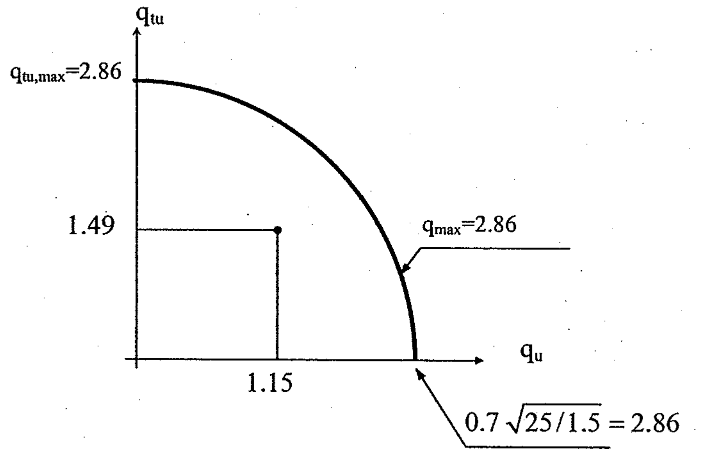
Step 2: Check the adequacy of the cross-section dimensions

$$q_{\max} = 0.70\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.70\sqrt{\frac{25}{1.5}} = 2.86\ N/mm^2 < 4.0\ N/mm^2$$

$$\sqrt{(q_u)^2 + (q_{tu})^2} \leq q_{\max}$$

$$\sqrt{(1.15)^2 + (1.49)^2} = 1.88 \leq 2.86 ......o.k$$

Since the previous equation is satisfied, the cross section dimensions are adequate for resisting combined shear and torsion.



Graphical representation of the maximum stresses due to shear and torsion.

Step 3: Design of closed stirrups for shear and torsion

Step 3.1: Area of stirrups for shear

The concrete shear strength q_{cu} equals

$$q_{cu} = 0.24\sqrt{\frac{25}{1.5}} = 0.98\ N/mm^2$$

Since the applied shear is greater than q_{cu}, shear reinforcement is needed

$$q_{su} = q_u - \frac{q_{cu}}{2} = 1.15 - \frac{0.98}{2} = 0.66\ N/mm^2$$

The spacing of the stirrups should be smaller of ph/8 (185) mm or 200 mm, **try a spacing of 150 mm**

$$A_{st} = \frac{q_{su} \times b \times s}{f_y/1.15} = \frac{0.66 \times 250 \times 150}{280/1.15} = 101.6\ mm^2$$

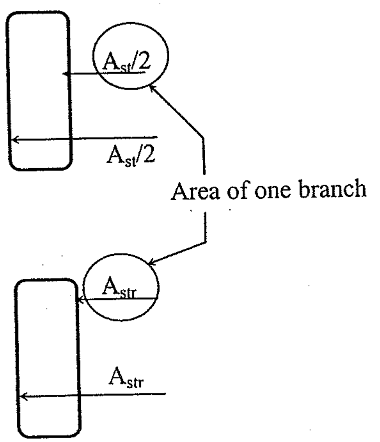
Area for one branch of the stirrup equals $A_{st}/2$=50.8 mm^2

Step 3.2: Area of stirrups for torsion

The area of one branch A_{str}

Using the same stirrup spacing of 150 mm, one gets

$$A_{str} = \frac{M_{tu} \times s}{2 \times A_o \times f_{yst}/\gamma_s} = \frac{16.1 \times 10^6 \times 150}{2 \times 82365 \times 280/1.15} = 60.2\ mm^2$$



Step 3.3: Stirrups for combined shear and torsion

Area of one branch for combined shear and torsion

$$= A_{str} + A_{st}/2 = 60.2 + 101.6/2 = 111\ mm^2$$

Choose ϕ 12 mm (113 mm^2)

$$A_{st,\min} = \frac{0.4}{f_{ys}} b \times s = \frac{0.40}{280} 250 \times 150 = 53.9\ mm^2$$

Total area chosen $= 2 \times 113 > A_{st,\min}$ *.......o.k*

Final design use ϕ 12/150 mm

Step 4: Design of longitudinal reinforcement for torsion

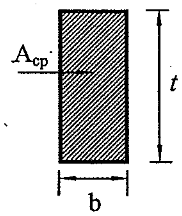
$$A_{sl} = \frac{A_{str} \times p_h}{s}\left(\frac{f_{yst}}{f_y}\right) = \frac{60.2 \times 1480}{150}\left(\frac{280}{360}\right) = 462\ mm^2$$

Calculate the minimum area for longitudinal reinforcement $A_{sl,min}$

$$A_{sl,\min} = \frac{0.40\sqrt{\frac{f_{cu}}{\gamma_c}} A_{cp}}{f_y/\gamma_s} - \frac{A_{str} \times p_h}{s}\left(\frac{f_{yst}}{f_y}\right)$$

There is a condition on this equation that $\frac{A_{str}}{s} \geq \frac{b}{6 \times f_{yst}}$

$$\frac{60.2}{150} \geq \frac{250}{6 \times 280} \ldots o.k$$



$$A_{sl,\min} = \frac{0.40\sqrt{\frac{25}{1.5}} \times 250 \times 650}{360/1.15} - \frac{60.2 \times 1480}{150}\left(\frac{280}{360}\right) = 385\ mm^2$$

Since $A_{sl} > A_{sl,min}$ ...o.k

Choose 6ϕ12 (678 mm^2)

Step 5: Longitudinal reinforcement for flexure

M_u=42 kN.m

$$R=\frac{M_u}{f_{cu} b d^2}=\frac{42\times10^6}{25\times250\times600^2}=0.0186$$

From (R-ω) curve it can be determined that ω=0.0224

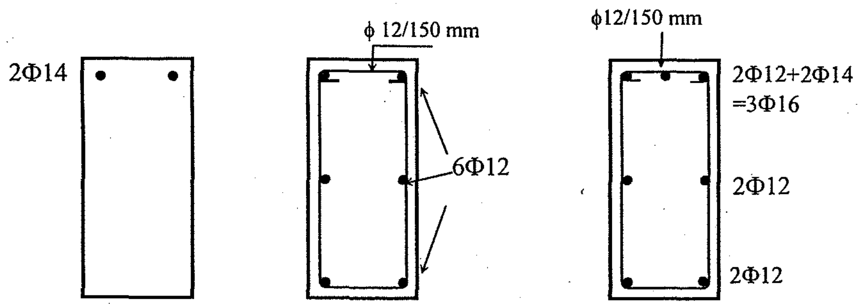
$$A_s=\omega\frac{f_{cu}}{f_y}b\times d=0.0224\frac{25}{360}\times250\times600=233\ mm^2$$

$$A_{s\min}=smaller\ of\begin{cases}\dfrac{0.225\sqrt{f_{cu}}}{f_y}b\ d=\dfrac{0.225\sqrt{25}}{360}\times250\times600=468\ mm^2\\ 1.3A_s=1.3\times233=303\,mm^2\end{cases}$$

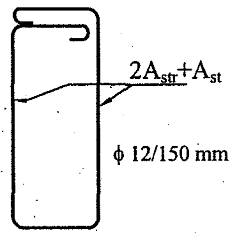
but not less than $\frac{0.15}{100}\times250\times600=225\ mm^2$ →→use (2Φ 14, 308 mm²)

Step 6: Reinforcement details

The maximum spacing between longitudinal torsional reinforcement is 300 mm. It should be noted that the longitudinal torsional reinforcement that will be placed at the top part of the section is added to flexural reinforcement as shown in the figure below.



Flexural reinforcement | Shear & torsion reinforcement | Flexure, shear & torsion reinforcement



Stirrup detail

Example 8.5

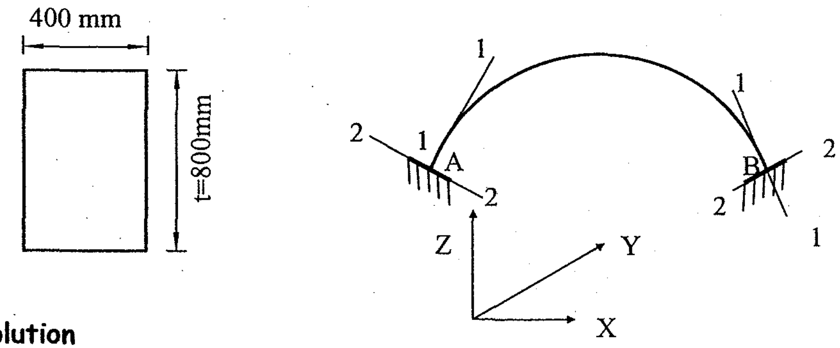
The curved beam shown in figure has a width of 400 mm and a thickness of 800mm. The beam is subjected to uniformly distributed load. Computer analysis[1] of the beam reveals that the maximum shear force and torsional moment at the support A are :

Q_u = 612 kN & M_{tu} = 40 kN.m

f_{cu} = 30 N/mm²

f_{yst} = 240 N/mm², f_y=400 N/mm²

Design the beam for shear and torsion



Solution

Step 1: Shear and torsional stresses

Step 1.1: Shear Stresses

d=750 mm

$$q_u=\frac{Q_u}{b\times d}=\frac{612\times1000}{400\times750}=2.04\ N/mm^2$$

Step 1.2: Torsional Stresses

Assume concrete cover of 40 mm to the centerline of the stirrup

x_1 = 400 –2 x 40 = 320 mm

y_1 = 800 –2 x 40 = 720 mm

$$p_h=2\times(x_1+y_1)=2\times(320+720)=2080\ mm$$

$$A_{oh}=x_1\cdot y_1=320\times720=230400\ mm^2$$

$$A_o=0.85A_{oh}=0.85\times230400=195840\ mm^2$$

[1] The beam was modeled using several frame elements connected together to approximate the curved shape using the computer program SAP 2000. The end support was restrained against torsional rotation by fixing the support joint in 1-1 direction. This was achieved after rotating the local axis in the Z direction to coincide with slope at both ends as shown in figure.

$$t_e = \frac{A_{oh}}{p_h} = \frac{230400}{2080} = 110.8\ mm$$

$$q_{tu} = \frac{M_{tu}}{2 \times A_o \times t_e} = \frac{40 \times 10^6}{2 \times 195840 \times 110.8} = 0.922\ N/mm^2$$

$$q_{tu,\min} = 0.06\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.06\sqrt{\frac{30}{1.5}} = 0.27\ \ Mpa$$

Since $q_{tu}(0.922) > q_{tumin}$ (0.27)then one has to design for torsion

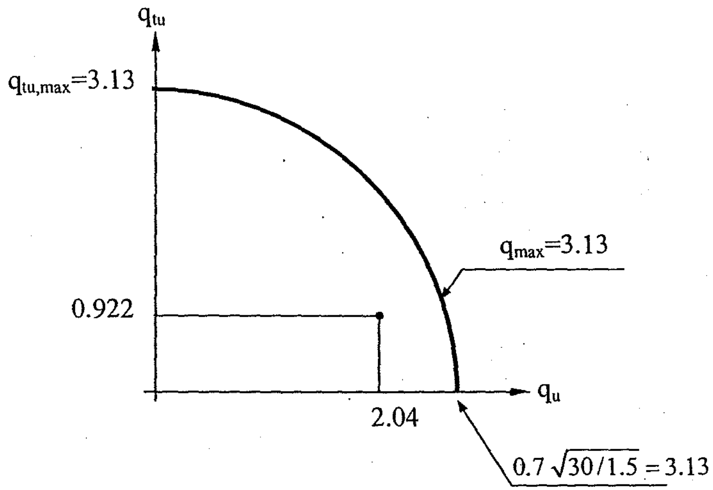
Step 2: Check the adequacy of the cross-section dimensions

$$q_{\max} = 0.70\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.70\sqrt{\frac{30}{1.5}} = 3.13\ N/mm^2 < 4.0\ N/mm^2$$

$$\sqrt{(q_u)^2 + (q_{tu})^2} \le q_{\max}$$

$$\sqrt{(2.04)^2 + (0.922)^2} = 2.37 \le 3.13......o.k$$

Since the previous equation is satisfied, the cross section dimensions are adequate for resisting combined shear and torsion.



Graphical representation of the maximum stresses due to shear and torsion.

Step 3: Design of closed stirrups for shear and torsion

Step 3.1: Area of stirrups for shear

The concrete shear strength q_{cu} equals

$$q_{cu} = 0.24\sqrt{\frac{30}{1.5}} = 1.07\ N/mm^2$$

Since the applied shear is greater than q_{cu} shear reinforcement is needed

$$q_{su} = q_u - \frac{q_{cu}}{2} = 2.04 - \frac{1.07}{2} = 1.505\ N/mm^2$$

The spacing of the stirrups should be smaller of $p_h/8$ (260) mm or 200 mm, **try spacing of 100 mm**

$$A_{st} = \frac{q_{su} \times b \times s}{f_{yst}/1.15} = \frac{1.505 \times 400 \times 100}{240/1.15} = 288.45\ mm^2$$

Step 3.2: Area of stirrups for torsion

The area of one branch A_{str}

$$A_{str} = \frac{M_{tu} \times s}{2 \times A_o \times f_{yst}/\gamma_s} = \frac{40 \times 10^6 \times 100}{2 \times 195840 \times 240/1.15} = 48.93\ mm^2$$

Step 3.3: Stirrups for combined shear and torsion

According to the code (4-2-2-1-6-b), beams with width greater or equal to 400 mm should be reinforced with stirrups having 4 branches. The four branches can be used in shear design. However, only the outermost branches (exterior) can be considered for torsion design.

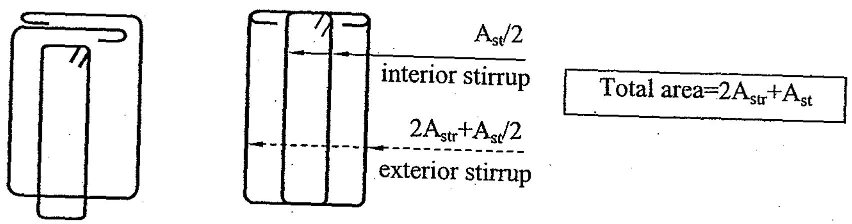
The area of one branch $=A_{st}/4 = 288/4 = 72\ mm^2$

Choose ϕ10/100 mm (=76 mm² inside stirrup)

The area of the exterior stirrups (one branch)$=A_{str} + A_{st(exterior)}/2$

$$= 48.93 + 72 = 121\ mm^2$$

Choose ϕ 14/100 mm (=154 mm² outside stirrup)



Check A_{smin}

$$A_{st,\min} = \frac{0.40}{f_y} b \times s = \frac{0.40}{240} 400 \times 100 = 66.67\ mm^2$$

$A_{st,chosen}$ =area of outside stirrup +area of the inside stirrup

$$A_{st,chosen} = 2 \times 154 + 2 \times 76 = 460\,mm^2 > A_{st,\min} \ldots\ldots o.k$$

Step 4: Design of Longitudinal reinforcement

$$A_{sl} = \frac{A_{str} \times p_h}{s}\left(\frac{f_{yst}}{f_y}\right) = \frac{48.93 \times 2080}{100}\left(\frac{240}{400}\right) = 610.7\ mm^2$$

Calculate the minimum area for longitudinal reinforcement $A_{sl,min}$

$$A_{sl,\min} = \frac{0.40\sqrt{\frac{f_{cu}}{\gamma_c}}A_{cp}}{f_y / \gamma_s} - \frac{A_{str} \times p_h}{s}\left(\frac{f_{yst}}{f_y}\right)$$

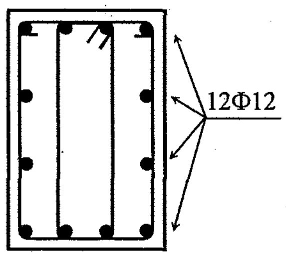
There is a condition on this equation that $\frac{A_{str}}{s} \geq \frac{b}{6 \times f_{yst}}$

$$\frac{48.93}{100} \geq \frac{400}{6 \times 240} \ldots o.k$$

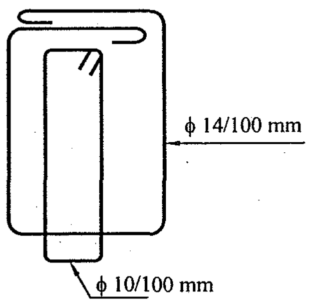
$$A_{sl,\min} = \frac{0.40\sqrt{\frac{30}{1.5}} \times 400 \times 800}{400/1.15} - \frac{48.93 \times 2080}{100}\left(\frac{240}{400}\right) = 1035.1\ mm^2$$

Since $A_{sl} < A_{sl,min}$ ...use $A_{sl,min}$ (1035 mm^2)

Choose 12ϕ12 (1356 mm^2)



Shear & torsion reinforcement



Stirrup detail

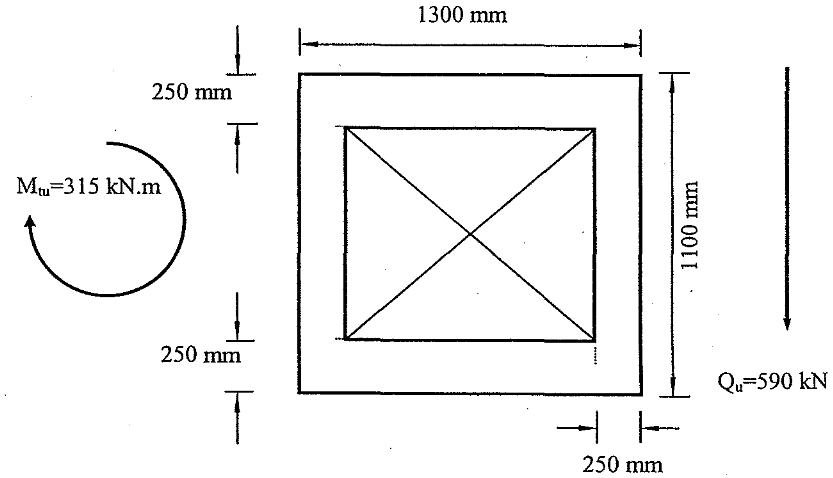
Example 8.6

The box section shown in figure is subjected to combined shear and torsion. Check the adequacy of the concrete dimensions and design both web and longitudinal reinforcement.

Data

f_{cu} = 30 N/mm^2

f_{yst} = 240 N/mm^2, f_y=400 N/mm^2



Solution

Step 1: Shear and Torsional Stresses

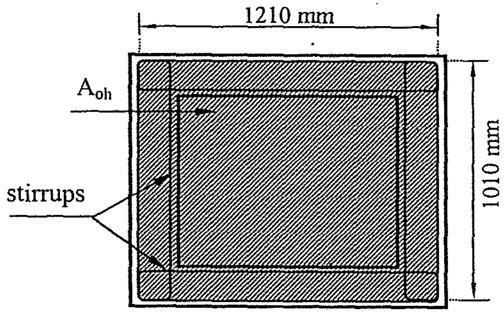
Step 1.1: Shear Stresses

For calculating shear stresses, only the web width will be considered thus: b=250+250=500mm

$$q_u = \frac{Q_u}{b \times d} = \frac{590 \times 1000}{500 \times 1050} = 1.124\ N/mm^2$$

Step 1.2: Torsional Stresses

Assume concrete cover of 45 mm to the *centerline* of the stirrup all around the cross section



$p_h = 2(1210 + 1010) = 4440\ mm$

$A_{oh} = 1210 \times 1010 = 1222100\ mm^2$

$A_o = 0.85 A_{oh} = 0.85 \times 1222100 = 1038785\ mm^2$

$t_e = \dfrac{A_{oh}}{p_h} = \dfrac{1222100}{4440}$ =275.2 mm>t_{actual}(250 mm)

Use $t_e = t_{actual}$=250 mm

M_{tu}=315 kN.m

$$q_{tu} = \frac{M_{tu}}{2 \times A_o \times t_e} = \frac{315 \times 10^6}{2 \times 1038785 \times 250} = 0.606\ N/mm^2$$

$$q_{tu,\min} = 0.06\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.06\sqrt{\frac{30}{1.5}} = 0.27$$

Since q_{tu}(0.606)>q_{tumin} (0.27) then torsion can not be neglected

Step 2: Check the adequacy of the cross-section dimensions

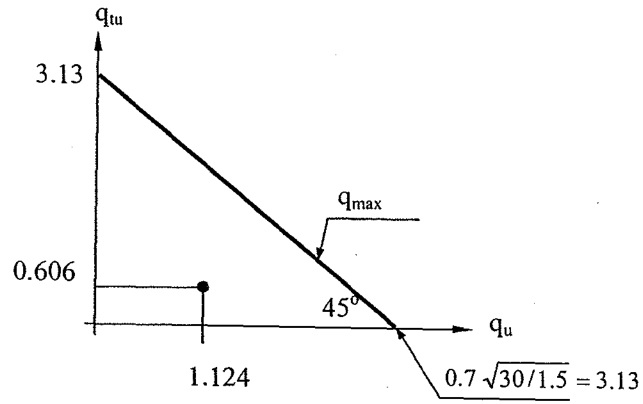
For box section, use the following equations is applied

$q_{max} = 0.7\ \sqrt{30/1.5} = 3.13 \quad \leq 4\ N/mm^2$

$q_{tu} + q_u \leq q_{max}$

$0.606 + 1.124 = 1.73 \leq 3.13$ ....$o.k$

Since the previous equation is satisfied, the cross section dimensions are adequate for resisting combined shear and torsion.



Graphical representation of the interaction of the maximum stresses due to shear and torsion for box sections

Step 3: Design of closed stirrups for shear and torsion

Step 3.1: Area of stirrups for shear

The concrete shear strength q_{cu} equals

$$q_{cu} = 0.24\sqrt{\frac{30}{1.5}} = 1.07\ N/mm^2$$

Since the applied shear (1.124)is greater than q_{cu} (1.07)shear reinforcement is needed

$$q_{su} = q_u - \frac{q_{cu}}{2} = 1.124 - \frac{1.07}{2} = 0.59\ \ N/mm^2$$

Assume spacing s=100 mm

$$A_{st} = \frac{q_{su} \times b \times s}{f_y/1.15} = \frac{0.59 \times 500 \times 100}{240/1.15} = 140.67\ mm^2$$

Since two stirrups is used and each one have two branches as shown in figure Area required for shear for one branch of the stirrup equals $A_{st}/4$=35.17 mm^2

Step 3.2: Area of stirrups for torsion

The area of one branch A_{str}

$$A_{str} = \frac{M_{tu} \times s}{2 \times A_o \times f_{yst} / \gamma_s} = \frac{315 \times 10^6 \times 100}{2 \times 1038785 \times 240 / 1.15} = 72.65\ mm^2$$

For box section the code permits (4-2-3-5-b) the use of reinforcement along the interior and exterior sides of each web if the wall thickness t_w is less or equal than the section width/6.

$\because t_w(250) > \frac{1100}{6}$ and $t_w(250) > \frac{1300}{6}$, only the external leg is considered in calculations of torsional reinforcement as shown in the figure below.

A_{str} for one branch $= 72.65\ mm^2$

Step 3.3: Stirrups for combined shear and torsion

A-Flanges

The area of the stirrups is required for torsion only (one branch) $= 72.65\ mm^2$

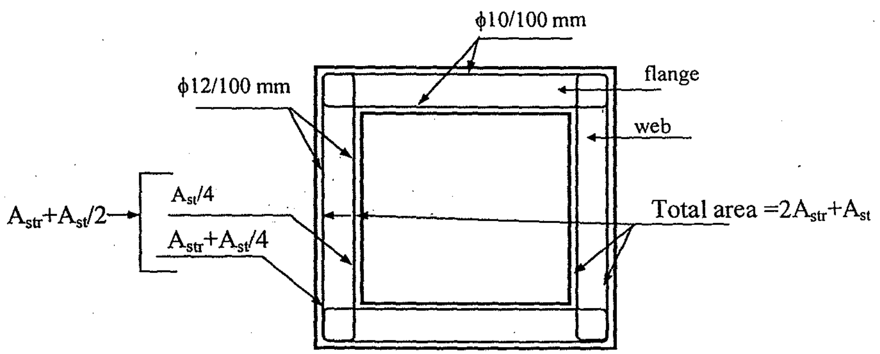
Thus choose ϕ 10/100 mm ($78.5\ mm^2$).

B-Webs

The area required for one branch of the exterior leg for shear and torsion $= A_{str} + A_{st}/4$
$= 72.65 + 35.17 = 107.8\ mm^2$

Thus choose ϕ12/100 mm ($113\ mm^2$)

The area required for one branch of the interior leg for shear $= 35.17\ mm^2$



Check A_{smin}

$$\left(2A_{str} + A_{st}\right)_{min} = \frac{0.40}{f_y} b \times s = \frac{0.40}{240} 500 \times 100 = 83.33\ mm^2$$

$A_{st,chosen} = 4 \times 113 = 452\ mm^2 > 83.33$ ......ok

Step 4: Design of Longitudinal reinforcement

$$A_{sl} = \frac{A_{str} \times p_h}{s} \left(\frac{f_{yst}}{f_y}\right) = \frac{72.65 \times 4440}{100} \left(\frac{240}{400}\right) = 1935\ mm^2$$

Calculate the minimum area for longitudinal reinforcement $A_{sl,min}$

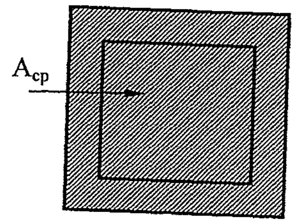
Since the chosen stirrup is for combined shear and torsion, use the calculated A_{str}

$$A_{sl,min} = \frac{0.40 \sqrt{\frac{f_{cu}}{\gamma_c}} A_{cp}}{f_y / \gamma_s} - \left(\frac{A_{str}}{s}\right) \times p_h \left(\frac{f_{yst}}{f_y}\right)$$

There is a condition on this equation that $\frac{A_{str}}{s} \geq \frac{b}{6 \times f_{yst}}$ (code 4-2-3-5-c)

$\frac{72.65}{100} < \frac{1300}{6 \times 240}$ thus use $\frac{b}{6 \times f_{yst}}$

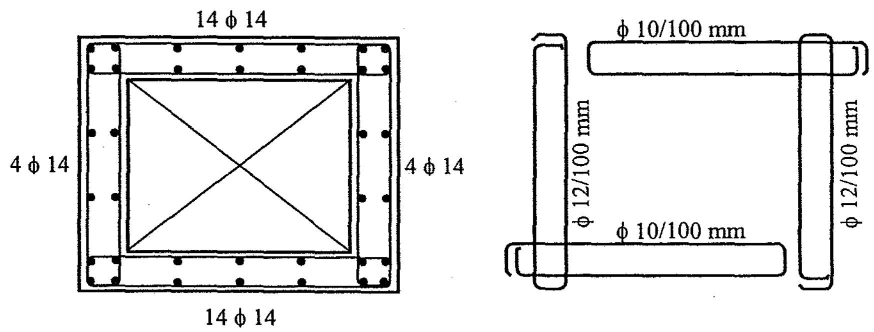
$A_{cp} = 1300 \times 1100 = 1430000\ mm^2$



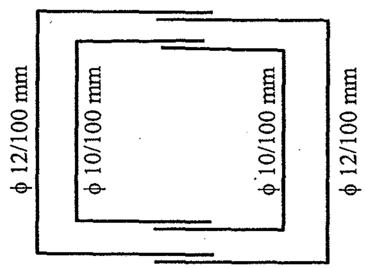
$$A_{sl,min} = \frac{0.40 \sqrt{\frac{30}{1.5}} \times 1430000}{400/1.15} - \frac{1300}{6 \times 240} \times 4440 \times \left(\frac{240}{400}\right) = 4949\ mm^2$$

Since $A_{sl} < A_{sl,min}$ use $A_{sl,min}$

The bar diameter chosen should be greater than 12mm or s/15(6.67 mm)
Choose 36 ϕ 14(5541 mm^2) such that the maximum spacing between longitudinal steel is less than 300 mm.



Note : Another alternative for stirrups arrangement is given below. Note also that the internal stirrup is taken as ϕ 10/100 mm since it is only resist shear stresses.



Alternative stirrups detail

Example 8.7

The Figure below shows a box section that constitutes the cross-section of the girder of a road-way bridge. Structural analysis of the bridge revealed that the critical section of the girder near the support is subject to the following straining actions:

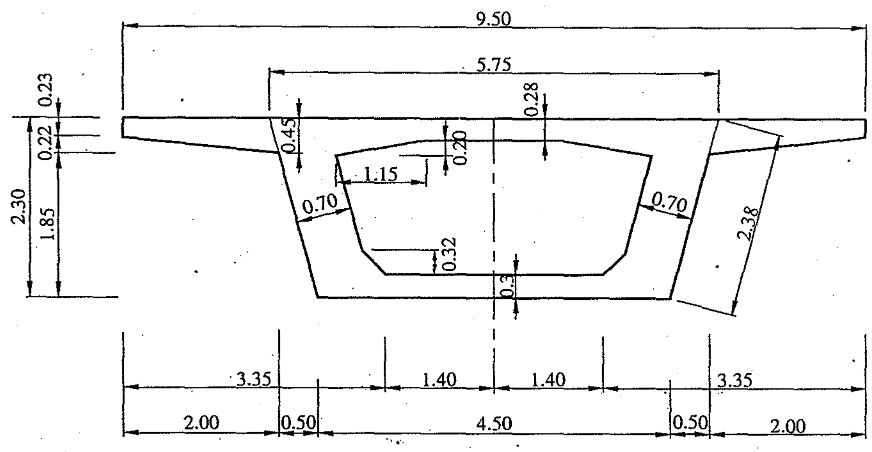
$Q_u = 6060$ kN

$M_{tu}=11700$ kN.m

It is required to carryout a design for the combined shear and torsion for that section. The material properties are as follows:

$f_{cu}= 35$ N/mm^2

$f_y = 360$ N/mm^2

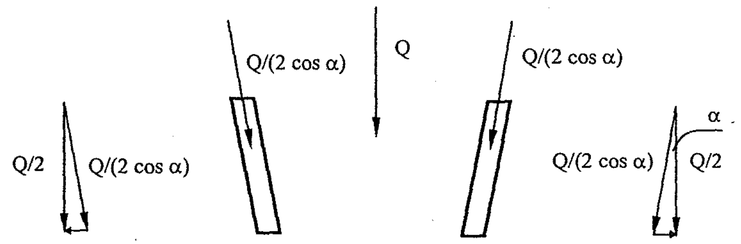


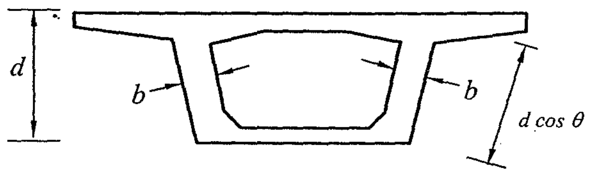
Solution

Step 1: Shear and Torsional Stresses

Step 1.1 Shear Stresses

The applied vertical shear force is resisted by the internal shear stresses developed in each web as shown in the following figure.





$$q_u = \frac{Q_u/2\times\cos\alpha}{b\times d/\cos\alpha} = \frac{Q_u/2}{b\times d}$$

Assume a concrete cover of 80 mm to the centerline of the longitudinal reinforcement

$$d = 2300 - 80 = 2220\ mm$$

$$q_u = \frac{Q_u}{b\times d} = \frac{6060/2\times1000}{700\times2220} = 1.95\ N/mm^2$$

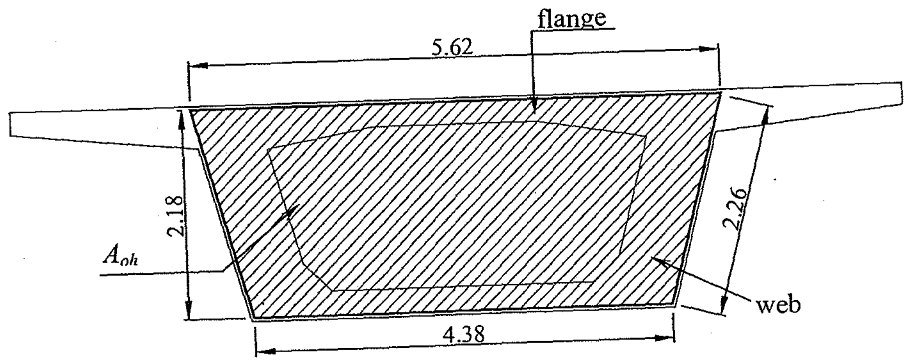
Note: Since the angle α is relatively small, the horizontal component can be neglected.

Step 1.2 Torsional Stresses

Assume a concrete cover of 60 mm to the *centerline* of the transversal steel around the cross section. As illustrated in the figure shown below

$$A_{oh} = 2.18\,\frac{4.38+5.62}{2} = 10.9\ m^2$$

$$p_h = 4.38 + 5.62 + 2.26\times2 = 14.52\ m$$



$$t_e = \frac{A_{oh}}{p_h} = \frac{10.9\times1000}{14.52} = 750\ mm$$

Since the effective thickness (t_e) is less than both the web thickness and the flange thickness, use the actual thickness.

Use $t_e = t_{actual}$

$$q_{tu(web)} = \frac{M_{tu}}{2\times A_o\times t_e} = \frac{11700\times10^6}{2\times(0.85\times10.9\times10^6)\times700} = 0.902\ N/mm^2$$

The top flange is more critical because its thickness is smaller than the bottom one. Thus the torsion stress in the slab (flange) equals

$$q_{tu(flange)} = \frac{M_{tu}}{2\times A_o\times t_e} = \frac{11700\times10^6}{2\times(0.85\times10.9\times10^6)\times280} = 2.25\ N/mm^2$$

$$q_{tu,\min} = 0.06\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.06\sqrt{\frac{35}{1.5}} = 0.29\ N/mm^2$$

Since $q_{tu} > q_{tumin}$ then torsion can not be neglected

Step 2: Check the adequacy of the cross-section dimensions

It should be noted that the flanges of the box-section (the top and bottom flanges) are subjected to shear stresses due to torsion only, while the webs are subjected to shear stresses due to combined shear and torsion.

For the flanges (top or bottom flanges)

The top flange is more critical because its thickness is smaller than the bottom one.

$$q_{u,\max} = 0.70\sqrt{\frac{f_{cu}}{\gamma_c}} = 0.70\times\sqrt{\frac{35}{1.5}} = 3.38\ N/mm^2\ < 4\ N/mm^2$$

$$q_{u,\max} = 3.38\ N/mm^2$$

Since q_{tu} (2.25)<$q_{tu,max}$, the flange dimensions are adequate for torsion.

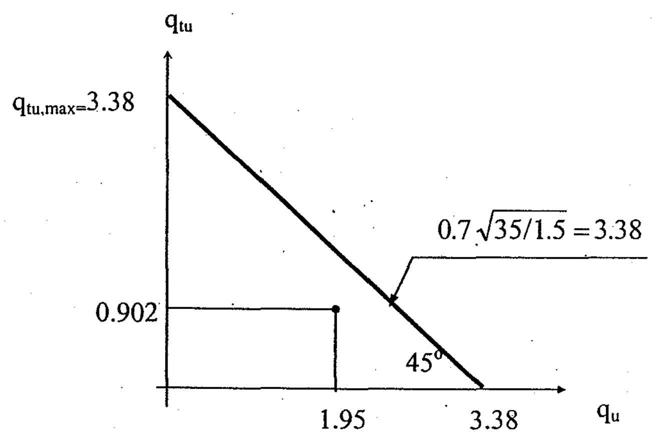
For the webs

For box sections that subjected to combined shear and torsion, the following relation should be satisfied

$$q_{tu} + q_u \le q_{u\max} \le 3.38\ N/mm^2$$

$$0.902 + 1.95 = 2.85 \le 3.38\ \ ....o.k$$

Since $q_u < q_{u,max}$ and $q_{tu} < q_{tu,max}$, the cross-section dimensions are adequate.



Graphical representation of the interaction of the maximum stresses due to shear and torsion for box sections

Step 3: Design of closed stirrups for shear and torsion

Step 3.1: Area of stirrups for shear

The concrete shear strength q_{cu} equals

$$q_{cu} = 0.24\sqrt{\frac{35}{1.5}} = 1.16\ N/mm^2$$

Since the applied shear is greater than q_{cu}, stirrups for shear are needed

$$q_{su} = q_u - \frac{q_{cu}}{2} = 1.95 - \frac{1.16}{2} = 1.37\ \ N/mm^2$$

Assume spacing s=100 mm

The area of stirrups for the two webs equals

$$A_{st} = \frac{q_{su}\times b\times s}{f_y/1.15} = \frac{1.37\times(2\times700)\times100}{360/1.15} = 612\ mm^2$$

For one web = 612/2=306 mm^2

Area of one branch=305.9/2=153 mm^2

Step 3.2: Area of stirrups for torsion

$$A_{str} = \frac{M_{tu}\times s}{2\times A_o\times f_{yst}/\gamma_s} = \frac{11700\times10^6\times100}{2\times(0.85\times10.9\times10^6)\times360/1.15} = 201\ mm^2$$

For box section the code permits (4-2-3-5-b) the use of reinforcement along the interior and exterior sides of each web if the wall thickness t_w is less or equal than the section width/6.

$$\because t_w(700) < \frac{b_{avg}}{6} < \frac{(4.5+5.75)\times1000/2}{6}$$

The area of the stirrups for torsion can be divided on the two sides.

Area of one branch A_{str}= 201/2 =100.5 mm^2

Step 3.3: Stirrups for combined shear and torsion

A-Flanges

The area of the stirrups is required for torsion only (one branch) =153 mm^2

Thus choose Φ 14/100 mm (154 mm^2). The top flange is subjected to direct live load that causes other straining action in the transverse detraction. The designer should be aware that the chosen reinforcement for torsion might be increased to take care of the additional stresses due to other straining actions.

B-Webs

Area required for shear and torsion for one branch $= A_{str}+A_{st}=100.5+153=253.5\ mm^2$

Thus choose ϕ 18/100 mm (*254 mm²*)

Check $A_{st\ min}$ (for the two webs)

$$2A_{str}+A_{st})_{min}=\frac{0.40}{f_y}b\times s=\frac{0.40}{360}(2\times700)\times100=156\ mm^2$$

$$A_{st,chosen}=4\times254=1016\ mm^2>156\ .....ok$$

Check $A_{st\ min}$ (for the two flanges)

$$2A_{str}+A_{st})_{min}=\frac{0.40}{f_y}b\times s=\frac{0.40}{360}(280+300)\times100=64.4\ mm^2$$

$$A_{st,chosen}=4\times154=616\ mm^2>64.84\ ......ok$$

Final design two stirrups ϕ18/100 mm (two branches) in the webs and two stirrups ϕ14/100 mm (two branches) in the flanges

Step 4 Design of Longitudinal reinforcement

$$A_{sl}=\frac{A_{str}\times p_h}{s}\left(\frac{f_{yst}}{f_y}\right)=\frac{201\times14.52\times1000}{100}\left(\frac{360}{360}\right)=29286\ mm^2$$

Calculate the minimum area for longitudinal reinforcement $A_{sl,min}$(code Eq. 4-53-b). Since the chosen stirrup is for combined shear and torsion, use the calculated A_{str}

$$A_{sl,min}=\frac{0.4\sqrt{\frac{f_{cu}}{\gamma_c}}A_{cp}}{f_y/\gamma_s}-\left(\frac{A_{str}}{s}\right)\times p_h\left(\frac{f_{yst}}{f_y}\right)$$

$$A_{cp}=2300\ \frac{5750+4500}{2}=11787500\ mm^2$$

There is a condition on this equation that $\frac{A_{str}}{s}\geq\frac{b_{avg}}{6\times f_{yst}}$ (code 4-2-3-5-c)

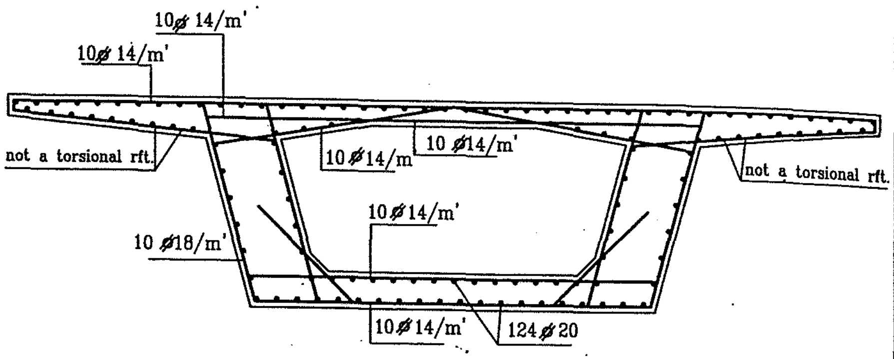
$\frac{201}{100}<\frac{0.5(4500+5750)}{7\times360/1.15}$ thus use $\frac{b_{avg}}{6\times f_{yst}}$

$$A_{sl,min}=\frac{0.4\sqrt{\frac{35}{1.5}}\times11787500}{360/1.15}-\frac{0.5(4500+5750)}{6\times360}\times14.52\times1000\left(\frac{360}{360}\right)=38304.04\ mm^2$$

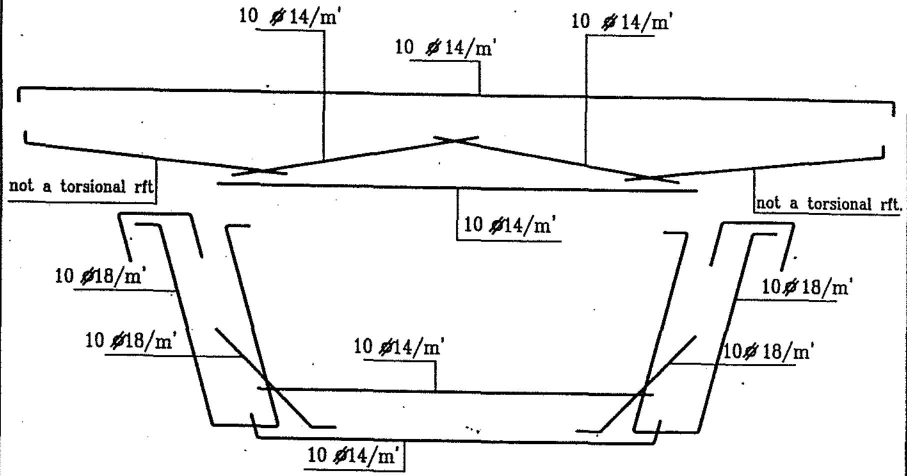
Since $A_{sl}<A_{sl,min}$ use $A_{sl,min}$

The bar diameter chosen should be greater than 12mm

Choose 124 Φ 20 such that the maximum spacing between longitudinal steel is less than 300 mm. Reinforcement details for the cross section are shown in Fig EX 8.7.



Note: Longitudinal flexural reinforcement is not shown



Reinforcement details

APPENDIX A

Design Charts for Sections Subjected to Flexure

Area of Steel Bars in cm^2 (used in Egypt)

Φ	Weight	Cross sectional area (cm^2)											
mm	kg/m'	1	2	3	4	5	6	7	8	9	10	11	12
6	0.222	0.28	0.57	0.85	1.13	1.41	1.70	1.98	2.26	2.54	2.83	3.11	3.39
8	0.395	0.50	1.01	1.51	2.01	2.51	3.02	3.52	4.02	4.52	5.03	5.53	6.03
10	0.617	0.79	1.57	2.36	3.14	3.93	4.71	5.50	6.28	7.07	7.85	8.64	9.42
12	0.888	1.13	2.26	3.39	4.52	5.65	6.79	7.92	9.05	10.18	11.31	12.44	13.57
14	1.208	1.54	3.08	4.62	6.16	7.70	9.24	10.78	12.32	13.85	15.39	16.93	18.47
16	1.578	2.01	4.02	6.03	8.04	10.05	12.06	14.07	16.08	18.10	20.11	22.12	24.13
18	1.998	2.54	5.09	7.63	10.18	12.72	15.27	17.81	20.36	22.90	25.45	27.99	30.54
20	2.466	3.14	6.28	9.42	12.57	15.71	18.85	21.99	25.13	28.27	31.42	34.56	37.70
22	2.984	3.80	7.60	11.40	15.21	19.01	22.81	26.61	30.41	34.21	38.01	41.81	45.62
25	3.853	4.91	9.82	14.73	19.63	24.54	29.45	34.36	39.27	44.18	49.09	54.00	58.90
28	4.834	6.16	12.32	18.47	24.63	30.79	36.95	43.10	49.26	55.42	61.58	67.73	73.89
32	6.313	8.04	16.08	24.13	32.17	40.21	48.25	56.30	64.34	72.38	80.42	88.47	96.51
38	8.903	11.34	22.68	34.02	45.36	56.71	68.05	79.39	90.73	102.1	113.4	124.8	136.1

Area of Other Steel Bars in cm^2

Φ	Weight	Cross sectional area (cm^2)											
mm	kg/m'	1	2	3	4	5	6	7	8	9	10	11	12
6	0.222	0.28	0.57	0.85	1.13	1.41	1.70	1.98	2.26	2.54	2.83	3.11	3.39
8	0.395	0.50	1.01	1.51	2.01	2.51	3.02	3.52	4.02	4.52	5.03	5.53	6.03
10	0.617	0.79	1.57	2.36	3.14	3.93	4.71	5.50	6.28	7.07	7.85	8.64	9.42
13	1.042	1.33	2.65	3.98	5.31	6.64	7.96	9.29	10.62	11.95	13.27	14.60	15.93
16	1.578	2.01	4.02	6.03	8.04	10.05	12.06	14.07	16.08	18.10	20.11	22.12	24.13
19	2.226	2.84	5.67	8.51	11.34	14.18	17.01	19.85	22.68	25.52	28.35	31.19	34.02
22	2.984	3.80	7.60	11.40	15.21	19.01	22.81	26.61	30.41	34.21	38.01	41.81	45.62
25	3.853	4.91	9.82	14.73	19.63	24.54	29.45	34.36	39.27	44.18	49.09	54.00	58.90
28	4.834	6.16	12.32	18.47	24.63	30.79	36.95	43.10	49.26	55.42	61.58	67.73	73.89
32	6.313	8.04	16.08	24.13	32.17	40.21	48.25	56.30	64.34	72.38	80.42	88.47	96.5
38	8.903	11.34	22.68	34.02	45.36	56.71	68.05	79.39	90.73	102.1	113.4	124.8	136.1

Area of Steel Bars in mm^2 (used in Egypt)

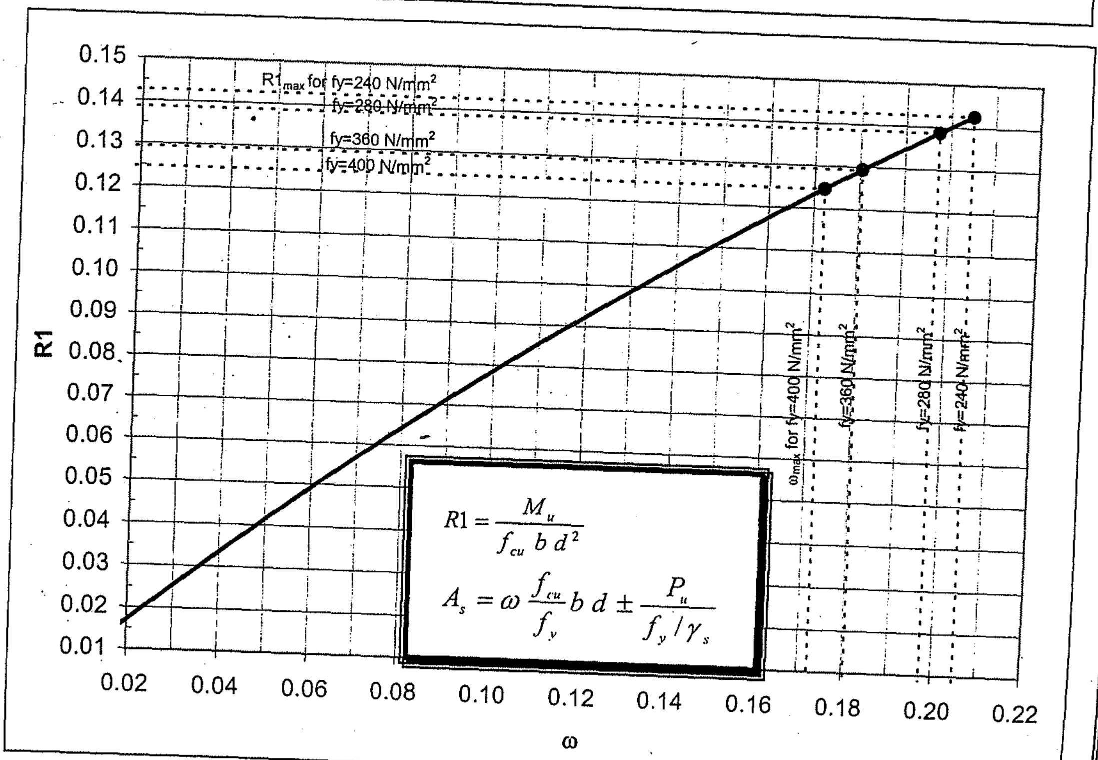
Φ	Weight	Cross sectional area (mm^2)											
mm	kg/m'	1	2	3	4	5	6	7	8	9	10	11	12
6	0.222	28.3	56.5	84.8	113	141	170	198	226	254	283	311	339
8	0.395	50.3	101	151	201	251	302	352	402	452	503	553	603
10	0.617	78.5	157	236	314	393	471	550	628	707	785	864	942
12	0.888	113	226	339	452	565	679	792	905	1018	1131	1244	1357
14	1.208	154	308	462	616	770	924	1078	1232	1385	1539	1693	1847
16	1.578	201	402	603	804	1005	1206	1407	1608	1810	2011	2212	2413
18	1.998	254	509	763	1018	1272	1527	1781	2036	2290	2545	2799	3054
20	2.466	314	628	942	1257	1571	1885	2199	2513	2827	3142	3456	3770
22	2.984	380	760	1140	1521	1901	2281	2661	3041	3421	3801	4181	4562
25	3.853	491	982	1473	1963	2454	2945	3436	3927	4418	4909	5400	5890
28	4.834	616	1232	1847	2463	3079	3695	4310	4926	5542	6158	6773	7389
32	6.313	804	1608	2413	3217	4021	4825	5630	6434	7238	8042	8847	9651
38	8.903	1134	2268	3402	4536	5671	6805	7939	9073	10207	11341	12475	13609

Area of Other Steel Bars in mm^2

Φ	Weight	Cross sectional area (mm^2)											
mm	kg/m'	1	2	3	4	5	6	7	8	9	10	11	12
6	0.222	28.3	56.5	84.8	113.1	141.4	170	198	226	254	283	311	339
8	0.395	50.3	100.5	151	201	251	302	352	402	452	503	553	603
10	0.617	79	157	236	314	393	471	550	628	707	785	864	942
13	1.042	133	265	398	531	664	796	929	1062	1195	1327	1460	1593
16	1.578	201	402	603	804	1005	1206	1407	1608	1810	2011	2212	2413
19	2.226	284	567	851	1134	1418	1701	1985	2268	2552	2835	3119	3402
22	2.984	380	760	1140	1521	1901	2281	2661	3041	3421	3801	4181	4562
25	3.853	491	982	1473	1963	2454	2945	3436	3927	4418	4909	5400	5890
28	4.834	616	1232	1847	2463	3079	3695	4310	4926	5542	6158	6773	7389
32	6.313	804	1608	2413	3217	4021	4825	5630	6434	7238	8042	8847	9651
38	8.903	1134	2268	3402	4536	5671	6805	7939	9073	10207	11341	12475	13609

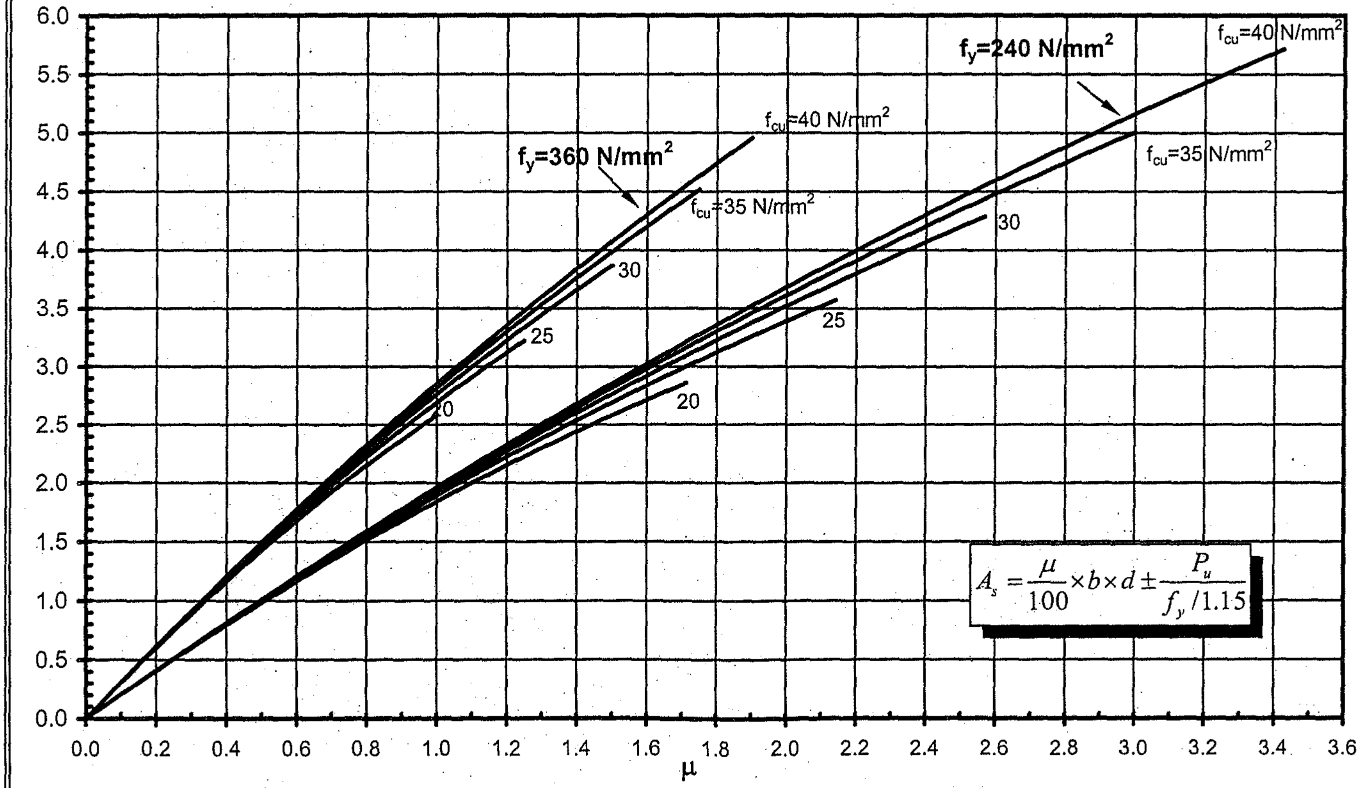
DESIGN CHART FOR SECTIONS SUBJECTED TO SIMPLE BENDING (Table 4-1)

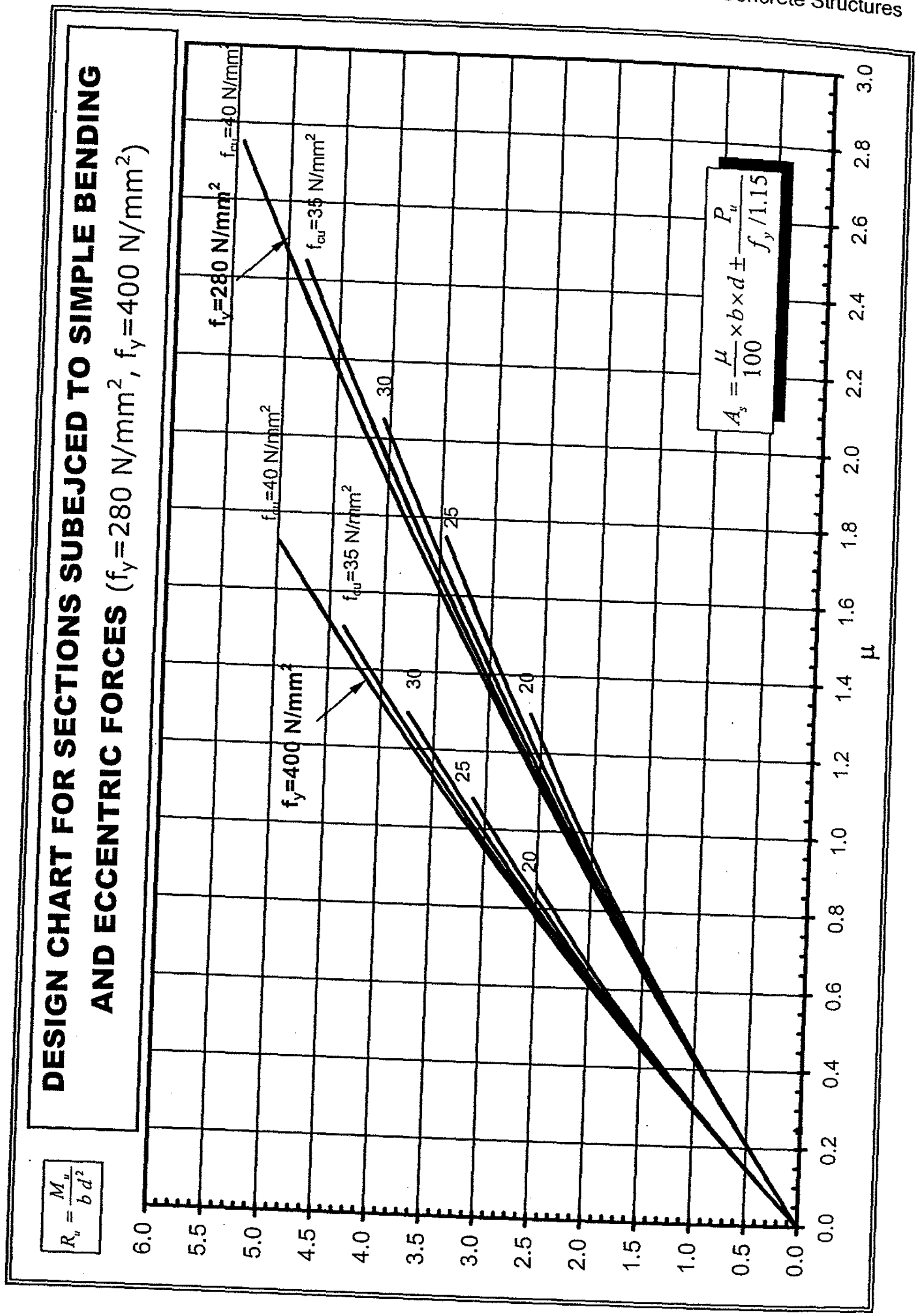
R1	ω	
0.015	0.018	
0.020	0.024	
0.025	0.030	
0.030	0.036	
0.035	0.042	
0.040	0.048	
0.045	0.055	
0.050	0.061	
0.055	0.068	
0.060	0.074	
0.065	0.081	
0.070	0.088	
0.075	0.095	
0.080	0.102	
0.085	0.109	
0.090	0.117	
0.095	0.124	
0.100	0.132	
0.105	0.140	
0.110	0.148	
0.115	0.156	
0.120	0.164	$R1_{max}$
0.125	0.173	fy=400
0.129	0.180	fy=360
0.139	0.198	fy=280
0.143	0.206	fy=240



$$R_u = \frac{M_u}{b\, d^2}$$

DESIGN CHART FOR SECTIONS SUBEJCED TO SIMPLE BENDING AND ECCENTRIC FORCES (f_y=240 N/mm², f_y=360 N/mm²)





Design Tables for Sections Subjected to Simple Bending (R-μ)

$R_u = \frac{M_u}{b\,d^2}$	μ (%) values for fy=240 N/mm^2				
	f_{cu}=40	f_{cu}=35	f_{cu}=30	f_{cu}=25	f_{cu}=20
0.5	0.25	0.25	0.25	0.25	0.25
0.6	0.29	0.29	0.29	0.30	0.30
0.7	0.34	0.34	0.34	0.35	0.35
0.8	0.39	0.39	0.40	0.40	0.40
0.9	0.44	0.44	0.45	0.45	0.46
1.0	0.49	0.50	0.50	0.50	0.51
1.1	0.54	0.55	0.55	0.56	0.56
1.2	0.60	0.60	0.60	0.61	0.62
1.3	0.65	0.65	0.66	0.66	0.68
1.4	0.70	0.70	0.71	0.72	0.73
1.5	0.75	0.76	0.76	0.77	0.79
1.6	0.80	0.81	0.82	0.83	0.85
1.7	0.86	0.86	0.87	0.89	0.91
1.8	0.91	0.92	0.93	0.95	0.97
1.9	0.96	0.97	0.99	1.00	1.04
2.0	1.02	1.03	1.04	1.06	1.10
2.1	1.07	1.08	1.10	1.12	1.16
2.2	1.13	1.14	1.16	1.19	1.23
2.3	1.18	1.20	1.22	1.25	1.30
2.4	1.24	1.26	1.28	1.31	1.37
2.5	1.30	1.31	1.34	1.37	1.44
2.6	1.35	1.37	1.40	1.44	1.51
2.7	1.41	1.43	1.46	1.51	1.59
2.8	1.47	1.49	1.52	1.57	1.67
2.9	1.53	1.55	1.59	1.64	
3.0	1.58	1.61	1.65	1.71	
3.1	1.64	1.67	1.71	1.78	
3.2	1.70	1.73	1.78	1.85	
3.3	1.76	1.80	1.85	1.93	
3.4	1.82	1.86	1.91	2.00	
3.5	1.88	1.92	1.98	2.08	
3.6	1.95	1.99	2.05	2.16	
3.7	2.01	2.05	2.12		
3.8	2.07	2.12	2.20		
3.9	2.13	2.19	2.27		
4.0	2.20	2.26	2.34		
4.1	2.26	2.33	2.42		
4.2	2.33	2.40	2.50		
4.3	2.40	2.47	2.58		
4.4	2.46	2.54			
4.5	2.53	2.61			
4.6	2.60	2.69			
4.7	2.67	2.76			
4.8	2.74	2.84			
4.9	2.81	2.92			
5.0	2.88	2.99			
5.1	2.95				
5.2	3.03				
5.3	3.10				
5.4	3.18				
5.5	3.25				
5.6	3.33				
5.7	3.41				

$$A_s = \frac{\mu}{100} b \times d \pm \frac{P_u}{f_y/1.15}$$

$R_u = \frac{M_u}{b\,d^2}$	μ(%) values for fy=360 N/mm^2				
	f_{cu}=40	f_{cu}=35	f_{cu}=30	f_{cu}=25	f_{cu}=20
0.46	0.15	0.15	0.15	0.15	0.15
0.5	0.16	0.16	0.16	0.16	0.16
0.6	0.19	0.20	0.20	0.20	0.20
0.7	0.23	0.23	0.23	0.23	0.23
0.8	0.26	0.26	0.26	0.27	0.27
0.9	0.30	0.30	0.30	0.30	0.30
1.0	0.33	0.33	0.33	0.34	0.34
1.1	0.36	0.36	0.37	0.37	0.38
1.2	0.40	0.40	0.40	0.41	0.41
1.3	0.43	0.43	0.44	0.44	0.45
1.4	0.47	0.47	0.47	0.48	0.49
1.5	0.50	0.50	0.51	0.52	0.53
1.6	0.54	0.54	0.55	0.55	0.57
1.7	0.57	0.58	0.58	0.59	0.61
1.8	0.61	0.61	0.62	0.63	0.65
1.9	0.64	0.65	0.66	0.67	0.69
2.0	0.68	0.69	0.70	0.71	0.73
2.1	0.72	0.72	0.73	0.75	0.78
2.2	0.75	0.76	0.77	0.79	0.82
2.3	0.79	0.80	0.81	0.83	0.87
2.4	0.83	0.84	0.85	0.87	0.91
2.5	0.86	0.88	0.89	0.92	0.96
2.6	0.90	0.91	0.93	0.96	1.01
2.7	0.94	0.95	0.97	1.00	
2.8	0.98	0.99	1.01	1.05	
2.9	1.02	1.03	1.06	1.09	
3.0	1.06	1.07	1.10	1.14	
3.1	1.10	1.11	1.14	1.19	
3.2	1.14	1.16	1.19	1.24	
3.3	1.18	1.20	1.23	1.29	
3.4	1.22	1.24	1.28		
3.5	1.26	1.28	1.32		
3.6	1.30	1.33	1.37		
3.7	1.34	1.37	1.42		
3.8	1.38	1.41	1.46		
3.9	1.42	1.46	1.51		
4.0	1.47	1.50			
4.1	1.51	1.55			
4.2	1.55	1.60			
4.3	1.60	1.64			
4.4	1.64	1.69			
4.5	1.69	1.74			
4.6	1.73	1.79			
4.7	1.78				
4.8	1.83				
4.9	1.87				
5.0	1.92				
5.1	1.97				
5.17	2.00				

$$A_s = \frac{\mu}{100} b \times d \pm \frac{P_u}{f_y/1.15}$$

1. Assume μ =0.5-1%
2. Determine R from the table
3. Compute d then compute A_s

Design Tables for sections subjected to simple bending(R-μ)

$R_u = \frac{M_u}{b d^2}$	μ (%) values for fy=280 N/mm²				
	f_{cu}=40	f_{cu}=35	f_{cu}=30	f_{cu}=25	f_{cu}=20
0.6	0.25	0.25	0.25	0.25	0.26
0.7	0.29	0.29	0.30	0.30	0.30
0.8	0.34	0.34	0.34	0.34	0.34
0.9	0.38	0.38	0.38	0.39	0.39
1.0	0.42	0.42	0.43	0.43	0.44
1.1	0.47	0.47	0.47	0.48	0.48
1.2	0.51	0.51	0.52	0.52	0.53
1.3	0.55	0.56	0.56	0.57	0.58
1.4	0.60	0.60	0.61	0.62	0.63
1.5	0.64	0.65	0.66	0.66	0.68
1.6	0.69	0.69	0.70	0.71	0.73
1.7	0.74	0.74	0.75	0.76	0.78
1.8	0.78	0.79	0.80	0.81	0.83
1.9	0.83	0.83	0.85	0.86	0.89
2.0	0.87	0.88	0.89	0.91	0.94
2.1	0.92	0.93	0.94	0.96	1.00
2.2	0.97	0.98	0.99	1.02	1.06
2.3	1.01	1.03	1.04	1.07	1.11
2.4	1.06	1.08	1.09	1.12	1.17
2.5	1.11	1.13	1.15	1.18	1.23
2.6	1.16	1.18	1.20	1.23	1.30
2.7	1.21	1.23	1.25	1.29	1.36
2.8	1.26	1.28	1.30	1.35	
2.9	1.31	1.33	1.36	1.41	
3.0	1.36	1.38	1.41	1.47	
3.1	1.41	1.43	1.47	1.53	
3.2	1.46	1.49	1.53	1.59	
3.3	1.51	1.54	1.58	1.65	
3.4	1.56	1.59	1.64	1.72	
3.5	1.62	1.65	1.70		
3.6	1.67	1.70	1.76		
3.7	1.72	1.76	1.82		
3.8	1.78	1.82	1.88		
3.9	1.83	1.88	1.95		
4.0	1.88	1.93	2.01		
4.1	1.94	1.99	2.08		
4.2	2.00	2.05			
4.3	2.05	2.11			
4.4	2.11	2.18			
4.5	2.17	2.24			
4.6	2.23	2.30			
4.7	2.29	2.37			
4.8	2.35	2.43			
4.9	2.41				
5.0	2.47				
5.1	2.53				
5.2	2.59				
5.3	2.66				
5.4	2.72				
5.5	2.79				

$$A_s = \frac{\mu}{100} b \times d + \frac{P_u}{f_y/1.15}$$

$R_u = \frac{M_u}{b d^2}$	μ(%) values for fy=400 N/mm²				
	f_{cu}=40	f_{cu}=35	f_{cu}=30	f_{cu}=25	f_{cu}=20
0.5	0.15	0.15	0.15	0.15	0.15
0.6	0.18	0.18	0.18	0.18	0.18
0.7	0.21	0.21	0.21	0.21	0.21
0.8	0.24	0.24	0.24	0.24	0.24
0.9	0.27	0.27	0.27	0.27	0.27
1.0	0.30	0.30	0.30	0.30	0.31
1.1	0.33	0.33	0.33	0.33	0.34
1.2	0.36	0.36	0.36	0.37	0.37
1.3	0.39	0.39	0.39	0.40	0.41
1.4	0.42	0.42	0.43	0.43	0.44
1.5	0.45	0.45	0.46	0.46	0.48
1.6	0.48	0.49	0.49	0.50	0.51
1.7	0.51	0.52	0.52	0.53	0.55
1.8	0.55	0.55	0.56	0.57	0.58
1.9	0.58	0.58	0.59	0.60	0.62
2.0	0.61	0.62	0.63	0.64	0.66
2.1	0.64	0.65	0.66	0.67	0.70
2.2	0.68	0.68	0.70	0.71	0.74
2.3	0.71	0.72	0.73	0.75	0.78
2.4	0.74	0.75	0.77	0.79	0.82
2.5	0.78	0.79	0.80	0.82	0.86
2.6	0.81	0.82	0.84	0.86	
2.7	0.85	0.86	0.88	0.90	
2.8	0.88	0.89	0.91	0.94	
2.9	0.92	0.93	0.95	0.98	
3.0	0.95	0.97	0.99	1.03	
3.1	0.99	1.00	1.03	1.07	
3.2	1.02	1.04	1.07	1.11	
3.3	1.06	1.08	1.11		
3.4	1.09	1.12	1.15		
3.5	1.13	1.15	1.19		
3.6	1.17	1.19	1.23		
3.7	1.21	1.23	1.27		
3.8	1.24	1.27	1.32		
3.9	1.28	1.31			
4.0	1.32	1.35			
4.1	1.36	1.40			
4.2	1.40	1.44			
4.3	1.44	1.48			
4.4	1.48	1.52			
4.5	1.52				
4.6	1.56				
4.7	1.60				
4.8	1.64				
4.9	1.69				
5.0	1.73				

$$A_s = \frac{\mu}{100} b \times d + \frac{P_u}{f_y/1.15}$$

1. Assume μ =0.5-1%
2. Determine R_u from the table
3. Compute d then compute As

DESIGN TABLES FOR SECTIONS SUBJECTED TO BENDING(Ku- μ)

μ (%)	K_u values for f_y=240 N/mm²				
	f_{cu}=20	f_{cu}=25	f_{cu}=30	f_{cu}=35	f_{cu}=40
0.25	1.405	1.401	1.398	1.396	1.395
0.30	1.287	1.282	1.279	1.277	1.275
0.35	1.195	1.190	1.186	1.184	1.182
0.40	1.121	1.116	1.112	1.109	1.108
0.45	1.060	1.054	1.050	1.048	1.046
0.50	1.009	1.003	0.999	0.996	0.994
0.55	0.965	0.958	0.954	0.951	0.949
0.60	0.927	0.920	0.915	0.912	0.910
0.65	0.893	0.886	0.881	0.878	0.875
0.70	0.863	0.856	0.851	0.847	0.845
0.75	0.837	0.829	0.824	0.820	0.817
0.80	0.813	0.805	0.799	0.795	0.793
0.85	0.791	0.783	0.777	0.773	0.770
0.90	0.771	0.762	0.757	0.753	0.750
0.95	0.753	0.744	0.738	0.734	0.731
1.00	0.737	0.727	0.721	0.717	0.713
1.05	0.721	0.711	0.705	0.701	0.697
1.10	0.707	0.697	0.690	0.686	0.682
1.15	0.694	0.683	0.676	0.672	0.668
1.20	0.681	0.671	0.664	0.659	0.655
1.25	0.670	0.659	0.652	0.647	0.643
1.30	0.659	0.648	0.640	0.635	0.632
1.35	0.649	0.637	0.630	0.625	0.621
1.40	0.640	0.628	0.620	0.614	0.611
1.45	0.631	0.618	0.610	0.605	0.601
1.50	0.622	0.610	0.601	0.596	0.592
1.55	0.614	0.601	0.593	0.587	0.583
1.60	0.607	0.593	0.585	0.579	0.575
1.65	0.600	0.586	0.577	0.571	0.567
1.70	0.593	0.579	0.570	0.564	0.559
1.75		0.572	0.563	0.557	0.552
1.80		0.566	0.556	0.550	0.545
1.85		0.560	0.550	0.544	0.539
1.90		0.554	0.544	0.537	0.533
1.95		0.548	0.538	0.532	0.527
2.00		0.543	0.533	0.526	0.521
2.10		0.533	0.522	0.515	0.510
2.20			0.513	0.505	0.500
2.30			0.504	0.496	0.491
2.40			0.496	0.488	0.482
2.50			0.488	0.480	0.474
2.60				0.472	0.466
2.70				0.465	0.459
2.80				0.459	0.452
2.90				0.453	0.446
3.00				0.447	0.440
3.10					0.434
3.20					0.429
3.30					0.424
3.40					0.419

μ (%)	K_u values for f_y=360 N/mm²				
	f_{cu}=20	f_{cu}=25	f_{cu}=30	f_{cu}=35	f_{cu}=40
0.15	1.479	1.475	1.472	1.470	1.469
0.20	1.287	1.282	1.279	1.277	1.275
0.25	1.156	1.151	1.147	1.145	1.143
0.30	1.060	1.054	1.050	1.048	1.046
0.35	0.986	0.980	0.976	0.973	0.970
0.40	0.927	0.920	0.915	0.912	0.910
0.45	0.878	0.870	0.866	0.862	0.860
0.50	0.837	0.829	0.824	0.820	0.817
0.55	0.802	0.793	0.788	0.784	0.781
0.60	0.771	0.762	0.757	0.753	0.750
0.65	0.745	0.735	0.729	0.725	0.722
0.70	0.721	0.711	0.705	0.701	0.697
0.75	0.700	0.690	0.683	0.679	0.675
0.80	0.681	0.671	0.664	0.659	0.655
0.85	0.665	0.653	0.646	0.641	0.637
0.90	0.649	0.637	0.630	0.625	0.621
0.95	0.635	0.623	0.615	0.610	0.606
1.00	0.622	0.610	0.601	0.596	0.592
1.05		0.597	0.589	0.583	0.579
1.10		0.586	0.577	0.571	0.567
1.15		0.575	0.566	0.560	0.556
1.20		0.566	0.556	0.550	0.545
1.25		0.557	0.547	0.540	0.536
1.30			0.538	0.532	0.527
1.35			0.530	0.523	0.518
1.40			0.522	0.515	0.510
1.45			0.515	0.508	0.502
1.50			0.508	0.501	0.495
1.55				0.494	0.488
1.60				0.488	0.482
1.65				0.482	0.476
1.70				0.476	0.470
1.75				0.470	0.464
1.80					0.459
1.85					0.454
1.90					0.449
1.95					0.444
2.00					0.440

1-Assume μ (0.5–1%)
2-From the table determine K_u
3-Determine the beam depth d

$$d = K_u \sqrt{\frac{M_u}{b}} \qquad A_s = \frac{\mu}{100} b\, d \pm \frac{P_u}{f_y/\gamma_s}$$

DESIGN TABLES FOR SECTIONS SUBJECTED TO BENDING (Ku- μ)

μ (%)	K_u values for f_y=280 N/mm²				
	f_{cu}=20	f_{cu}=25	f_{cu}=30	f_{cu}=35	f_{cu}=40
0.25	1.304	1.300	1.297	1.294	1.293
0.30	1.195	1.190	1.186	1.184	1.182
0.35	1.110	1.105	1.101	1.098	1.096
0.40	1.042	1.036	1.032	1.029	1.027
0.45	0.986	0.980	0.976	0.973	0.970
0.50	0.939	0.932	0.928	0.925	0.922
0.55	0.898	0.891	0.887	0.883	0.881
0.60	0.863	0.856	0.851	0.847	0.845
0.65	0.833	0.825	0.819	0.816	0.813
0.70	0.805	0.797	0.792	0.788	0.785
0.75	0.781	0.772	0.767	0.763	0.760
0.80	0.759	0.750	0.744	0.740	0.737
0.85	0.739	0.730	0.724	0.719	0.716
0.90	0.721	0.711	0.705	0.701	0.697
0.95	0.705	0.694	0.688	0.683	0.680
1.00	0.690	0.679	0.672	0.667	0.664
1.05	0.676	0.665	0.658	0.653	0.649
1.10	0.663	0.651	0.644	0.639	0.635
1.15	0.651	0.639	0.632	0.626	0.623
1.20	0.640	0.628	0.620	0.614	0.611
1.25	0.629	0.617	0.609	0.603	0.599
1.30	0.620	0.607	0.599	0.593	0.589
1.35	0.611	0.597	0.589	0.583	0.579
1.40	0.602	0.588	0.580	0.574	0.569
1.45		0.580	0.571	0.565	0.561
1.50		0.572	0.563	0.557	0.552
1.55		0.565	0.555	0.549	0.544
1.60		0.558	0.548	0.542	0.537
1.65		0.551	0.541	0.534	0.530
1.70		0.545	0.535	0.528	0.523
1.75		0.539	0.528	0.521	0.516
1.80			0.522	0.515	0.510
1.85			0.517	0.509	0.504
1.90			0.511	0.504	0.498
1.95			0.506	0.498	0.493
2.00			0.501	0.493	0.488
2.05			0.496	0.488	0.483
2.10			0.492	0.484	0.478
2.15				0.479	0.473
2.20				0.475	0.469
2.25				0.470	0.464
2.30				0.466	0.460
2.35				0.463	0.456
2.40				0.459	0.452
2.45				0.455	0.449
2.50					0.445
2.55					0.442
2.60					0.438
2.65					0.435
2.70					0.432

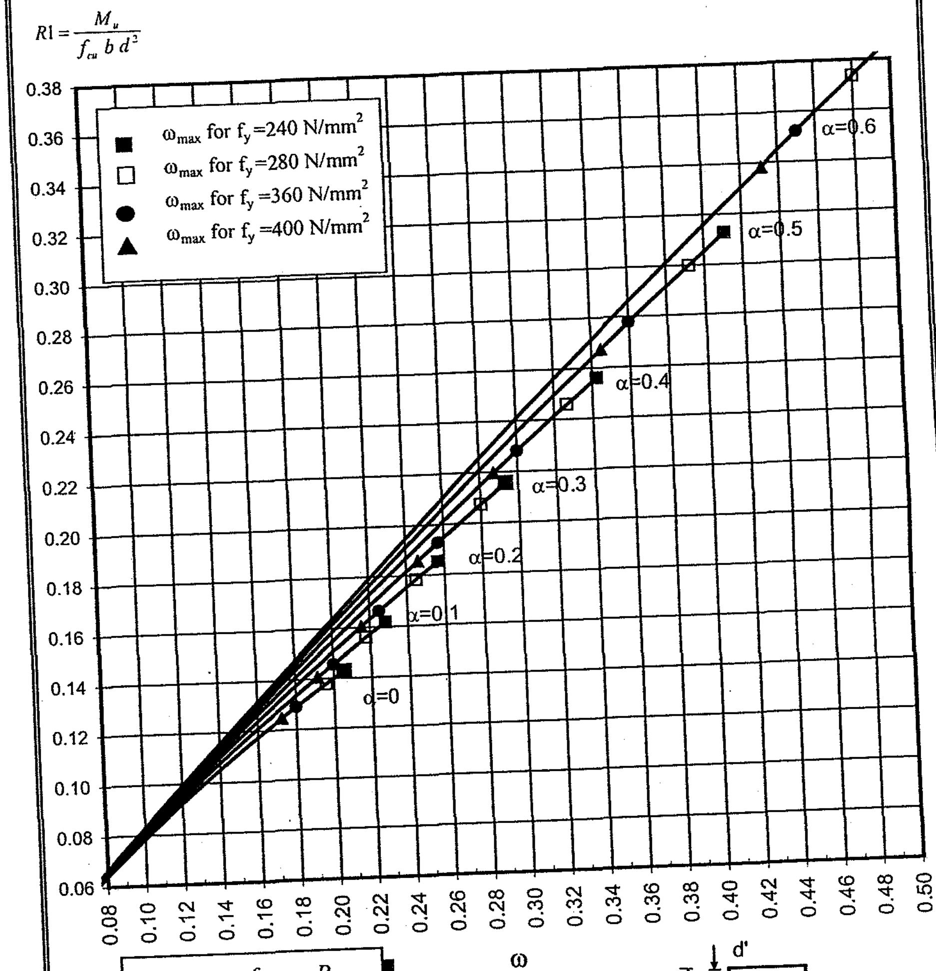
μ (%)	K_u values for f_y=400 N/mm²				
	f_{cu}=20	f_{cu}=25	f_{cu}=30	f_{cu}=35	f_{cu}=40
0.15	1.405	1.401	1.398	1.396	1.395
0.20	1.223	1.218	1.215	1.213	1.211
0.25	1.099	1.094	1.090	1.088	1.086
0.30	1.009	1.003	0.999	0.996	0.994
0.35	0.939	0.932	0.928	0.925	0.922
0.40	0.883	0.876	0.871	0.867	0.865
0.45	0.837	0.829	0.824	0.820	0.817
0.50	0.798	0.790	0.784	0.780	0.777
0.55	0.765	0.756	0.750	0.746	0.743
0.60	0.737	0.727	0.721	0.717	0.713
0.65	0.712	0.702	0.695	0.690	0.687
0.70	0.690	0.679	0.672	0.667	0.664
0.75	0.670	0.659	0.652	0.647	0.643
0.80	0.652	0.641	0.633	0.628	0.624
0.85		0.624	0.617	0.611	0.607
0.90		0.610	0.601	0.596	0.592
0.95		0.596	0.588	0.582	0.577
1.00		0.584	0.575	0.569	0.564
1.05		0.572	0.563	0.557	0.552
1.10			0.552	0.546	0.541
1.15			0.542	0.535	0.531
1.20			0.533	0.526	0.521
1.25			0.524	0.517	0.512
1.30			0.516	0.508	0.503
1.35				0.501	0.495
1.40				0.493	0.488
1.45				0.486	0.480
1.50				0.480	0.474
1.55					0.467
1.60					0.461
1.65					0.456
1.70					0.450

1-Assume μ (0.5–1%)
2-From the table determine K_u
3-Determine the beam depth d

$$d = K_u \sqrt{\frac{M_u}{b}} \qquad A_s = \frac{\mu}{100}\, b\, d \pm \frac{P_u}{f_y/\gamma_s}$$

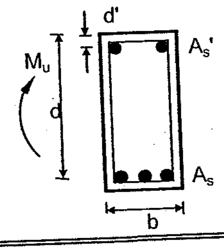
DESIGN CHART FOR DOUBLY REINFORCED SECTIONS SUBEJCED TO SIMPLE BENDING.

All types of steel (table 4-1). d'/d=0.05



$$A_s = \omega\, b\, d\, \frac{f_{cu}}{f_y} \pm \frac{P_u}{f_y/\gamma_s}$$

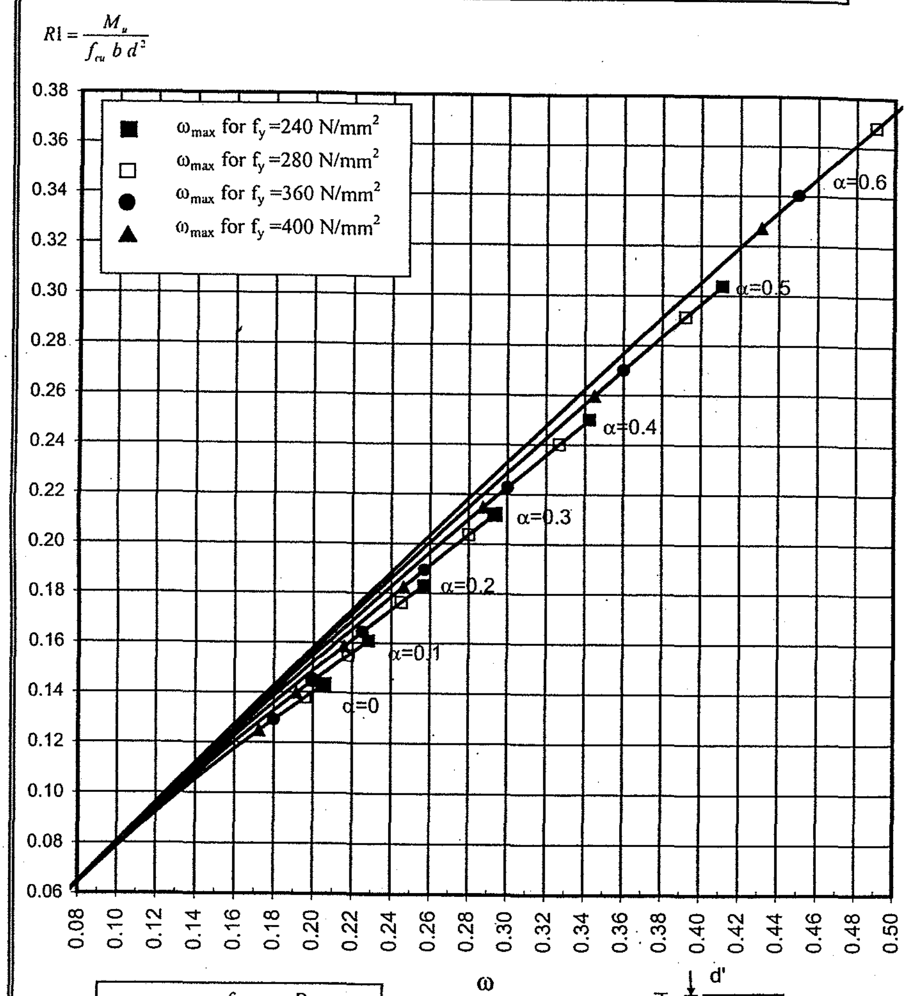
$$A'_s = \alpha\, \omega\, b\, d\, \frac{f_{cu}}{f_y}$$



In case P_u=0, $A'_s = \alpha \times A_s$

DESIGN CHART FOR DOUBLY REINFORCED SECTIONS SUBEJCED TO SIMPLE BENDING.

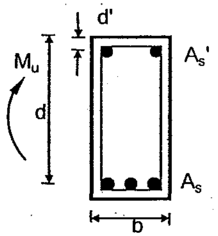
All types of steel (table 4-1). d'/d=0.10



$$A_s = \omega\, b\, d\, \frac{f_{cu}}{f_y} \pm \frac{P_u}{f_y/\gamma_s}$$

$$A_s' = \alpha\, \omega\, b\, d\, \frac{f_{cu}}{f_y}$$

In case P_u=0, $A_s' = \alpha \times A_s$



DOUBLY REINFORCED SECTIONS SUBEJCED TO SIMPLE BENDING

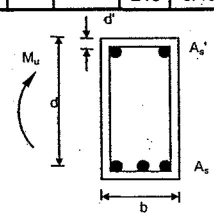
All types of steel (Table 4-1). d'/d=0.05

$R1 = \frac{M_u}{f_{cu}\, b\, d^2}$	ω											
	α = 0.0		α = 0.1		α = 0.2		α = 0.3		α = 0.4		α = 0.5	
0.01		0.012		0.012		0.012		0.012		0.012		0.012
0.02		0.023		0.023		0.023		0.024		0.024		0.024
0.03		0.036		0.036		0.036		0.036		0.036		0.036
0.04		0.048		0.048		0.048		0.048		0.048		0.048
0.05		0.061		0.061		0.060		0.060		0.060		0.060
0.06		0.074		0.073		0.073		0.072		0.072		0.072
0.07		0.088		0.087		0.086		0.085		0.085		0.084
0.08		0.102		0.100		0.099		0.098		0.097		0.097
0.09		0.117		0.114		0.112		0.111		0.110		0.109
0.10		0.132		0.128		0.126		0.124		0.123		0.122
0.11		0.147		0.143		0.140		0.137		0.136		0.134
0.12	400	0.164		0.158		0.154		0.151		0.149		0.147
0.13	360	0.181		0.174		0.169		0.165		0.162		0.160
0.14	240/280	0.199	360/400	0.190		0.184		0.179		0.175		0.172
0.15		0.221	280	0.207		0.199		0.193		0.189		0.185
0.16			240	0.225	360/400	0.215		0.207		0.202		0.198
0.17						0.231		0.222		0.216		0.212
0.18					240/280	0.247	400	0.237		0.230		0.225
0.19						0.265	360	0.252		0.244		0.238
0.20							280	0.268		0.258		0.252
0.21							240	0.284		0.273		0.265
0.22									400	0.288		0.279
0.23									360	0.302		0.292
0.24									280	0.317		0.306
0.25									240	0.333		0.320
0.26											400	0.334
0.27												0.349
0.28											360	0.363
0.29												0.377
0.30											280	0.392
0.31											240	0.407

$$R1 = \frac{M_u}{f_{cu}\, b\, d^2} \qquad \omega_{max} = \frac{\mu_{max}}{1-\alpha}\frac{f_y}{f_{cu}}$$

$$A_s = \omega\, b\, d\, \frac{f_{cu}}{f_y} \pm \frac{P_u}{f_y/\gamma_s} \qquad A_s' = \alpha\, \omega\, b\, d\, \frac{f_{cu}}{f_y}$$

In case P_u=0, $A_s' = \alpha \times A_s$



DOUBLY REINFORCED SECTIONS SUBEJCED TO SIMPLE BENDING
All types of steel (Table 4-1). d'/d=0.10

$R1=\frac{M_u}{f_{cu}\,b\,d^2}$	ω											
	α = 0.0		α = 0.1		α = 0.2		α = 0.3		α = 0.4		α = 0.5	
0.01		0.012		0.012		0.012		0.012		0.012		0.012
0.02		0.023		0.024		0.024		0.024		0.024		0.024
0.03		0.036		0.036		0.036		0.036		0.036		0.036
0.04		0.048		0.048		0.048		0.048		0.049		0.049
0.05		0.061		0.061		0.061		0.061		0.061		0.061
0.06		0.074		0.074		0.074		0.074		0.074		0.074
0.07		0.088		0.087		0.087		0.087		0.087		0.087
0.08		0.102		0.101		0.100		0.100		0.099		0.099
0.09		0.117		0.115		0.114		0.113		0.112		0.112
0.10		0.132		0.129		0.128		0.126		0.125		0.125
0.11		0.147		0.144		0.142		0.140		0.139		0.138
0.12	400	0.164		0.159		0.156		0.154		0.152		0.151
0.13	360	0.181		0.175		0.171		0.168		0.166		0.164
0.14	240/280	0.199	360/400	0.192		0.186		0.182		0.179		0.177
0.15			280	0.209		0.202		0.197		0.193		0.191
0.16			240	0.226	400/360	0.218		0.211		0.207		0.204
0.17					280	0.234		0.227		0.221		0.218
0.18					240	0.251	400	0.242		0.236		0.231
0.19							360	0.258		0.250		0.245
0.20							280	0.274		0.265		0.259
0.21							240	0.290	400	0.280		0.273
0.22									360	0.295		0.287
0.23										0.311		0.302
0.24									280	0.326		0.316
0.25									240	0.341		0.330
0.26											400	0.345
0.27											360	0.360
0.28												0.375
0.29											280	0.390
0.30											240	0.405

$$R1=\frac{M_u}{f_{cu}\,b\,d^2} \qquad \omega_{max}=\frac{\mu_{max}}{1-\alpha}\,\frac{f_y}{f_{cu}}$$

$$A_s=\omega\,b\,d\,\frac{f_{cu}}{f_y}\pm\frac{P_u}{f_y/\gamma_s} \qquad A_s'=\alpha\,\omega\,b\,d\,\frac{f_{cu}}{f_y}$$

In case P_u=0, $A_s'=\alpha\times A_s$

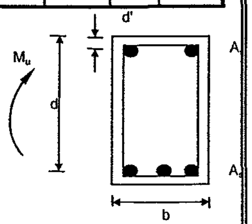


Table C: Design of doubly reinforced section for different d'/d ratios

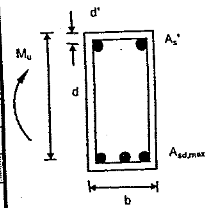
f_y N/mm^2	ω_{dmax}	α	$R1_{dmax}=\frac{M_{u\,dmax}}{f_{cu}\,b\,d^2}$					
			d'/d=0.05	d'/d=0.075	d'/d=0.1	d'/d=0.15	d'/d=0.175	d'/d=0.2
240	0.205	0.00	0.143	0.143	0.143	0.143	0.143	0.143
	0.216	0.05	0.152	0.152	0.152	0.151	0.151	0.151
	0.228	0.10	0.162	0.162	0.161	0.160	0.160	0.159
	0.242	0.15	0.173	0.172	0.172	0.170	0.169	0.169
	0.257	0.20	0.186	0.185	0.183	0.181	0.180	0.179
	0.274	0.25	0.200	0.198	0.197	0.194	0.192	0.191
	0.293	0.30	0.216	0.214	0.212	0.208	0.206	0.205
	0.316	0.35	0.235	0.232	0.230	0.225	0.223	0.220
	0.342	0.40	0.256	0.253	0.250	0.245	0.242	0.239
	0.374	0.45	0.282	0.278	0.275	0.268	0.264	0.260
	0.411	0.50	0.313	0.309	0.304	0.295	0.291	0.286
280	0.196	0.00	0.138	0.138	0.138	0.138	0.138	0.138
	0.206	0.05	0.147	0.146	0.146	0.146	0.146	0.145
	0.218	0.10	0.156	0.156	0.155	0.154	0.154	0.153
	0.231	0.15	0.167	0.166	0.165	0.164	0.163	0.162
	0.245	0.20	0.179	0.178	0.176	0.174	0.173	0.172
	0.261	0.25	0.192	0.191	0.189	0.186	0.185	0.184
	0.280	0.30	0.207	0.206	0.204	0.200	0.198	0.197
	0.302	0.35	0.225	0.223	0.221	0.216	0.214	0.212
	0.327	0.40	0.246	0.243	0.240	0.235	0.232	0.229
	0.356	0.45	0.271	0.267	0.264	0.257	0.253	0.250
	0.392	0.50	0.300	0.296	0.291	0.283	0.279	0.274
360	0.180	0.00	0.129	0.129	0.129	0.129	0.129	0.129
	0.189	0.05	0.137	0.137	0.137	0.136	0.136	0.136
	0.200	0.10	0.146	0.145	0.145	0.144	0.143	0.143
	0.212	0.15	0.155	0.155	0.154	0.153	0.152	0.151
	0.225	0.20	0.166	0.165	0.164	0.162	0.161	0.160
	0.240	0.25	0.179	0.177	0.176	0.173	0.172	0.171
	0.257	0.30	0.193	0.191	0.189	0.186	0.184	0.183
	0.277	0.35	0.209	0.207	0.205	0.201	0.199	0.197
	0.300	0.40	0.228	0.226	0.223	0.218	0.215	0.213
	0.327	0.45	0.251	0.248	0.244	0.238	0.235	0.232
	0.360	0.50	0.278	0.274	0.270	0.262	0.258	0.254
400	0.172	0.00	0.125	0.125	0.125	0.125	0.125	compression steel does not yield
	0.181	0.05	0.132	0.132	0.132	0.131	0.131	
	0.192	0.10	0.140	0.140	0.140	0.139	0.138	
	0.203	0.15	0.150	0.149	0.148	0.147	0.146	
	0.216	0.20	0.160	0.159	0.158	0.156	0.156	
	0.230	0.25	0.172	0.171	0.170	0.167	0.166	
	0.246	0.30	0.186	0.184	0.182	0.179	0.178	
	0.265	0.35	0.201	0.199	0.197	0.193	0.191	
	0.287	0.40	0.220	0.217	0.215	0.210	0.207	
	0.313	0.45	0.241	0.238	0.235	0.229	0.226	
	0.345	0.50	0.267	0.263	0.260	0.252	0.248	

1-Enter the table with: (f_y,d'/d,$R1_{dmax}$)
2- from the table get ω,α
3-sub in the equations to get $A_{s,dmax}$,A's

$$R1_{dmax}=\frac{M_{u\,dmax}}{f_{cu}\,b\,d^2}$$

$$A_{sd,max}=\omega_{dmax}\,\frac{f_{cu}}{f_y}\,b\,d$$

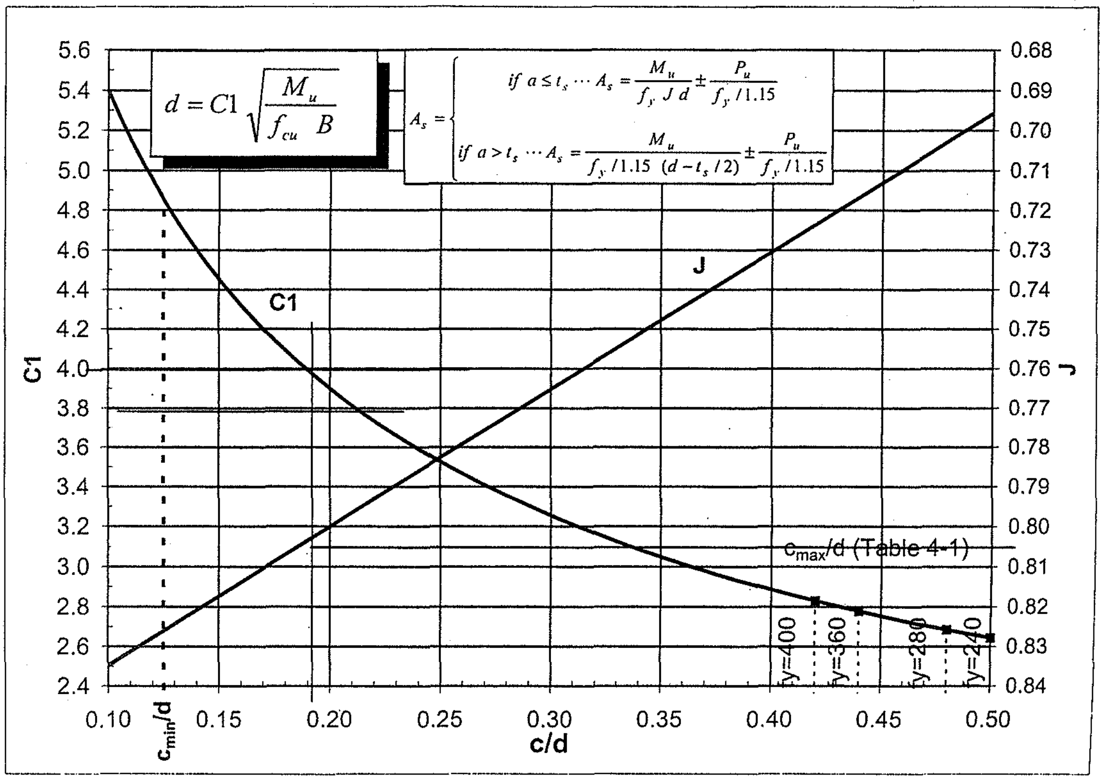
$$A_s'=\alpha\,A_{sd,max}$$



C1	J	c/d	c_{max}/d
2.65	0.696	0.500	fy=240
2.69	0.703	0.480	fy=280
2.78	0.717	0.440	fy=360
2.83	0.723	0.420	fy=400
2.90	0.732	0.395	
2.95	0.738	0.379	
3.00	0.743	0.364	
3.05	0.748	0.350	
3.10	0.753	0.337	
3.15	0.757	0.324	
3.20	0.761	0.312	
3.25	0.765	0.301	
3.30	0.768	0.291	
3.35	0.772	0.281	
3.40	0.775	0.272	
3.45	0.778	0.263	
3.50	0.781	0.254	
3.55	0.784	0.246	
3.60	0.787	0.239	
3.65	0.789	0.231	
3.70	0.791	0.225	
3.75	0.794	0.218	
3.80	0.796	0.212	
3.85	0.798	0.206	
3.90	0.800	0.200	
3.95	0.802	0.194	
4.00	0.804	0.189	
4.05	0.806	0.184	
4.10	0.807	0.179	
4.15	0.809	0.175	
4.20	0.810	0.170	
4.25	0.812	0.166	
4.30	0.813	0.162	
4.35	0.815	0.158	
4.40	0.816	0.154	
4.45	0.817	0.150	
4.50	0.818	0.147	
4.55	0.820	0.143	
4.60	0.821	0.140	
4.65	0.822	0.137	
4.70	0.823	0.134	
4.75	0.824	0.131	
4.80	0.825	0.128	
4.85	0.826	0.125	

DESIGN CHART FOR SECTIONS SUBJECTED TO SIMPLE BENDING (R and T-sections)

FOR ALL GRADES OF STEEL AND CONCRETE

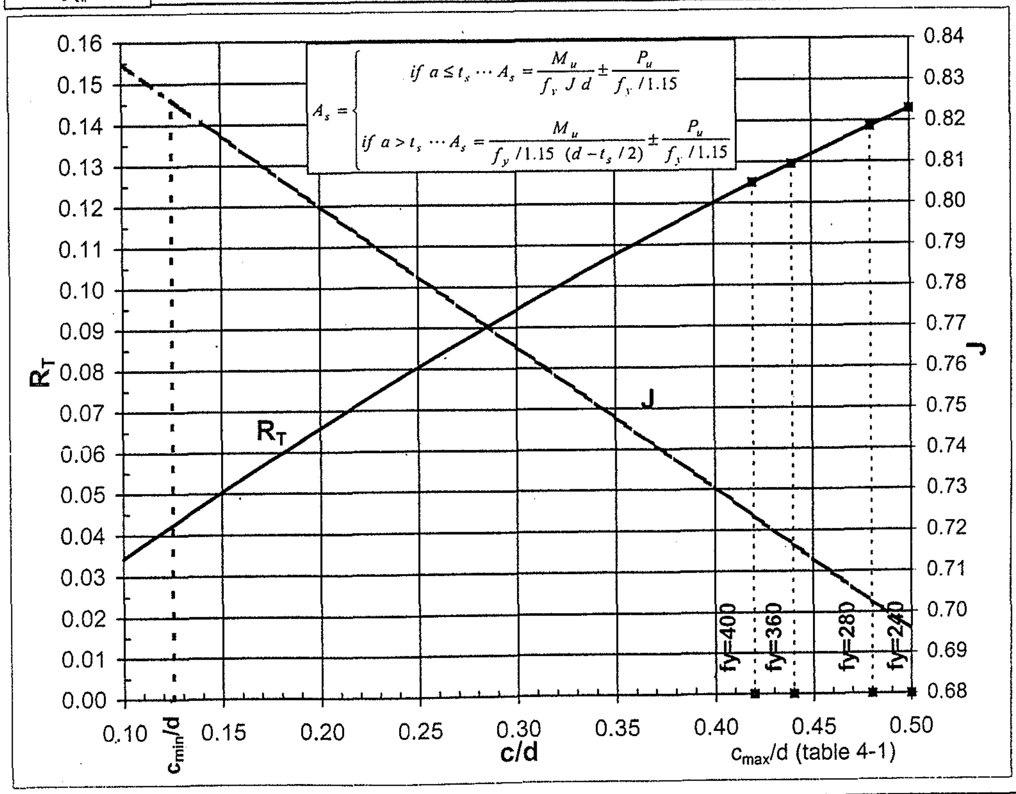


R_T	J	c/d	
0.042	0.827	0.124	
0.044	0.824	0.130	
0.046	0.822	0.136	
0.048	0.820	0.142	
0.050	0.818	0.149	
0.052	0.816	0.155	
0.054	0.813	0.162	
0.056	0.811	0.168	
0.058	0.809	0.174	
0.060	0.807	0.181	
0.062	0.804	0.188	
0.064	0.802	0.194	
0.066	0.800	0.201	
0.068	0.797	0.208	
0.070	0.795	0.214	
0.072	0.793	0.221	
0.074	0.790	0.228	
0.076	0.788	0.235	
0.078	0.786	0.242	
0.080	0.783	0.249	
0.082	0.781	0.256	
0.084	0.778	0.263	
0.086	0.776	0.270	
0.088	0.773	0.277	
0.090	0.771	0.284	
0.092	0.768	0.291	
0.094	0.766	0.299	
0.096	0.763	0.306	
0.098	0.760	0.314	
0.100	0.758	0.321	
0.102	0.755	0.329	
0.104	0.753	0.336	
0.106	0.750	0.344	
0.108	0.747	0.352	
0.110	0.745	0.360	
0.112	0.742	0.367	
0.114	0.739	0.375	
0.116	0.736	0.383	
0.118	0.733	0.392	
0.120	0.731	0.400	cmax/d
0.125	0.723	0.420	fy=400
0.130	0.717	0.440	fy=360
0.139	0.703	0.480	fy=280
0.143	0.696	0.500	fy=240

DESIGN CHART FOR SECTIONS SUBJECTED TO SIMPLE BENDING (T-sections)

$$R_T = \frac{M_u}{f_{cu}\ B\ d^2}$$

FOR ALL GRADES OF STEEL AND CONCRETE



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Units Conversion Table

To transform from | **To** | **Multiply by**

SI-units	French -units	factor
Concentrated loads		
1N	kg	0.1
1 kN	kg	100
1 kN	ton	0.1
Linear Loads /m'		
1 kN/m'	t/m'	0.1
Uniform Loads /m²		
kN/m^2	t/m^2	0.1
N/m^2	kg/m^2	0.1
kN/m^2	kg/m^2	100
Stress		
N/mm^2 (=1 MPa)	kg/cm^2	10
kN/m^2	kg/cm^2	0.01
kN/m^2	ton/m^2	0.1
Density		
N/m^3	kg/m^3	0.1
kN/m^3	ton/m^3	0.1
kN/m^3	kg/m^3	100
Moment		
kN.m	ton.m	0.1
N.mm	kg.cm	0.01
Area		
m^2	cm^2	10000
mm^2	cm^2	0.01

تـــم بحـــمـــد الله

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Professor Dr. Mashhour Ghoneim: is a Professor of Concrete Structures at Cairo University. He obtained his Ph.D. in Structural Engineering from the University of Alberta, Canada. He participated in the development of the shear and torsion design provisions of the American Concrete Institute Code (ACI 318). He supervised several researches for the degrees of M.Sc. and Ph.D. related to the behavior and stability of reinforced concrete members, particularly under seismic actions. He is the author of many technical papers in reinforced concrete.

Professor Ghoneim is a member of several professional committees including the Standing Committee for the Egyptian Code for Design and Construction of Concrete Structures, the Standing Committee for the Egyptian Code for the Use of FRP in Construction Fields, the Standing Committee for the Egyptian Code for Design and Planning of Bridges and Intersections and the Committee for the Egyptian Code for Masonry Works.

He participated in the design of many projects in Egypt and abroad. Some of the most notable of these projects are: the Library of Alexandria (the Bibliotheca Alexandrina), the bridge over Suez Canal, City Stars Complex in Cairo, San Stefano Grand Plaza in Alexandria and the Quay Walls of North Al-Sukhna port and Port Said East Port.

Dr. Mahmoud T. El-Mihilmy: is an Associate Professor in the Department of Structural Engineering at Cairo University. He holds two bachelor degrees, one in Architectural Engineering and another in Structural Engineering from Cairo University. He also has two Masters degrees in Architectural and Structural Engineering. Finally, he obtained his Ph.D. from Auburn University, USA, in Structural Engineering.

Dr. El-Mihilmy teaches undergraduate and graduate courses in the design of reinforced and prestressed concrete structures. He published numerous technical papers in respected journals worldwide. He is a member in the ACI, ASCE as well as the Egyptian Society of Engineers. He has served on many professional committees including the Egyptian Code for Design and Construction of Concrete Structures for the Development of Design Aids as well as the Egyptian Code for the Use of FRP in Construction Fields.

He was engaged in the design of a number of large-scale projects both on the international and national levels such as MGM-Hotel in Las Vegas, USA, and Conrad Hotel in Cairo. Finally, He was awarded the Ahmed Moharem Prize 2005 in Structural Engineering.

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